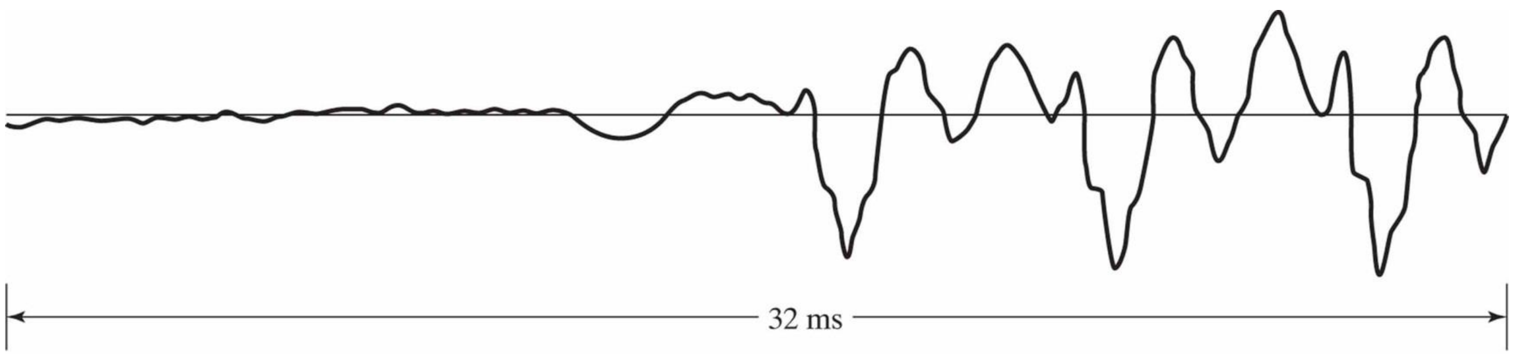
Digital Signal Processing

Chap 2.
Discrete-Time Signals and Systems

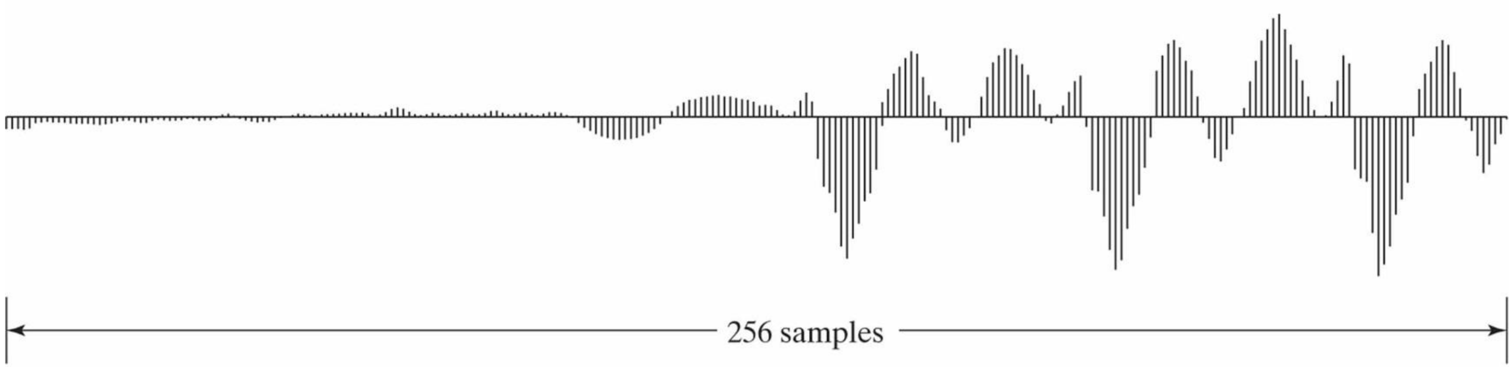
Chang-Su Kim

Discrete-Time Signals



(a)

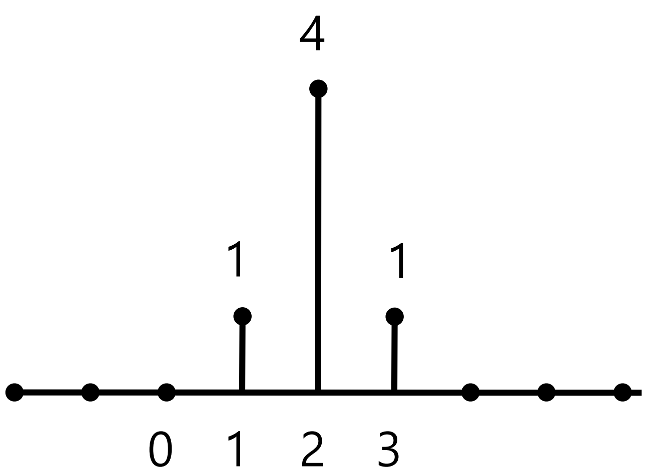
CT Signal



(b)

DT Signal

Representation



- Functional representation

$$x[n] = \begin{cases} 1, & n = 1,3 \\ 4, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

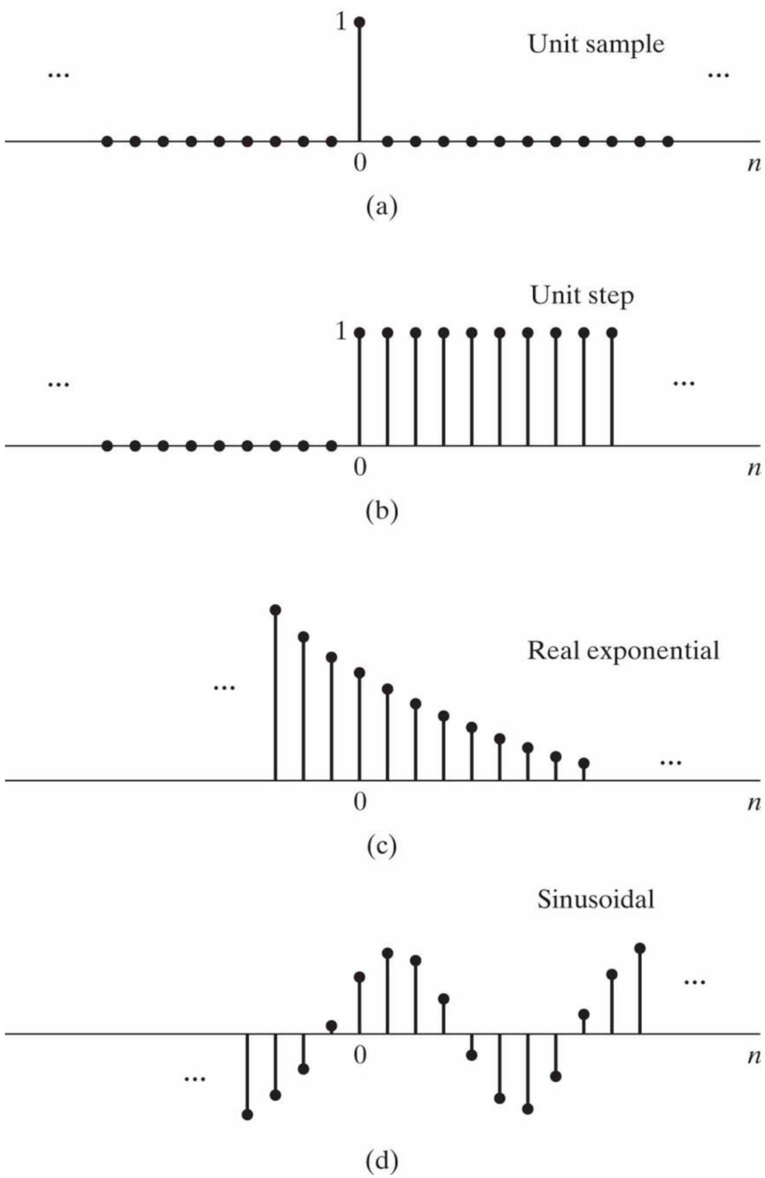
- Tabular representation

n	…	0	1	2	3	4	5	…
x[n]	…	0	1	4	1	0	0	0

- Sequence representation

$$x[n] = \{\dots, 0, \underset{\uparrow}{0}, 1, 4, 1, 0, 0, \dots\}$$

Elementary Sequences



- Unit sample sequence (impulse function, delta function)

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

- Unit step sequence

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

- Exponential sequence

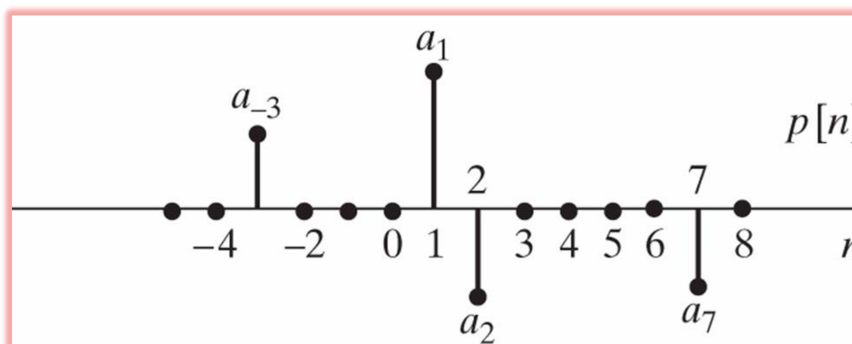
$$x[n] = A\alpha^n$$

- Sinusoidal sequence

$$x[n] = A\cos(\omega_0 n + \phi)$$

Properties of Impulse and Step Functions

1)	$\delta[n] = u[n] - u[n-1]$
2)	$u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$
3)	$x[n]\delta[n] = x[0]\delta[n]$
4)	$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$
5)	$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$



