3.2.

$$x[n] = \begin{cases} n, & 0 \le n \le N-1 \\ N, & N \le n \end{cases} = n\,u[n] - (n-N)u[n-N]$$

$$n\,x[n] \quad \Leftrightarrow \quad -z\frac{d}{dz}X(z) \Rightarrow n\,u[n] \Leftrightarrow -z\frac{d}{dz}\frac{1}{1-z^{-1}} \quad |z|>1$$

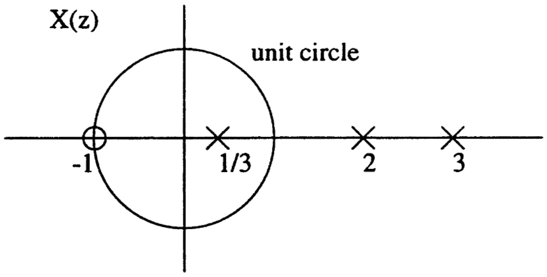
$$n\,u[n] \quad \Leftrightarrow \quad \frac{z^{-1}}{(1-z^{-1})^2} \quad |z|>1$$

$$x[n-n_0] \quad \Leftrightarrow \quad X(z)\cdot z^{-n_0} \Rightarrow (n-N)u[n-N] \Leftrightarrow \frac{z^{-N-1}}{(1-z^{-1})^2} \quad |z|>1$$

therefore

$$X(z) = \frac{z^{-1} - z^{-N-1}}{(1-z^{-1})^2} = \frac{z^{-1}(1-z^{-N})}{(1-z^{-1})^2}$$

3.4. The pole-zero plot of $X(z)$ appears below.



(a) For the Fourier transform of $x[n]$ to exist, the z-transform of $x[n]$ must have an ROC which includes the unit circle, therefore, $|\frac{1}{3}| < |z| < |2|$.
Since this ROC lies outside $\frac{1}{3}$, this pole contributes a right-sided sequence. Since the ROC lies inside 2 and 3, these poles contribute left-sided sequences. The overall $x[n]$ is therefore two-sided.

(b) Two-sided sequences have ROC's which look like washers. There are two possibilities. The ROC's corresponding to these are: $|\frac{1}{3}| < |z| < |2|$ and $|2| < |z| < |3|$.

(c) The ROC must be a connected region. For stability, the ROC must contain the unit circle. For causality the ROC must be outside the outermost pole. These conditions cannot be met by any of the possible ROC's of this pole-zero plot.

3.7. (a)

$$x[n] = u[-n-1] + \left(\frac{1}{2}\right)^n u[n]$$

$$\Rightarrow \quad X(z) = \frac{-1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} \qquad \frac{1}{2} < |z| < 1$$

Now to find $H(z)$ we simply use $H(z) = Y(z)/X(z)$; i.e.,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})} \cdot \frac{(1-z^{-1})(1-\frac{1}{2}z^{-1})}{-\frac{1}{2}z^{-1}} = \frac{1-z^{-1}}{1+z^{-1}}$$

$H(z)$ causal $\Rightarrow$ ROC $|z| > 1$.

(b) Since one of the poles of $X(z)$, which limited the ROC of $X(z)$ to be less than 1, is cancelled by the zero of $H(z)$, the ROC of $Y(z)$ is the region in the z-plane that satisfies the remaining two constraints $|z| > \frac{1}{2}$ and $|z| > 1$. Hence $Y(z)$ converges on $|z| > 1$.

