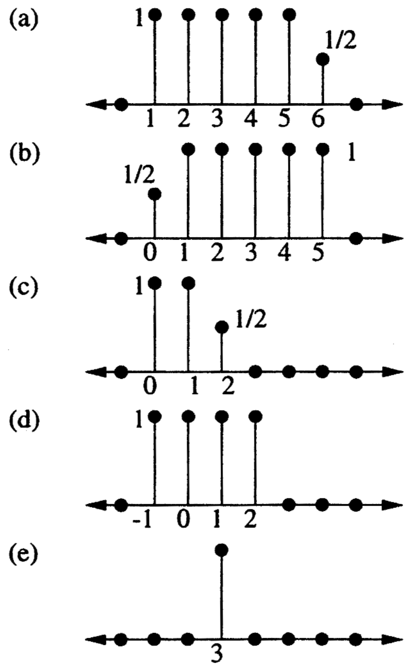
2.21.



2.22. For an LTI system, we use the convolution equation to obtain the output:

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

Let $n = m + N$:

$$\begin{aligned} y[m+N] &= \sum_{k=-\infty}^{\infty} x[m+N-k]h[k] \\ &= \sum_{k=-\infty}^{\infty} x[(m-k)+N]h[k] \end{aligned}$$

Since $x[n]$ is periodic, $x[n] = x[n+rN]$ for any integer r. Hence,

$$\begin{aligned} y[m+N] &= \sum_{k=-\infty}^{\infty} x[m-k]h[k] \\ &= y[m] \end{aligned}$$

So, the output must also be periodic with period N.

2.23. (a) Since $\cos(\pi n)$ only takes on values of +1 or -1, this transformation outputs the current value of $x[n]$ multiplied by either ± 1. $T(x[n]) = (-1)^n x[n]$.

- Hence, it is stable, because it doesn't change the magnitude of $x[n]$ and hence satisfies bounded-in/bounded-out stability.
- It is causal, because each output depends only on the current value of $x[n]$.
- It is linear. Let $y_1[n] = T(x_1[n]) = \cos(\pi n)x_1[n]$, and $y_2[n] = T(x_2[n]) = \cos(\pi n)x_2[n]$. Now

$$T(ax_1[n] + bx_2[n]) = \cos(\pi n)(ax_1[n] + bx_2[n]) = ay_1[n] + by_2[n]$$

- It is not time-invariant. If $y[n] = T(x[n]) = (-1)^n x[n]$, then $T(x[n-1]) = (-1)^n x[n-1] \neq y[n-1]$.

(b) This transformation simply "samples" $x[n]$ at location which can be expressed as k^2.

- The system is stable, since if $x[n]$ is bounded, $x[n^2]$ is also bounded.
- It is not causal. For example, $Tx[4] = x[16]$.
- It is linear. Let $y_1[n] = T(x_1[n]) = x_1[n^2]$, and $y_2[n] = T(x_2[n]) = x_2[n^2]$. Now

$$T(ax_1[n] + bx_2[n]) = ax_1[n^2] + bx_2[n^2]) = ay_1[n] + by_2[n]$$

- It is not time-invariant. If $y[n] = T(x[n]) = x[n^2]$, then $T(x[n-1]) = x[n^2 - 1] \neq y[n-1]$.

(c) First notice that

$$\sum_{k=0}^{\infty} \delta[n-k] = u[n]$$

So $T(x[n]) = x[n]u[n]$. This transformation is therefore stable, causal, linear, but not time-invariant.

To see that it is not time invariant, notice that $T(\delta[n]) = \delta[n]$, but $T(\delta[n+1]) = 0$.

(d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

- This is not stable. For example, $T(u[n]) = \infty$ for all n ¿ 1.
- It is not causal, since it sums *forward* in time.
- It is linear, since

$$\sum_{k=n-1}^{\infty} ax_1[k] + bx_2[k] = a \sum_{k=n-1}^{\infty} x_1[k] + b \sum_{k=n-1}^{\infty} x_2[k]$$

- It is time-invariant. Let

$$y[n] = T(x[n]) = \sum_{k=n-1}^{\infty} x[k],$$

then

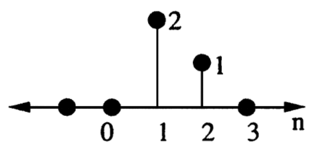
$$T(x[n-n_0]) = \sum_{k=n-n_0-1}^{\infty} x[k] = y[n-n_0]$$

2.25. We use the graphical approach to compute the convolution:

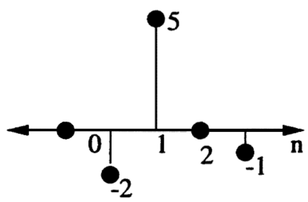
$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \end{aligned}$$