Properties of Exponential and Sinusoidal Sequences

- Exponential $x[n] = a^n$

$$= r^n e^{j\omega_0 n}$$

$$= r^n(\cos\omega_0 n + j\sin\omega_0 n)$$

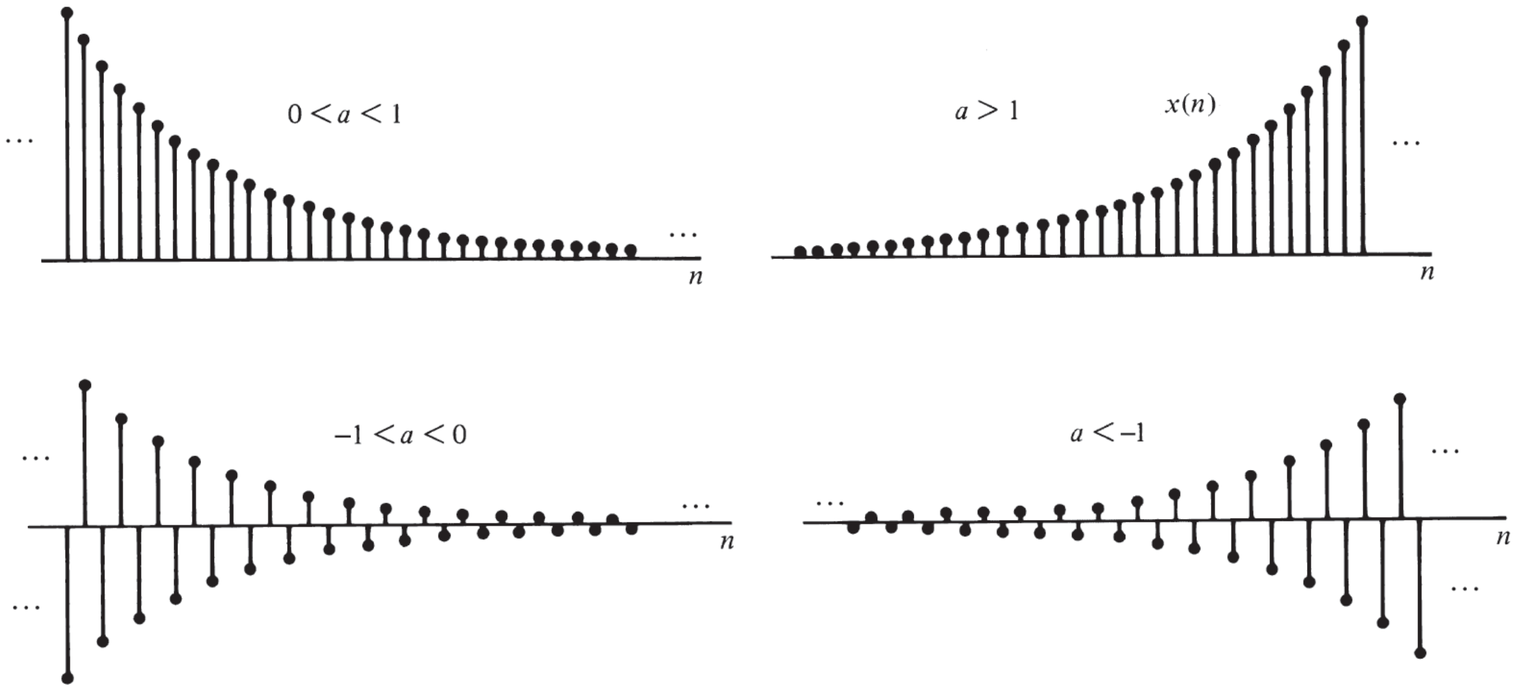
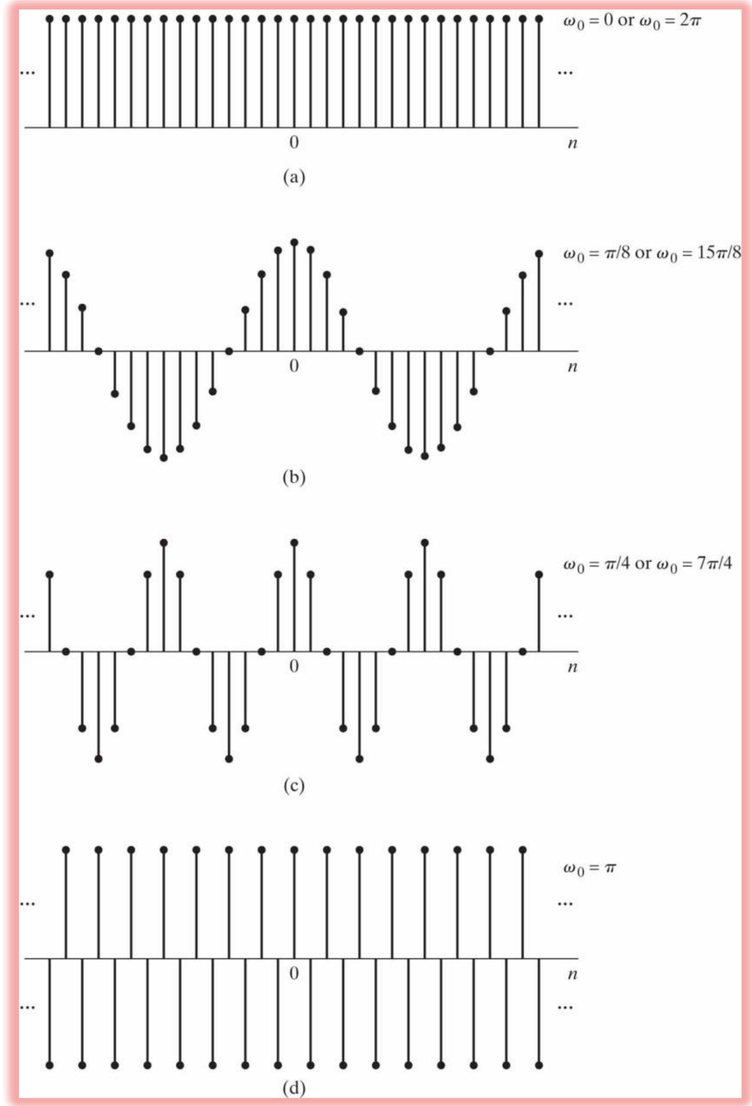


Figure 2.1.5 Graphical representation of exponential signals.

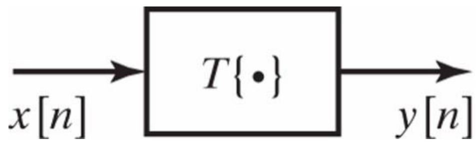
Properties of $e^{j\omega_0 n}$ and $\cos(\omega_0 n)$

- $\omega_0 + 2\pi = \omega_0$
- They are periodic only if $\omega_0 N = 2\pi k$

cf) Note the differences from the CT case



Discrete-Time Systems



$$y[n] = T\{x[n]\}$$

- Examples
  1. Ideal delay $\quad y[n] = x[n-2]$
  2. Moving average

$$y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1])$$

Memoryless Systems

- Memoryless Systems: The output $y[n]$ at any instance n depends only on the input value at the current time n, *i.e.* $y[n]$ is a function of $x[n]$

- Systems with Memory: The output $y[n]$ at an instance n depends on the input values at past and/or future time instances as well as the current time instance

- Examples:
  - A resistor: $y[n] = R\,x[n]$
  - A unit delay system: $y[n] = x[n-1]$
  - An accumulator:

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Proving or **Disproving** Mathematical Statement

- For a system to possess a given property, the property must hold for every possible input signal to the system

- **A counter example is sufficient to prove that a system does not possess a property**

- To prove that the system has the property, **we must prove that the property holds for every possible input signal**

Linear Systems

- A system is linear if it satisfies two properties.
  - Additivity: $x_1[n] + x_2[n] \Rightarrow y_1[n] + y_2[n]$
  - Homogeneity: $cx_1[n] \Rightarrow cy_1[n]$

- The two properties can be combined into a single property (linearity).

$$a_1 x_1[n] + a_2 x_2[n] \Rightarrow a_1\, y_1[n] + a_2 y_2[n]$$

- Examples
  - $y[n] = x^2[n]$
  - $y[n] = \log|x[n]|$
  - $y[n] = 2x[n] + 3$
  - $y[n] = \sum_{k=-\infty}^{n} x[k]$

Time-Invariant Systems

- A system is time-invariant if a delay (or a time-shift) in the input signal causes the same amount of delay in the output.

$$x[n-n_0] \Rightarrow y[n-n_0]$$

- Examples:
  - $y[n] = x^2[n]$
  - $y[n] = \sin|x[n]|$
  - $y[n] = x[2n]$
  - $y[n] = \sum_{k=-\infty}^{n} x[k]$

Causal Systems

- Causality: A system is causal if the output at any time instance depends only on the input values at the current and/or past time instances.

- Examples:
  - $y[n] = x[n] - x[n-1]$
  - $y[n] = x[n+1]$

- A memoryless system is always causal.

Stable Systems

- Stability: A system is stable if a <u>bounded input</u> yields a <u>bounded output</u> (BIBO).
  - In other words, if $|x[n]| < k_1$ then $|y[n]| < k_2$.