(c)

$$Y(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{3}}{1+z^{-1}} \qquad |z| > 1$$

Therefore,

$$y[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{3}(-1)^n u[n]$$

3.13.

$$\begin{aligned} G(z) &= \sin(z^{-1})(1+3z^{-2}+2z^{-4}) \\ &= (z^{-1} - \frac{z^{-3}}{3!} + \frac{z^{-5}}{5!} - \frac{z^{-7}}{7!})(1+3z^{-2}+2z^{-4}) \\ &= \sum_n g[n]z^{-n} \end{aligned}$$

$g[11]$ is simply the coefficient in front of z^{-11} in this power series expansion of $G(z)$:

$$g[11] = -\frac{1}{11!} + \frac{3}{9!} - \frac{2}{11!}$$

3.24. (a)

$$\begin{aligned}
y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\
&= \sum_{k=-\infty}^{\infty} \left(3\left(-\frac{1}{3}\right)^k u[k]\right) u[n-k] \\
&= \sum_{k=0}^{n} 3\left(-\frac{1}{3}\right)^k \\
&= \begin{cases} \frac{9}{4}\left(1-\left(-\frac{1}{3}\right)^{n+1}\right), & n \geq 0 \\ 0, & \text{otherwise} \end{cases}
\end{aligned}$$

(b)

$$\begin{aligned}
Y(z) &= H(z)X(z) \\
&= \frac{3}{1+\frac{1}{3}z^{-1}} \frac{1}{1-z^{-1}} \\
&= \frac{\frac{3}{4}}{1+\frac{1}{3}z^{-1}} + \frac{\frac{9}{4}}{1-z^{-1}} \\
y[n] &= \frac{3}{4}\left(-\frac{1}{3}\right)^n u[n] + \frac{9}{4}u[n] \\
&= \frac{9}{4}\left(1+\frac{1}{3}\left(-\frac{1}{3}\right)^n\right) u[n] \\
&= \frac{9}{4}\left(1-\left(-\frac{1}{3}\right)^{n+1}\right) u[n]
\end{aligned}$$

3.27.

$$X(z) = \frac{z^2}{(z-a)(z-b)} = \frac{z^2}{z^2-(a+b)z+ab}$$

Obtain a proper fraction:

$$\begin{array}{r|l} & 1 \\ \hline z^2-(a+b)z+ab & z^2 \\ & \underline{z^2 \quad -(a+b)z \quad +ab} \\ & \quad (a+b)z \quad -ab \end{array}$$

$$\begin{aligned} X(z) &= 1+\frac{(a+b)z-ab}{(z-a)(z-b)} = 1+\frac{\frac{(a+b)a-ab}{a-b}}{z-a}+\frac{\frac{(a+b)b-ab}{b-a}}{z-b} \\ &= 1+\frac{\frac{a^2}{a-b}}{z-a}-\frac{\frac{b^2}{a-b}}{z-b} = 1+\frac{1}{a-b}\left(\frac{a^2z^{-1}}{1-az^{-1}}-\frac{b^2z^{-1}}{1-bz^{-1}}\right) \\ x[n] &= \delta[n]+\frac{a^2}{a-b}a^{n-1}u[n-1]-\frac{b^2}{a-b}b^{n-1}u[n-1] \\ &= \delta[n]+\left(\frac{1}{a-b}\right)(a^{n+1}-b^{n+1})u[n-1] \end{aligned}$$

3.34. (a)

$$nx[n] \Leftrightarrow -z\frac{d}{dx}X(z)$$

$$x[n-n_0] \Leftrightarrow z^{-n_0}X(z)$$

$$X(z) = \frac{3z^{-3}}{(1-\frac{1}{4}z^{-1})^2} = 12z^{-2}\left[-z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\right]$$

$x[n]$ is left-sided. Therefore, $X(z)$ corresponds to:

$$x[n] = -12(n-2)\left(\frac{1}{4}\right)^{n-2}u[-n+1]$$

(b)

$$X(z) = \sin(z) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}z^{2k+1} \qquad \text{ROC includes } |z| = 1$$

Therefore,

$$x[n] = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}\delta[n+2k+1]$$

Which is stable.

