A random walk, a process in which a walker moves randomly from one node to another in a graph, can be used to analyze the underlying data structure of the graph. Whereas the conventional random walk theory describes the movements of a single walker (or agent), we propose a system of multiple random walkers (MRW) to simulate multiple agents on a graph simultaneously. Those agents traverse the graph according to a transition matrix, but they also interact with one another to achieve a desired goal. Our MRW system can support a variety of interactions by employing different restart rules. In particular, we develop the repulsive restart rule for data clustering. With this restart rule, as the random process continues, multiple agents repel one another and form their own dominant regions. Eventually, the power balance among the agents is achieved, and their distributions converge. By comparing the stationary distributions, clustering can be achieved.

Figure 1 illustrates an MRW process of double random walkers with the repulsive restart rule. In Figure 1(a), the top two rows show initial probability distributions of the red and blue walkers, which are randomly generated. The bottom row is the result of the clustering. The clustering result at $t = 0$ is meaningless. However, as the iteration goes on, each walker repels the other walker, while forming a dominant cluster region. Consequently, the probability distributions of the two walkers converge, respectively, and the power balance between the walkers is achieved. By comparing those probabilities at each node, we obtain the clustering result in Figure 1(f), which coincides with the intuitive clustering of the human visual system.

Figure 2 illustrates the repulsive MRW process in an image, which is shown in the lower right corner. In Figure 2(a), the top two rows show the initial probability distributions of the foreground and background walkers, based on the center prior, respectively. The bottom row is the segmentation result. At early stages, the foreground region is identified around the image center due to the initial distributions. However, as the iteration continues, the foreground walker explores nearby similar nodes, while competing with the background walker. The repulsive restart rule facilitates discriminative clustering. Finally, in Figure 2(e), the probability distributions converge, and we obtain a satisfactory segmentation result that extracts the bear faithfully.

We apply the proposed MRW system to the problem of segmenting similar images jointly. Recently, attempts to extract common foreground objects from a set of similar images have been made. This approach, called cosegmentation, was first addressed by Rother et al.\cite{3} and has been researched actively. For cosegmentation, we introduce the notion of concurrence distribution, which represents the similarity of each node in an image to foreground objects in the other images. Then, the MRW clustering is performed by incorporating the concurrence distribution into the repulsive restart rule. As exemplified in Figure 3, multiple executions of the inter-image concurrence computation and the intra-image MRW clustering discover foreground objects in images more clearly. Experimental results show that the proposed MRW algorithm improves the segmentation accuracy significantly, as compared with recent state-of-the-art cosegmentation techniques\cite{2, 4, 5, 6}, on the iCoseg dataset\cite{1}.

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