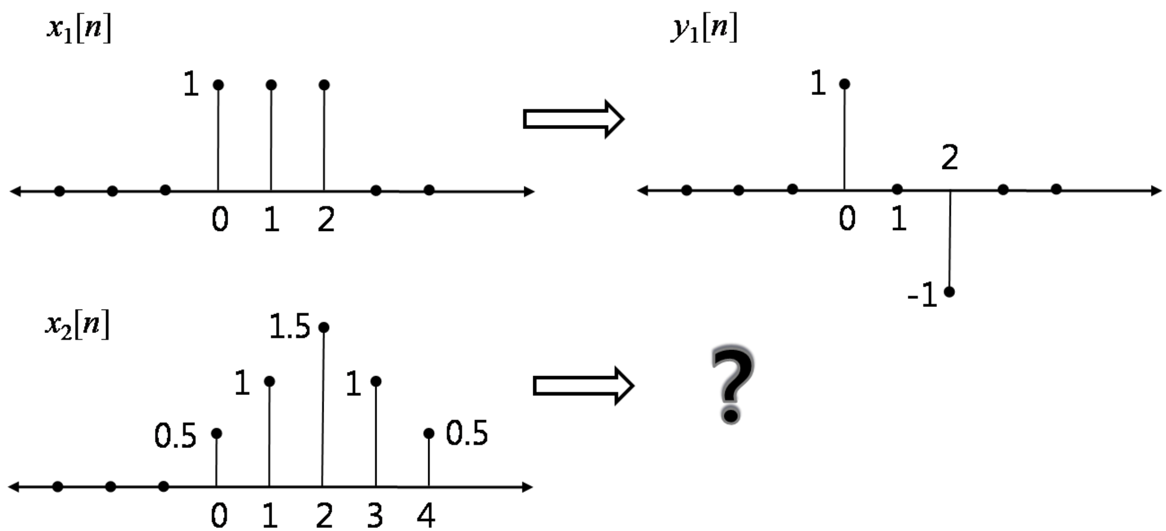
- Examples:
  - $y[n] = x^2[n]$
  - $y[n] = \sin|x[n]|$
  - $y[n] = x[2n]$
  - $y[n] = \sum_{k=-\infty}^{n} x[k]$

Linear Time-Invariant Systems and Their Properties

Divide and Conquer

- Divide an input signal into a sum of shifted scaled versions of an elementary signal
- If you know the system output in response to the elementary signal, you also know the output in response to the input signal

Ex) An LTI system processes $x_1[n]$ to make $y_1[n]$. The same system processes another input $x_2[n]$ to make $y_2[n]$. Plot $y_2[n]$.



