4.1.

$$
\begin{aligned}
x[n] &= x_c(nT) \\
&= \sin\left(2\pi(100)n\frac{1}{400}\right) \\
&= \sin\left(\frac{\pi}{2}n\right)
\end{aligned}
$$

4.4. (a) Letting $T = 1/100$ gives

$$\begin{aligned} x[n] &= x_c(nT) \\ &= \sin\left(20\pi n \frac{1}{100}\right) + \cos\left(40\pi n \frac{1}{100}\right) \\ &= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \end{aligned}$$

(b) No, another choice is $T = 11/100$:

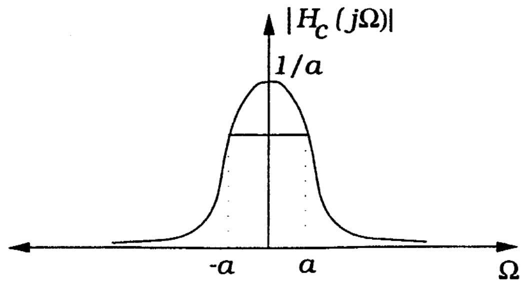
$$\begin{aligned} x[n] &= x_c(nT) \\ &= \sin\left(20\pi n \frac{11}{100}\right) + \cos\left(40\pi n \frac{11}{100}\right) \\ &= \sin\left(\frac{11\pi n}{5}\right) + \cos\left(\frac{22\pi n}{5}\right) \\ &= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \end{aligned}$$

4.6. (a) The Fourier transform of the filter impulse response

$$
\begin{aligned}
H_c(j\Omega) &= \int_{-\infty}^{\infty} h_c(t)e^{-j\Omega t}\,dt \\
&= \int_{0}^{\infty} a^{-at}e^{-j\Omega t}\,dt \\
&= \frac{1}{a+j\Omega}
\end{aligned}
$$

So, we take the magnitude

$$|H_c(j\Omega)| = \left(\frac{1}{a^2+\Omega^2}\right)^{\frac{1}{2}}$$



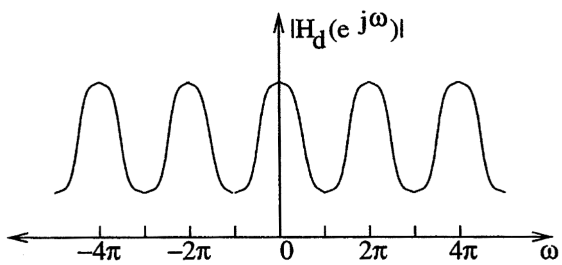
(b) Sampling the filter impulse response in (a), the discrete-time filter is described by

$$
\begin{aligned}
h_d[n] &= Te^{-anT}u[n] \\
H_d(e^{j\omega}) &= \sum_{n=0}^{\infty} Te^{-anT}e^{-j\omega n} \\
&= \frac{T}{1-e^{-aT}e^{-j\omega}}
\end{aligned}
$$

Taking the magnitude of this response

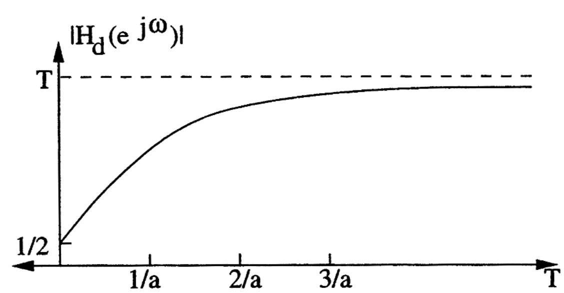
$$|H_d(e^{j\omega})| = \frac{T}{(1-2e^{-aT}\cos(\omega)+e^{-2aT})^{\frac{1}{2}}}.$$

Note that the frequency response of the discrete-time filter is periodic, with period 2π.



(c) The minimum occurs at $\omega = \pi$. The corresponding value of the frequency response magnitude is

$$
\begin{aligned}
|H_d(e^{j\pi})| &= \frac{T}{(1+2e^{-aT}+e^{-2aT})^{\frac{1}{2}}} \\
&= \frac{T}{1+e^{-aT}}.
\end{aligned}
$$



4.9. (a) Since $X(e^{j\omega}) = X(e^{j(\omega-\pi)})$, $X(e^{j\omega})$ is periodic with period π.

(b) Using the inverse DTFT,

$$\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j(\omega-\pi)})e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j(\omega+\pi)n}d\omega \\ &= \frac{1}{2\pi}e^{j\pi n}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega \\ &= (-1)^n x[n]. \end{aligned}$$

All odd samples of $x[n] = 0$, because $x[n] = -x[n]$. Hence $x[3] = 0$.

(c) Yes, $y[n]$ contains all even samples of $x[n]$, and all odd samples of $x[n]$ are 0.