(c)

$$X(z) = \frac{z^7-2}{1-z^{-7}} = z^7 - \frac{1}{1-z^{-7}} \qquad |z| > 1$$

$$X(z) = z^7 - \sum_{n=0}^{\infty}z^{-7n}$$

Therefore,

$$x[n] = \delta[n+7] - \sum_{n=0}^{\infty}\delta[n-7k]$$

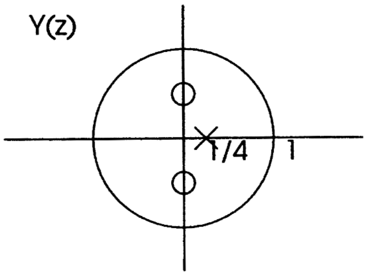
3.39. From pole-zero diagram

$$X(z) = \frac{z^2+1}{z-\frac{1}{2}}$$

(a)

$$y[n] = \left(\frac{1}{2}\right)^n x[n] \Rightarrow Y(z) = X(2z) = \frac{4z^2+1}{2z-\frac{1}{2}}$$

zeros $\pm\frac{1}{2}j$
poles $\frac{1}{4}, \infty$



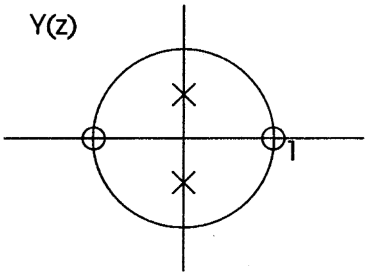
(b)

$$w[n] = \cos\left(\frac{\pi n}{2}\right) x[n] = \frac{1}{2}(e^{j\pi n/2} + e^{-j\pi n/2})x[n]$$

$$W(z) = \frac{1}{2}X(e^{-j\pi/2}z) + \frac{1}{2}X(e^{j\pi/2}z) = \frac{1}{2}X(-jz) + \frac{1}{2}X(jz)$$

$$W(z) = \frac{1}{2}\left(\frac{-z^2+1}{-jz-\frac{1}{2}}\right) + \frac{1}{2}\left(\frac{-z^2+1}{jz-\frac{1}{2}}\right) = \frac{z^2-1}{2(z^2+\frac{1}{4})}$$

poles at $\pm\frac{1}{2}j$
zeros at ± 1



3.43.

$$X(z) = \frac{\frac{1}{3}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{1}{4}}{1 - 2z^{-1}}$$

has poles at $z = \frac{1}{2}$ and $z = 2$.

Since the unit circle is in the region of convergence $X(z)$ and $x[n]$ have both a causal and an anticausal part. The causal part is "outside" the pole at $\frac{1}{2}$. The anticausal part is "inside" the pole at 2, therefore, $x[0]$ is the sum of the two parts

$$x[0] = \lim_{z \to \infty} \frac{\frac{1}{3}}{1 - \frac{1}{2}z^{-1}} + \lim_{z \to 0} \frac{\frac{1}{4}z}{z - 2} = \frac{1}{3} + 0 = \frac{1}{3}$$

3.45. (a) Yes, $h[n]$ is BIBO stable if its ROC includes the unit circle. Hence, the system is stable if $r_{min} < 1$ and $r_{max} > 1$.

(b) Let's consider the system step by step.

(i) First, $v[n] = \alpha^{-n}x[n]$. By taking the z-transform of both sides, $V(z) = X(\alpha z)$.

(ii) Second, $v[n]$ is filtered to get $w[n]$. So $W(z) = H(z)V(z) = H(z)X(\alpha z)$.

(iii) Finally, $y[n] = \alpha^n w[n]$. In the z-transform domain, $Y(z) = W(z/\alpha) = H(z/\alpha)X(z)$.

In conclusion, the system is LTI, with system function $G(z) = H(z/\alpha)$ and $g[n] = \alpha^n h[n]$.

(c) The ROC of $G(z)$ is $\alpha r_{min} < |z| < \alpha r_{max}$. We want $r_{min} < 1/\alpha$ and $r_{max} > 1/\alpha$ for the system to be stable.