Representing Signals in Terms of Impulses

- Sifting property

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$= \cdots$$

$$+x[-2]\delta[n+2]$$

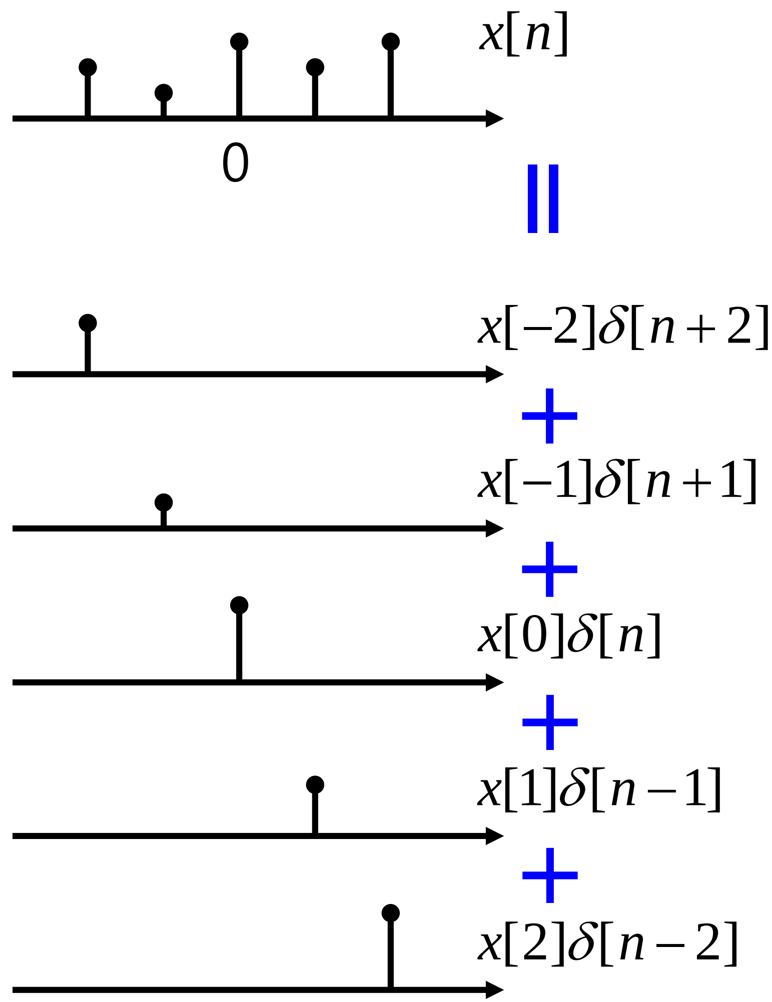
$$+x[-1]\delta[n+1]$$

$$+x[0]\delta[n]$$

$$+x[1]\delta[n-1]$$

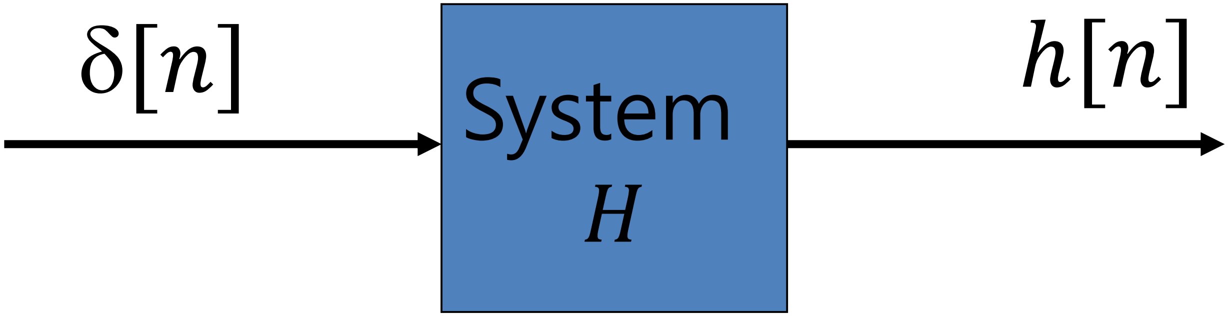
$$+x[2]\delta[n-2]$$

$$+\cdots$$



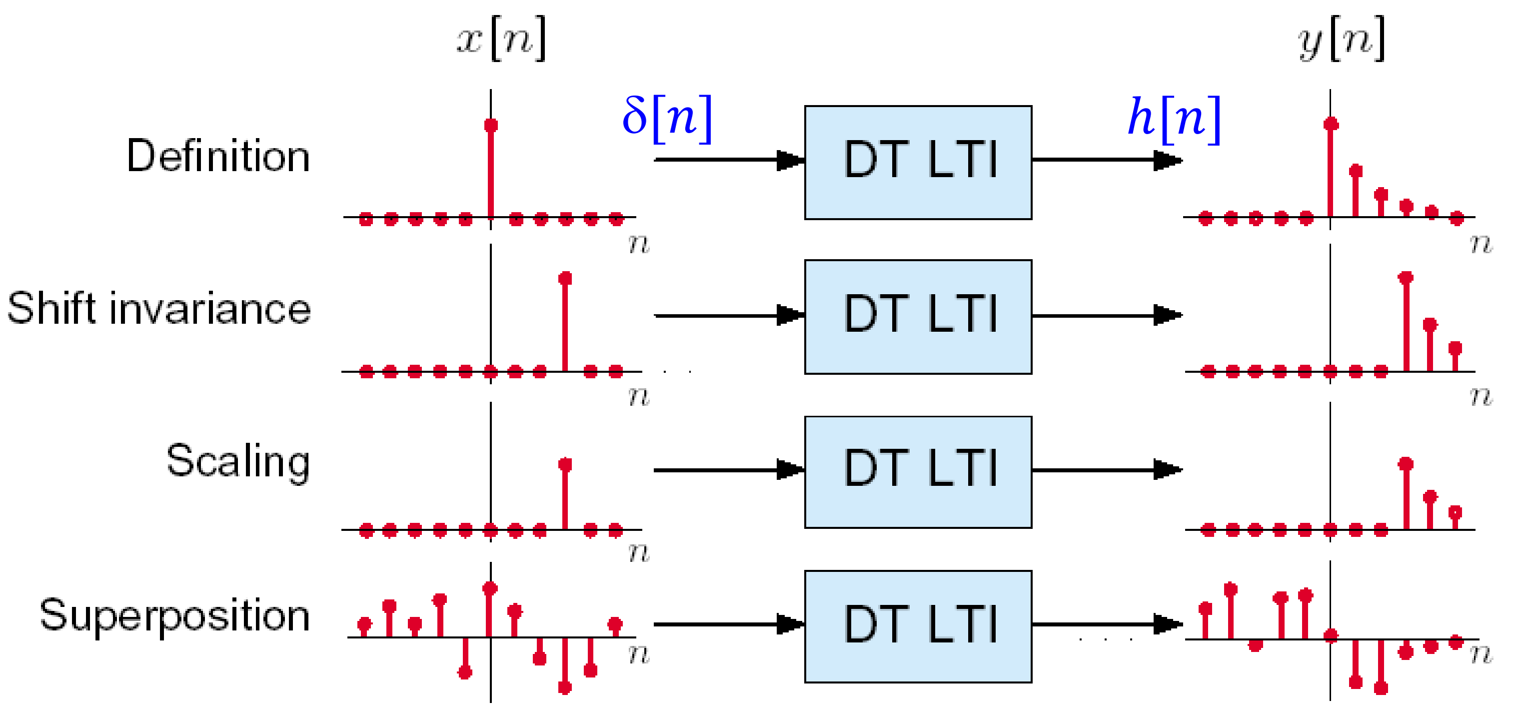
Impulse Response

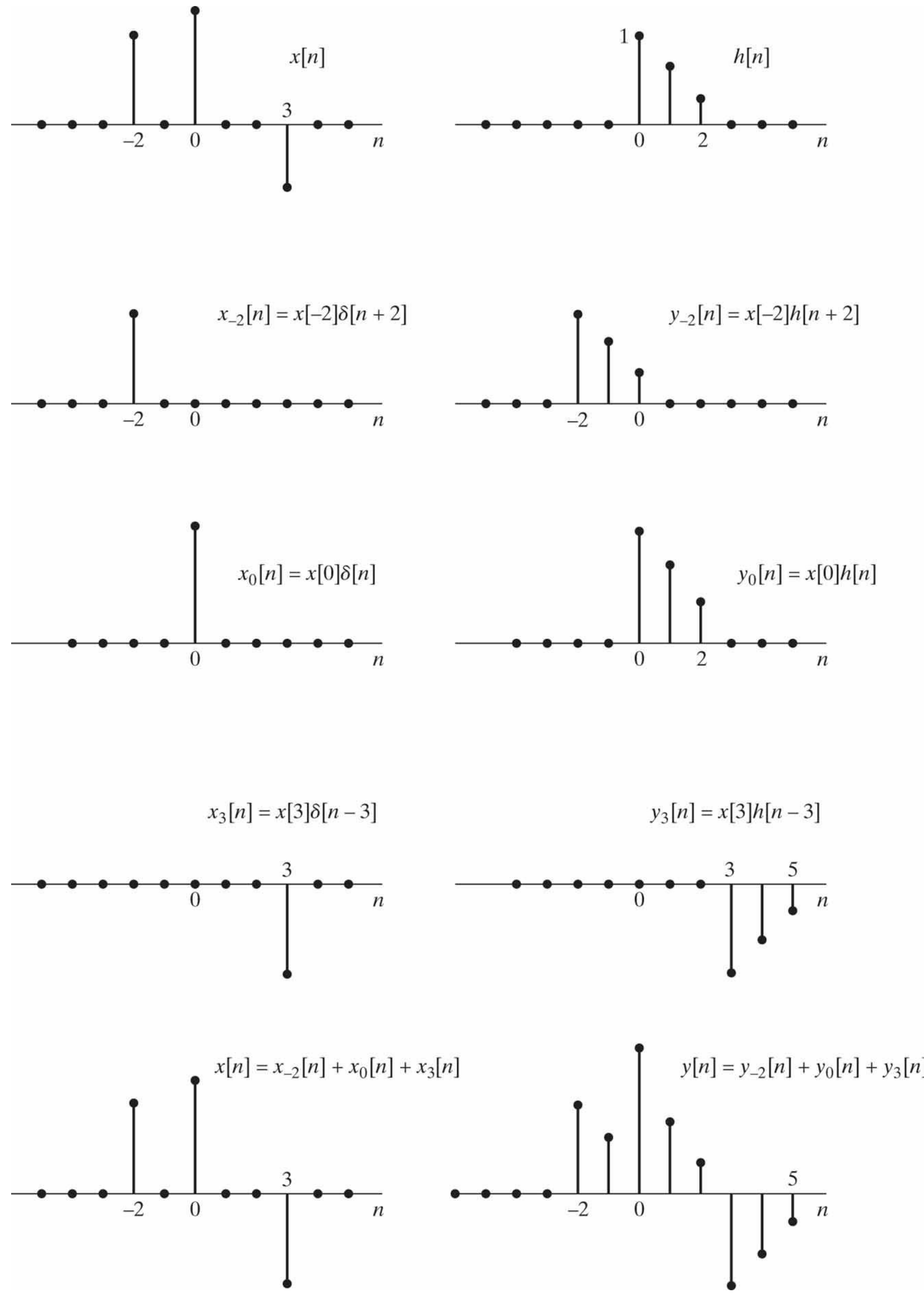
- The response of a system H to the unit impulse $\delta[n]$ is called the impulse response, which is denoted by $h[n]$
  - $h[n] = H\{\delta[n]\}$



Convolution Sum

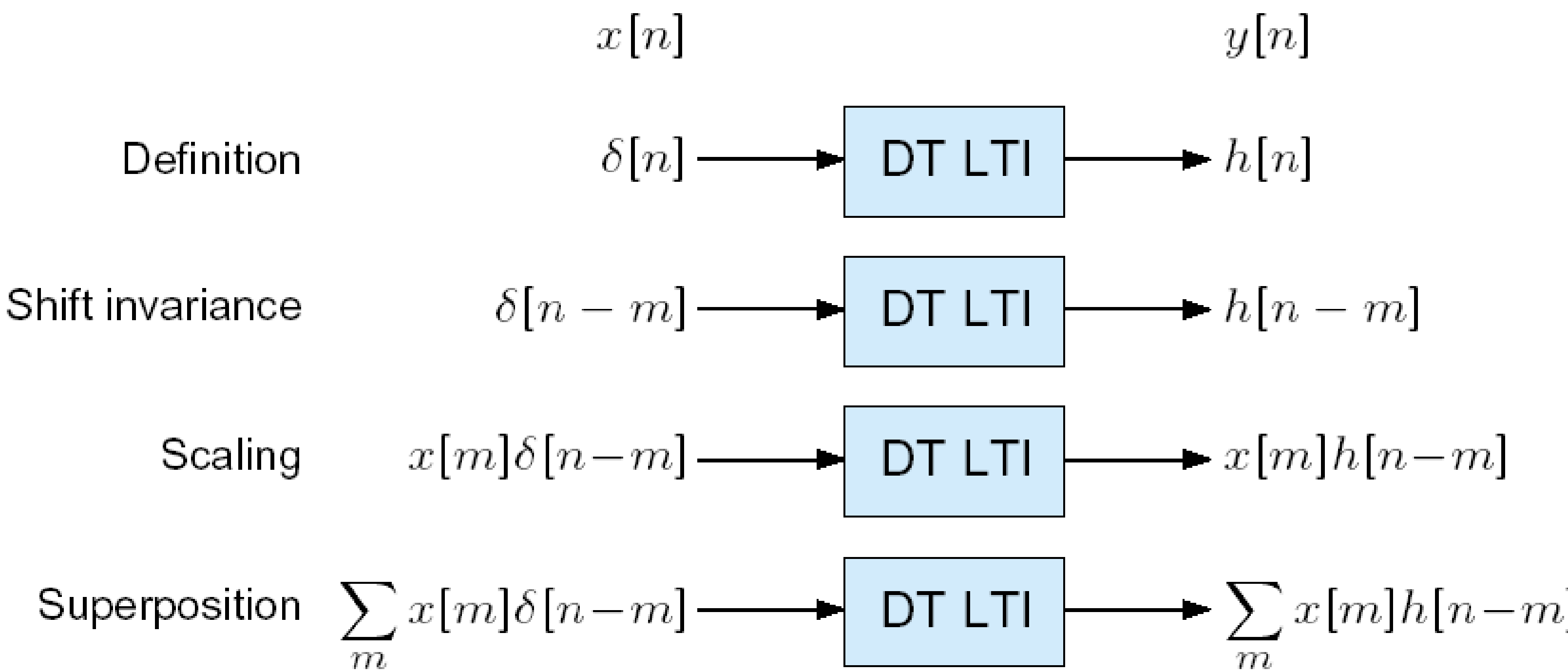
- Let $h[n]$ be the impulse response of an LTI system.
- Given $h[n]$, we can compute the response $y[n]$ of the system to any input signal $x[n]$.




x[n]
–2
0
3
n
1
h[n]
0
2
n
x_{–2}[n] = x[–2]δ[n + 2]
–2
0
n
y_{–2}[n] = x[–2]h[n + 2]
–2
0
n
x_0[n] = x[0]δ[n]
0
n
y_0[n] = x[0]h[n]
0
2
n
x_3[n] = x[3]δ[n – 3]
3
0
n
y_3[n] = x[3]h[n – 3]
3
5
0
n
x[n] = x_{–2}[n] + x_0[n] + x_3[n]
3
–2
0
n
y[n] = y_{–2}[n] + y_0[n] + y_3[n]
5
–2
0
n

Convolution Sum

- Let $h[n]$ be the impulse response of an LTI system.
- Given $h[n]$, we can compute the response $y[n]$ of the system to any input signal $x[n]$.



Convolution Sum

- Let $h[n]$ be the impulse response of an LTI system.
- Given $h[n]$, we can compute the response $y[n]$ of the system to any input signal $x[n]$.