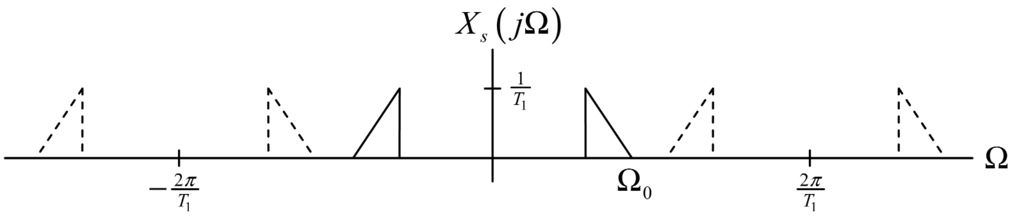
$$x[n] = \begin{cases} y[n/2], & n \text{ even} \\ 0, & \text{otherwise} \end{cases}$$

4.21

A. The impulse-train signal $x_s(t)$ has spectrum $X_s(j\Omega)$ given by

$$X_s(j\Omega) = \tfrac{1}{T} \sum_{k=-\infty}^{\infty} X\left[j\left(\Omega - k\tfrac{2\pi}{T_1}\right)\right].$$

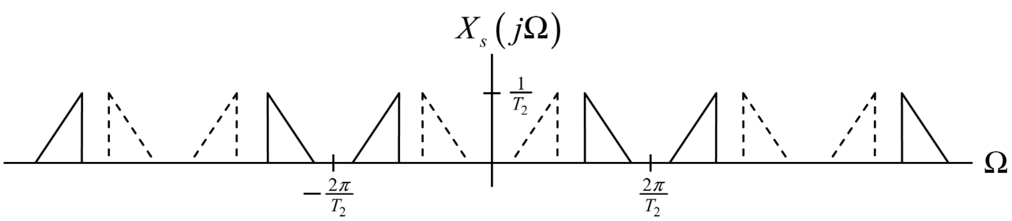
An example is shown below.



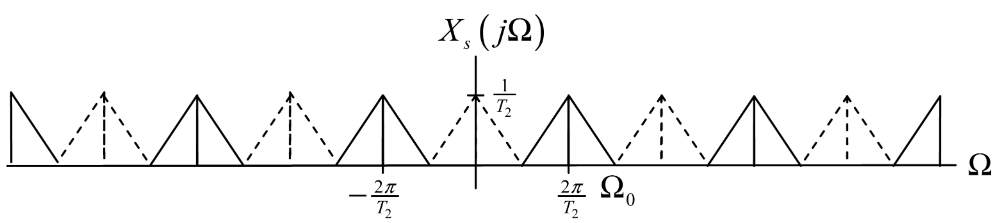
We will have $x_r(t) = x_c(t)$ provided $T_1 \le \frac{\pi}{\Omega_0}$.

B. We will have $x_o(t) = x_c(t)$ under any of the following circumstances:

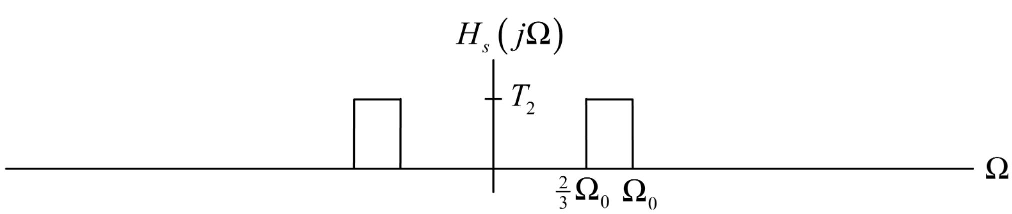
1. As illustrated above, $T_2 \le \frac{\pi}{\Omega_0}$.
2. As illustrated below, $\frac{1.5\pi}{\Omega_0} \le T_2 \le \frac{2\pi}{\Omega_0}$.



3. As illustrated below, $T_2 = \frac{3\pi}{\Omega_0}$.

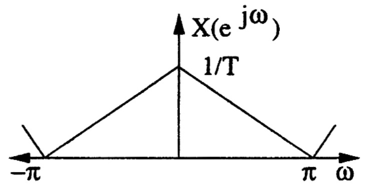


The frequency response of the filter that is needed to recover $x_c(t)$ is shown below.

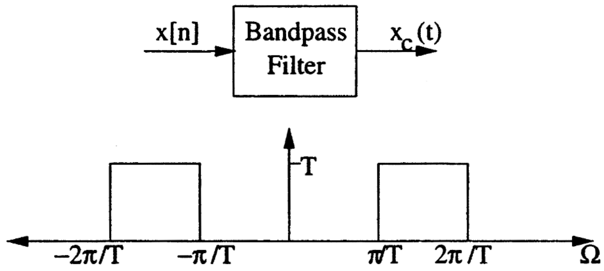


4.23. (a)

$$\omega = \Omega T, \quad T = \frac{2\pi}{\Omega_0}$$



(b) To recover simply filter out the undesired parts of $X(e^{j\omega})$.



(c)

$$T \leq \frac{2\pi}{\Omega_0}$$