$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]}$$

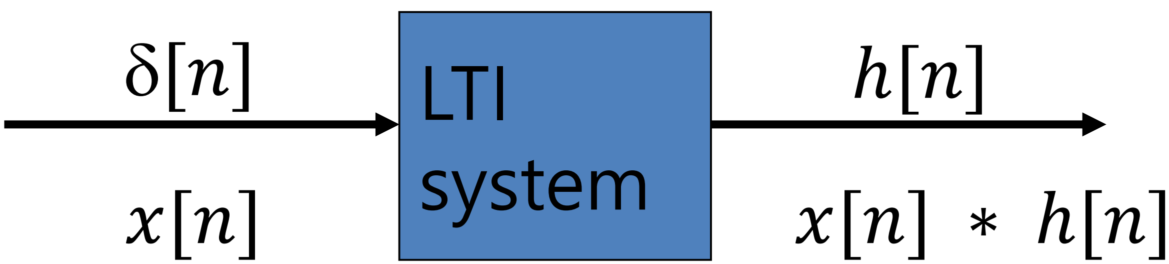
$$\begin{aligned}
y[n] &= H[x[n]] \\
&= H\left[\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right] \\
&= \sum_{k=-\infty}^{\infty} H\left[x[k]\delta[n-k]\right] \\
&= \sum_{k=-\infty}^{\infty} x[k]H\left[\delta[n-k]\right] \\
&= \sum_{k=-\infty}^{\infty} x[k]h[n-k]
\end{aligned}$$

Convolution Sum

- Notation for convolution sum

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- The characteristic of an LTI system is completely determined by its impulse response.



Convolution Sum

- To compute the convolution sum

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Step 1 Plot x and h vs k since the convolution sum is on k.

Step 2 Flip $h[k]$ around the vertical axis to obtain $h[-k]$.

Step 3 Shift $h[-k]$ by n to obtain $h[n-k]$.

Step 4 Multiply to obtain $x[k]h[n-k]$.

Step 5 Sum on k to compute $\sum x[k]h[n-k]$.

Step 6 Change n and repeat **Steps 3-6**.

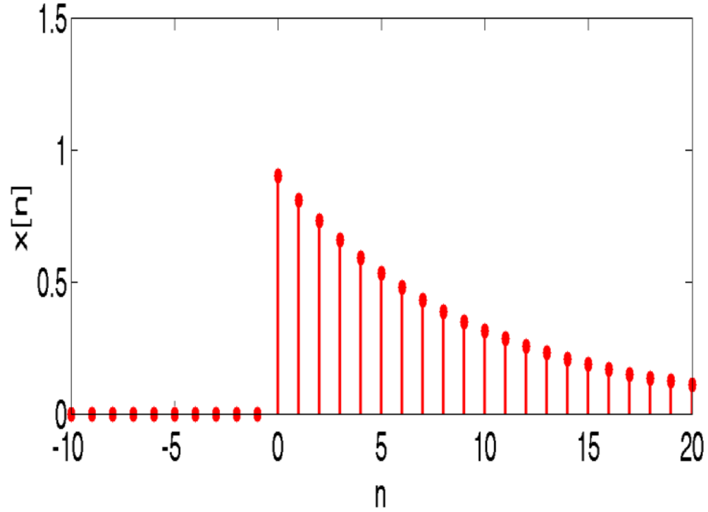
Example

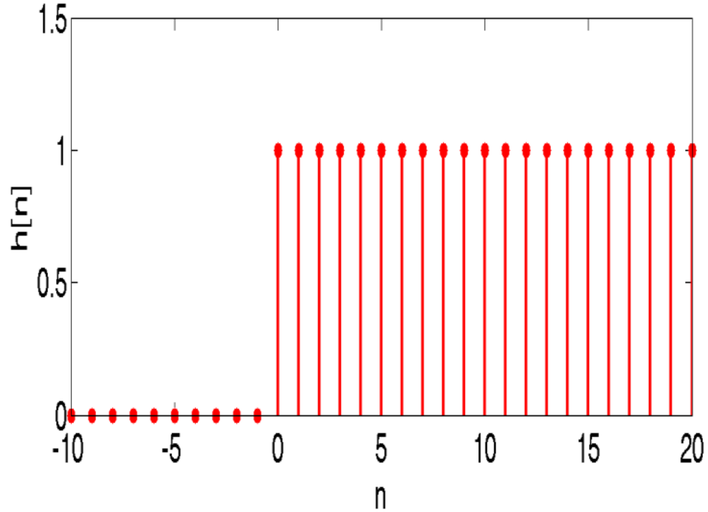
- Consider an LTI system that has an impulse response $h[n] = u[n]$

- What is the response when an input signal is given by
$$x[n] = a^n u[n]$$
where $0 < a < 1$?

- For $n \geq 0$,
$$y[n] = \sum_{k=0}^{n} \alpha^k$$
$$= \frac{1-\alpha^{n+1}}{1-\alpha}$$

- Therefore,
$$y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right) u[n]$$





Example

- Consider an LTI system that has an impulse response
  $$h[n] = u[n] - u[n-N]$$

- What is the response when an input signal is given by
  $$x[n] = a^n u[n]$$
  where $0 < a < 1$?

Properties of Convolution

- Identity property

$$x[n] * \delta[n] = x[n]$$

- Shifting property

$$x[n] * \delta[n-k] \ = \ x[n-k]$$

Properties of Convolution

- Commutative property

$$x[n] * h[n] = h[n] * x[n]$$

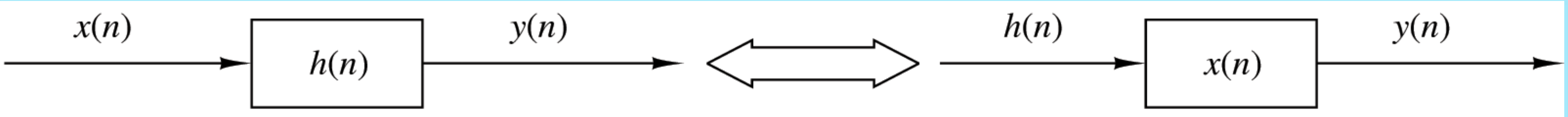


Figure 2.3.4 Interpretation of the commutative property of convolution.

Properties of Convolution

- Associative property

$$\{x[n] * h_1[n]\} * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$$

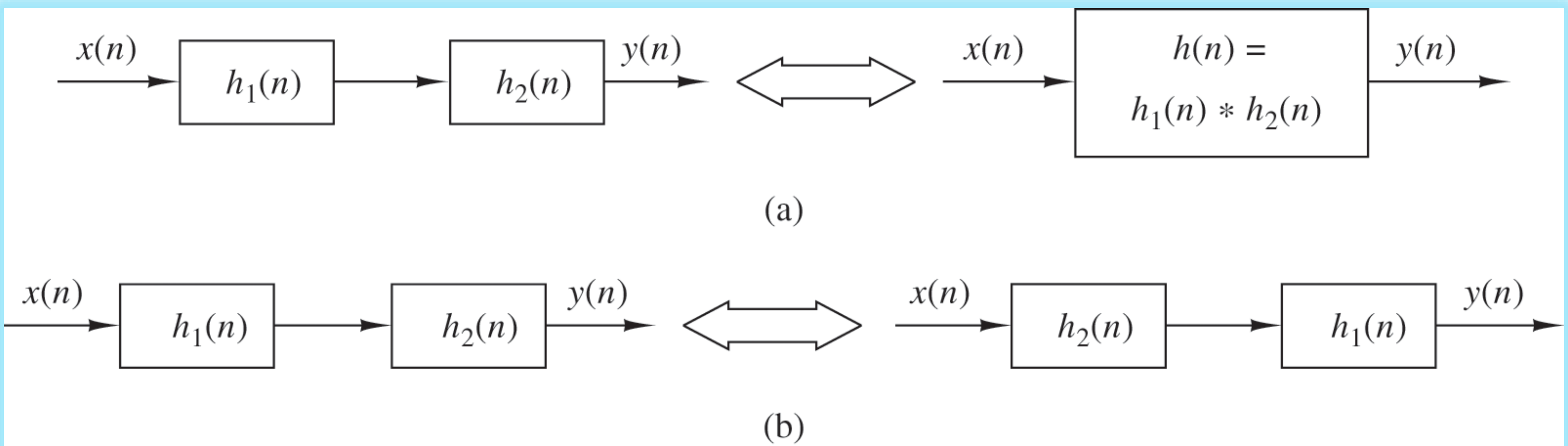


Figure 2.3.5 Implications of the associative (a) and the associative and commutative (b) properties of convolution.

Properties of Convolution

- Distributive property
  $x[n] * [h_1[n] + h_2[n]] = x[n] * h_1[n] + x[n] * h_2[n]$

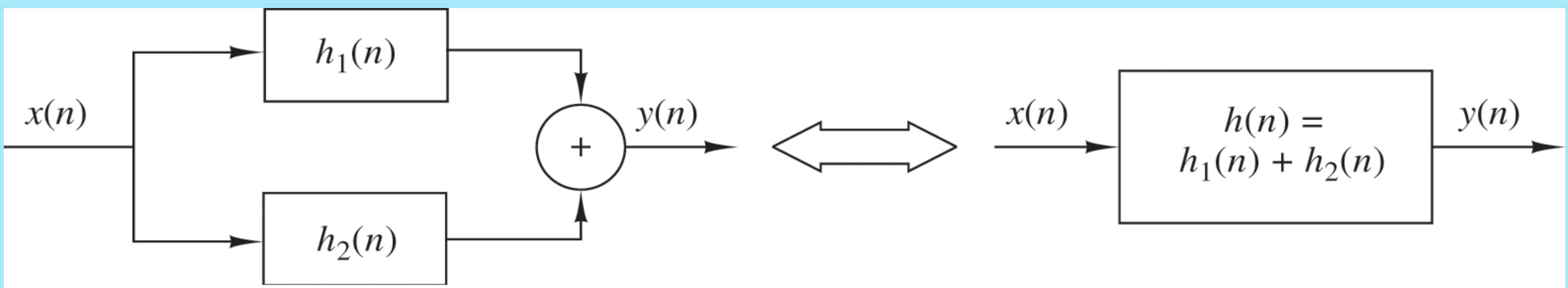


Figure 2.3.6 Interpretation of the distributive property of convolution: two LTI systems connected in parallel can be replaced by a single system with $h(n) = h_1(n) + h_2(n)$.

Causality of LTI Systems

- A system is causal if its output depends only on the past and present values of the input signal.

- Consider the following for a causal LTI system:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

  – Because of causality h[n-k] must be zero for $k > n$.

  – In other words, $h[n] = 0$ for $n < 0$.

Causality of LTI Systems

- So the convolution sum for a causal LTI system becomes

$$y[n] = \sum_{k=-\infty}^{n} x[k]h[n-k] = \sum_{k=0}^{\infty} h[k]x[n-k]$$

- So, if a given system is causal, one can infer that its impulse response is zero for negative time values, and use the above simpler convolution formulas.

Stability of LTI Systems

- A system is stable if a bounded input yields a bounded output (BIBO). In other words, if $|x[n]| < M_x$ then $|y[n]| < M_y$.

- Note that

$$\left|y[n]\right| = \left|\sum_{k=-\infty}^{\infty} x[n-k]h[k]\right| \le \sum_{k=-\infty}^{\infty} \left|x[n-k]\right|\left|h[k]\right| \le M_x \sum_{k=-\infty}^{\infty} \left|h[k]\right|$$

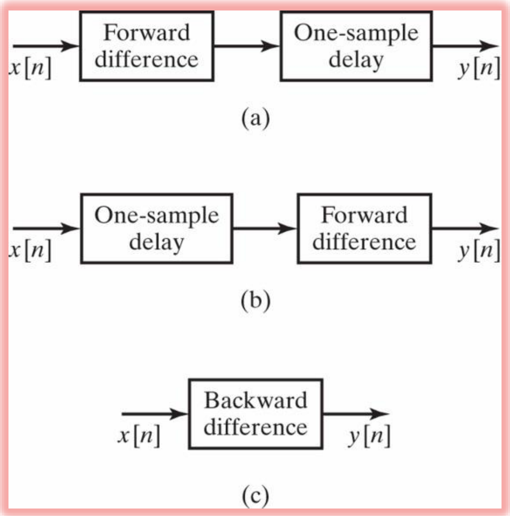
- Therefore, a system is stable if

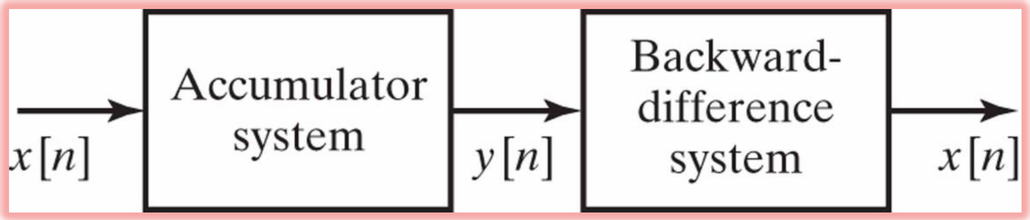
$$\sum_{k=-\infty}^{\infty} \left|h[k]\right| < \infty$$

Examples

System	Impulse response	Causal	Stable
$y[n] = x[n - n_d]$			
$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$			
$y[n] = \sum_{k=-\infty}^{n} x[k]$			
$y[n] = x[n+1] - x[n]$			
$y[n] = x[n] - x[n-1]$			

Examples


Forward difference
One-sample delay
x[n]
y[n]
(a)
One-sample delay
Forward difference
x[n]
y[n]
(b)
Backward difference
x[n]
y[n]
(c)


Accumulator system
Backward-difference system
x[n]
y[n]
x[n]

Constant-Coefficient Difference Equations (CCDE)

Discrete-Time Systems

- Block diagram representation

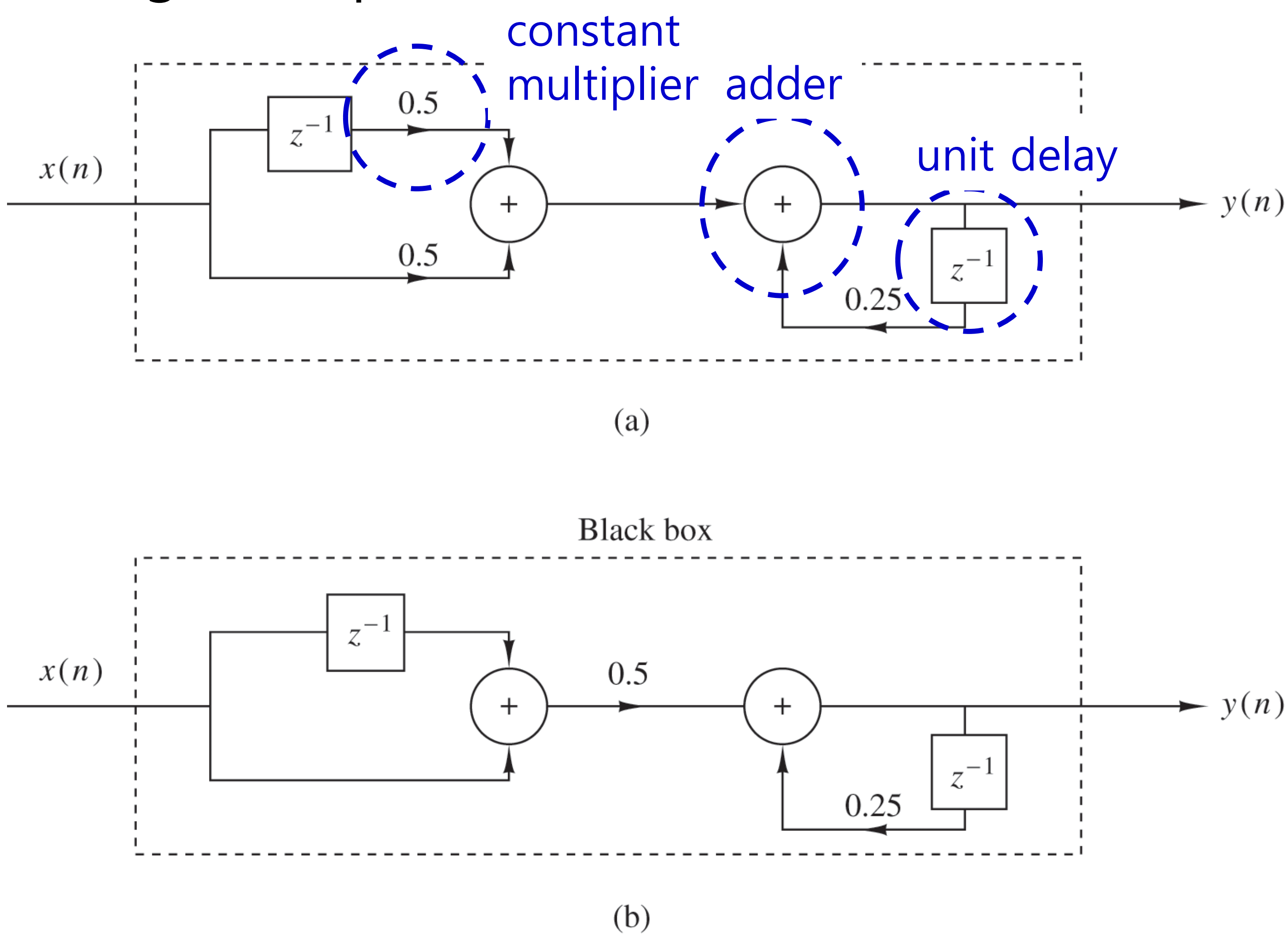
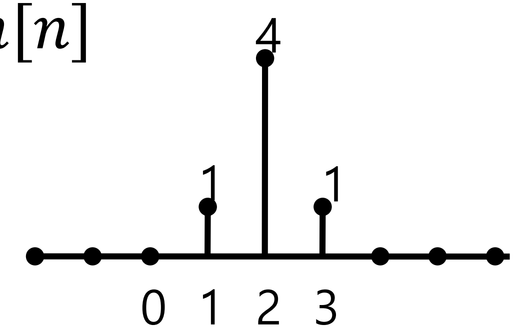


Figure 2.2.7 Block diagram realizations of the system $y(n) = 0.25y(n-1) + 0.5x(n) + 0.5x(n-1)$.

Recursive Systems

- If an impulse response has a finite duration, the system is an FIR system. Otherwise, an IIR system.
- An FIR system can be implemented directly using a finite number of adders, multipliers and delays.
  - e.g.) Implement the system with $h[n]$



Recursive Systems

- Can you directly implement the cumulative averaging system?

$$y(n) = \frac{1}{n+1} \sum_{k=0}^{n} x(k), \quad n = 0,1,\ldots$$

- It can be implemented in a **recursive** manner with a feedback loop
  - Past output values are used to compute a current output value

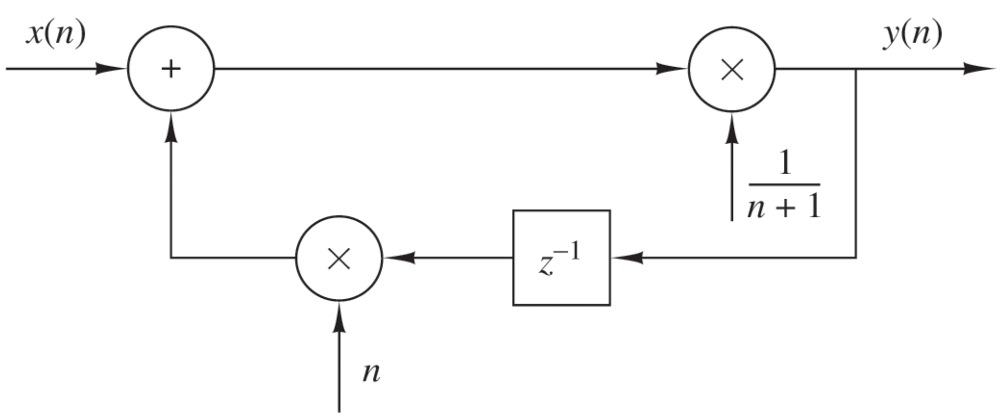


Figure 2.4.1 Realization of a recursive cumulative averaging system.

Constant-Coefficient Difference Equations (CCDE)

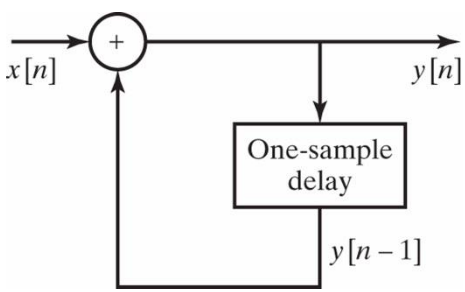
$$\boxed{\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]}$$

- The equation defines a recursive system, which processes an input $x[n]$ to make the output $y[n]$
- N is the order of the equation or the corresponding system

Constant-Coefficient Difference Equations (CCDE)

$$\boxed{\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]}$$

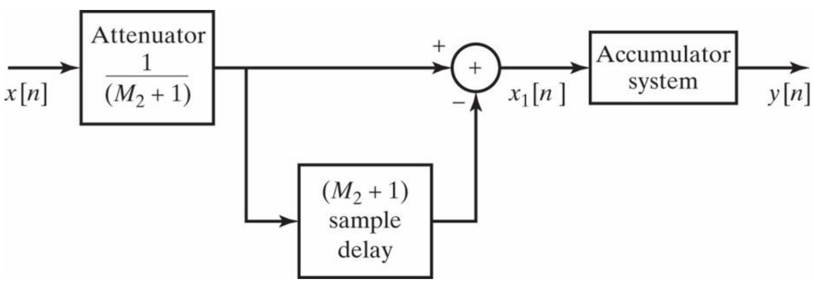
- Example 1: Accumulator $y[n] = \sum_{k=-\infty}^{n} x[k]$



Constant-Coefficient Difference Equations (CCDE)

$$\boxed{\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]}$$

- Example 1: MA System $y[n] = \frac{1}{M_2+1}\sum_{k=0}^{M_2} x[n-k]$



Constant-Coefficient Difference Equations (CCDE)

$$\boxed{\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]}$$

- Suppose that $x[n]$ is given, and we want to get $y[n]$ for $n \geq 0$. Which information do we need further?

Constant-Coefficient Difference Equations (CCDE)

$$\boxed{\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]}$$

- **Initial rest condition**
  - If $x[n]$ starts at $n = n_0$, i.e., $x[n] = 0$ when $n < n_0$, then $y[n] = 0$ when $n < n_0$.
  - Alternatively, the initial values are

    $y[n_0 - 1] = y[n_0 - 2] = \cdots = y[n_0 - N] = 0.$

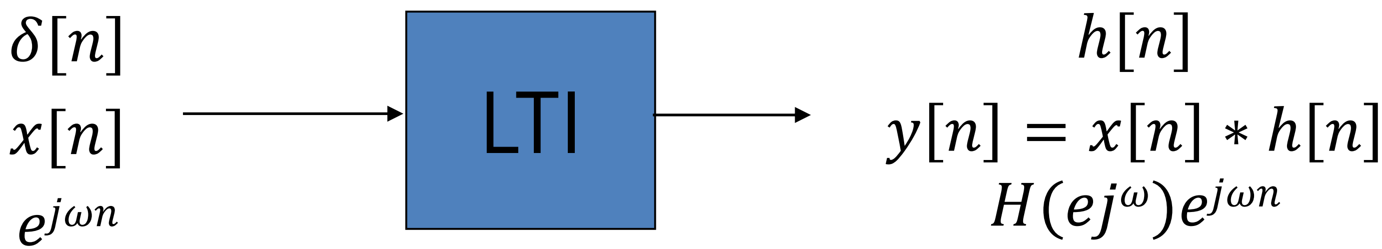
- If we assume **the initial rest condition**, then **the system described by the equation is LTI**.

Constant-Coefficient Difference Equations (CCDE)

- More details will be studied later, especially in Chap 6.

Frequency Domain Representation of Discrete-Time Signals and Systems

Eigenfunctions for LTI Systems



- $e^{j\omega n}$ is an eigenfunction of LTI systems
- Its eigenvalue is given by the Fourier transform of impulse response, $H(e^{j\omega})$, which is called *frequency response*

$$H\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

Eigenfunctions for LTI Systems

$$x[n] = \sum_k \alpha_k \, e^{j\omega_k n} \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n] = \sum_k \alpha_k \, H(e^{j\omega_k}) e^{j\omega_k n}$$

Frequency Response

- Ex) Determine the output sequence of the system with impulse response

$$h[n] = \frac{1}{2^n} u[n]$$

when the input is a complex exponential

$$x[n] = Ae^{j\frac{\pi}{2}n}$$

Frequency Response

- Ex) Determine the magnitude and phase of $H(e^{j\omega})$ for the three-point moving average (MA) system

$$y[n] = \frac{1}{3}\{x[n+1] + x[n] + x[n-1]\}.$$

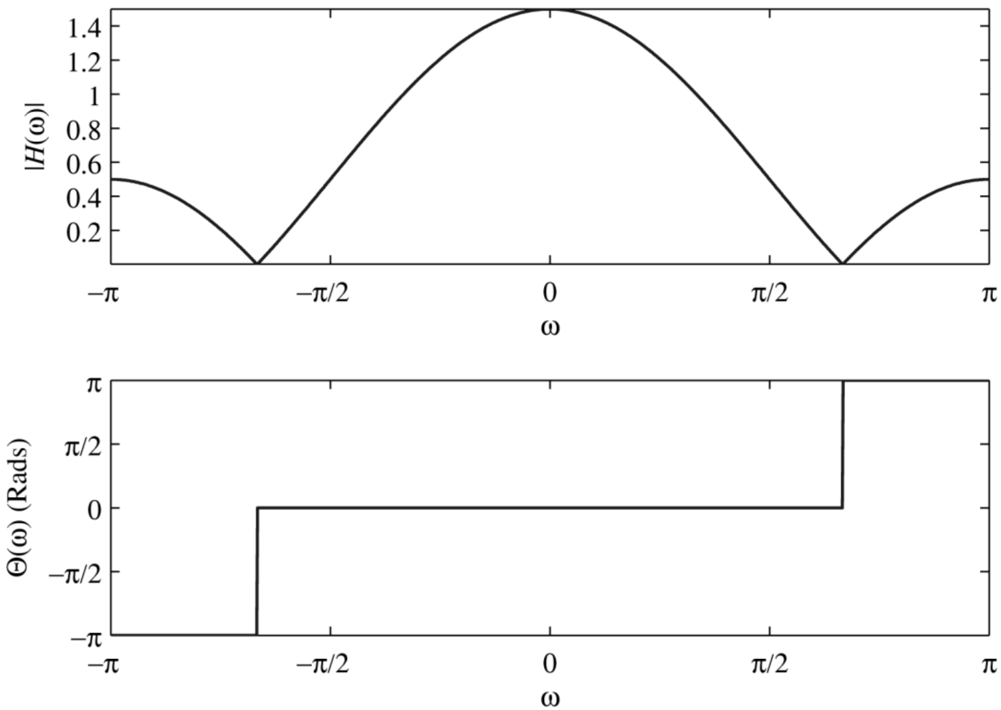


Figure 5.1.1 Magnitude and phase responses for the MA system in Example 5.1.2.

Sinusoidal Input

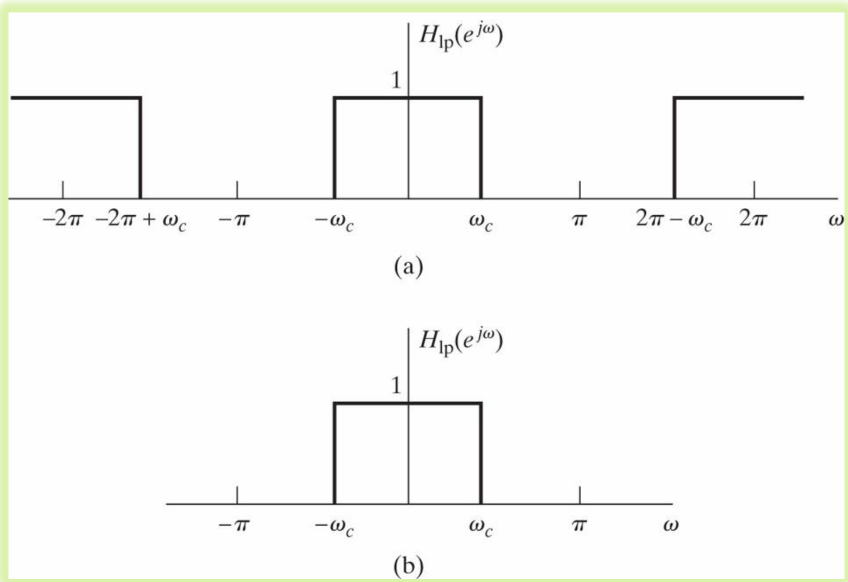
- Assuming that $h[n]$ is real, we have the input-output relationship

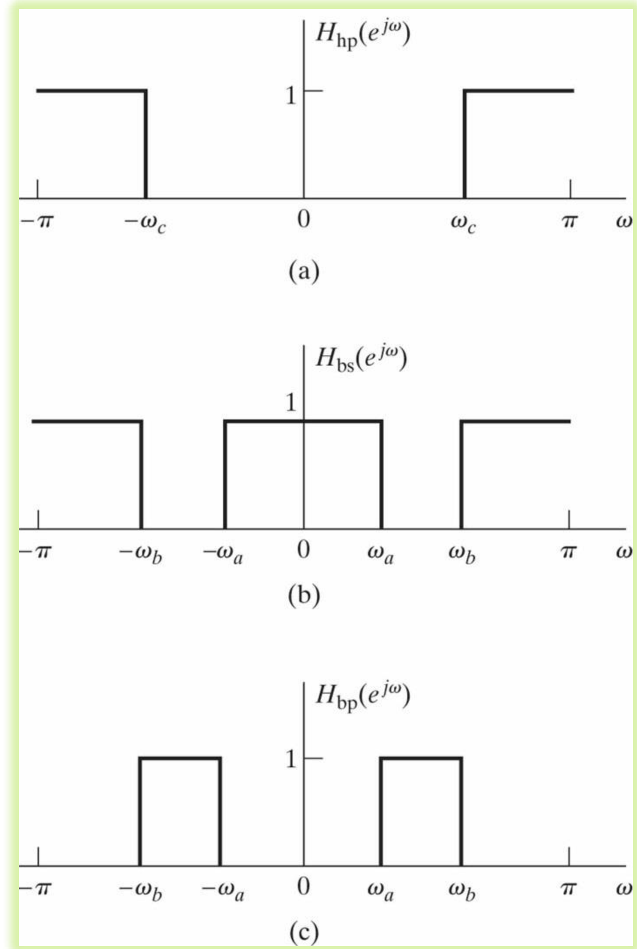
$$A\cos(\omega_0 n+\phi) \longrightarrow \boxed{\text{LTI}} \longrightarrow A\left|H\left(e^{j\omega_0}\right)\right|\cos(\omega_0 n+\phi+\theta(\omega_0))$$

$$H\left(e^{j\omega}\right)=\left|H\left(e^{j\omega}\right)\right|e^{j\theta(\omega)}$$

1. The amplitude is multiplied by $\left|H\left(e^{j\omega}\right)\right|$
2. The output has a phase lag relative to the input by an amount $\theta(\omega)=\angle H(e^{j\omega})$

Ideal Filters





$$H\left(e^{j\omega}\right) = H\left(e^{j(\omega+2\pi r)}\right)$$

Representation of Sequences by Fourier Transforms

Discrete-Time Fourier Transform

$$\boxed{\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega \\ X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \end{aligned}}$$

- DTFT can be derived from DTFS (discrete-time Fourier series)
- Frequency response is the DTFT of impulse response
- The existence of $X\left(e^{j\omega}\right)$
  - A sufficient condition: $x[n]$ is absolutely summable
  - We avoid rigorous conditions/proofs and use well-known Fourier transform pairs

Fourier Transform Pairs

TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n-n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
4. $a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
5. $u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
6. $(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
7. $\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2e^{-j2\omega}}$
8. $\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c, \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0+2\pi k)$
11. $\cos(\omega_0 n+\phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k)+\pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)]$

Symmetry Property

TABLE 2.1 SYMMETRY PROPERTIES OF THE FOURIER TRANSFORM

Sequence $x[n]$	Fourier Transform $X(e^{j\omega})$
1. $x^*[n]$	$X^*(e^{-j\omega})$
2. $x^*[-n]$	$X^*(e^{j\omega})$
3. $\mathcal{R}e\{x[n]\}$	$X_e(e^{j\omega})$ (conjugate-symmetric part of $X(e^{j\omega})$)
4. $j\mathcal{I}m\{x[n]\}$	$X_o(e^{j\omega})$ (conjugate-antisymmetric part of $X(e^{j\omega})$)
5. $x_e[n]$ (conjugate-symmetric part of $x[n]$)	$X_R(e^{j\omega}) = \mathcal{R}e\{X(e^{j\omega})\}$
6. $x_o[n]$ (conjugate-antisymmetric part of $x[n]$)	$jX_I(e^{j\omega}) = j\mathcal{I}m\{X(e^{j\omega})\}$
The following properties apply only when $x[n]$ is real:	
7. Any real $x[n]$	$X(e^{j\omega}) = X^*(e^{-j\omega})$ (Fourier transform is conjugate symmetric)
8. Any real $x[n]$	$X_R(e^{j\omega}) = X_R(e^{-j\omega})$ (real part is even)
9. Any real $x[n]$	$X_I(e^{j\omega}) = -X_I(e^{-j\omega})$ (imaginary part is odd)
10. Any real $x[n]$	$\lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert$ (magnitude is even)
11. Any real $x[n]$	$\angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$ (phase is odd)
12. $x_e[n]$ (even part of $x[n]$)	$X_R(e^{j\omega})$
13. $x_o[n]$ (odd part of $x[n]$)	$jX_I(e^{j\omega})$

Symmetry Property

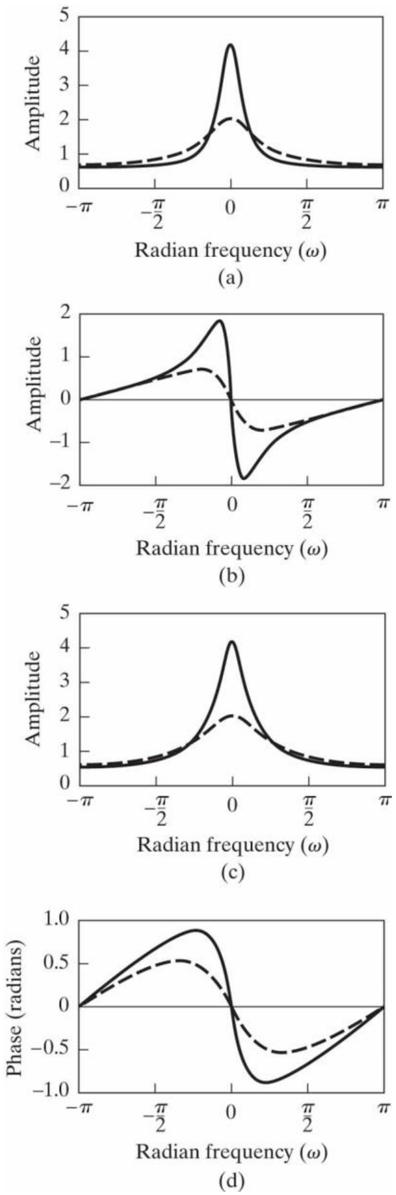


Figure 2.22 Frequency response for a system with impulse response $h[n] = a^n u[n]$. $a > 0$; $a = 0.75$ (solid curve) and $a = 0.5$ (dashed curve). (a) Real part. (b) Imaginary part. (c) Magnitude. (d) Phase.

Fourier Transform Theorems

TABLE 2.2 FOURIER TRANSFORM THEOREMS

Sequence	Fourier Transform
$x[n]$	$X(e^{j\omega})$
$y[n]$	$Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Parseval's theorem:	
8. $\sum_{n=-\infty}^{\infty} \|x[n]\|^2 = \dfrac{1}{2\pi}\int_{-\pi}^{\pi} \|X(e^{j\omega})\|^2 d\omega$	
9. $\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \dfrac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$	

Convolution Theorem

- $y[n] = x[n] * h[n] \;\Rightarrow\; Y\left(e^{j\omega}\right) = X\left(e^{j\omega}\right)H\left(e^{j\omega}\right)$

Convolution Theorem: Another Perspective

$$x[n] = \sum_k \alpha_k \, e^{j\omega_k n} \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n] = \sum_k \alpha_k \, H(e^{j\omega_k}) e^{j\omega_k n}$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$y[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) H(e^{j\omega}) e^{j\omega n} d\omega$$

Examples

- $x[n] = a^n u[n-5]$. What is $X(e^{j\omega})$?

Examples

- $X(e^{j\omega}) = \frac{1}{(1-ae^{-j\omega})(1-ae^{-j\omega})}$. What is $x[n]$?

Examples

- $X(e^{j\omega}) = \frac{1}{(1-ae^{-j\omega})(1-be^{-j\omega})}$. What is $x[n]$?

Examples

- Determine the impulse response $h[n]$ of a highpass filter with frequency response

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & \omega_c < |\omega| < \pi, \\ 0, & |\omega| < \omega_c. \end{cases}$$

Examples

- Determine the frequency response and the impulse response of a system described by a CCDE

$$y[n] - \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-1].$$