(a) $y[n] = x[n] * h[n]$

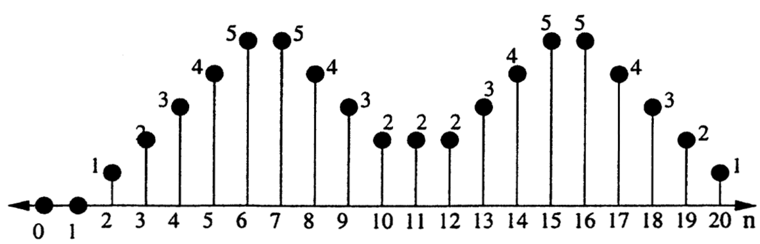
$$y[n] = \delta[n-1] * h[n] = h[n-1]$$



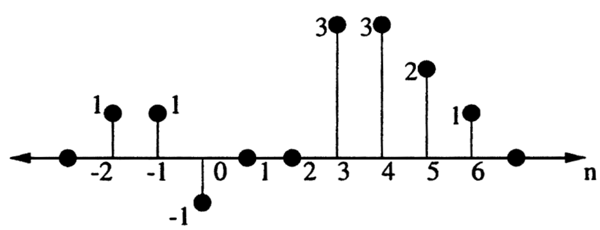
(b) $y[n] = x[n] * h[n]$



(c) $y[n] = x[n] * h[n]$



(d) $y[n] = x[n] * h[n]$



2.26. The response of the system to a delayed step:

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= \sum_{k=-\infty}^{\infty} u[k-4]h[n-k] \end{aligned}$$

$$y[n] = \sum_{k=4}^{\infty} h[n-k]$$

Evaluating the above summation:

For $n < 4$: $y[n] = 0$
For $n = 4$: $y[n] = h[0] = 1$
For $n = 5$: $y[n] = h[1] + h[0] = 2$
For $n = 6$: $y[n] = h[2] + h[1] + h[0] = 3$
For $n = 7$: $y[n] = h[3] + h[2] + h[1] + h[0] = 4$
For $n = 8$: $y[n] = h[4] + h[3] + h[2] + h[1] + h[0] = 2$
For $n \geq 9$: $y[n] = h[5] + h[4] + h[3] + h[2] + h[1] + h[0] = 0$

2.29. • System A:

$$x[n] = (\frac{1}{2})^n$$

This input is an eigenfunction of an LTI system. That is, if the system is linear, the output will be a replica of the input, scaled by a complex constant.
Since $y[n] = (\frac{1}{4})^n$, System A is NOT LTI.

• System B:

$$x[n] = e^{jn/8}u[n]$$

The Fourier transform of $x[n]$ is

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} e^{jn/8}u[n]e^{-j\omega n} \\ &= \sum_{n=0}^{\infty} e^{-j(\omega-\frac{1}{8})n} \\ &= \frac{1}{1-e^{-j(\omega-\frac{1}{8})}}. \end{aligned}$$

The output is $y[n] = 2x[n]$, thus

$$Y(e^{j\omega}) = \frac{2}{1-e^{-j(\omega-\frac{1}{8})}}.$$

Therefore, the frequency response of the system is

$$\begin{aligned} H(e^{j\omega}) &= \frac{Y(e^{j\omega})}{X(e^{j\omega})} \\ &= 2. \end{aligned}$$

Hence, the system is a linear amplifier. We conclude that System B is LTI, and unique.

• System C: Since $x[n] = e^{jn/8}$ is an eigenfunction of an LTI system, we would expect the output to be given by

$$y[n] = \gamma e^{jn/8},$$

where γ is some complex constant, if System C were indeed LTI. The given output, $y[n] = 2e^{jn/8}$, indicates that this is so.
Hence, System C is LTI. However, it is not unique, since the only constraint is that

$$H(e^{j\omega})|_{\omega=1/8} = 2.$$

2.30

A. LTI systems are stable iff $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ (the summation should converge).

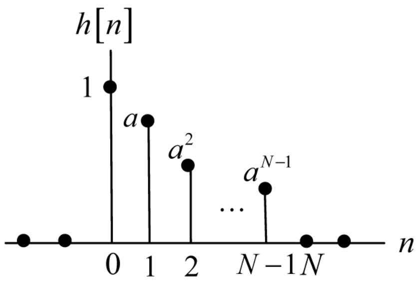
Then

$$S = \sum_{n=-\infty}^{\infty} |a|^n u[n]$$
$$= \sum_{n=0}^{\infty} |a|^n$$

S will converge only when $|a| < 1$ and $S = \frac{1}{1-|a|} < \infty$.

Therefore the system is stable for $|a| < 1$.

B. $y[n] = ay[n-1] + x[n] - a^N x[n-N]$. Therefore,
$h[n] = ah[n-1] + \delta[n] - a^N \delta[n-N]$.
Since the system is causal, $h[-1] = 0$. Then
$h[0] = 0 + 1 - 0 = 1$
$h[1] = a$, $h[2] = a^2$, $h[N] = a^N - a^N = 0$
$h[N+1] = a \times 0 + 0 - 0 = 0$.



C. We see that even though it is a recursive system (with feedback), its impulse respor
finite in length. The length of $h[n]$ is N terms. Hence this system is FIR.

D. FIR systems are always stable as the sum $\sum_{n=-\infty}^{\infty} |h[n]|$ has at most a finite number of nonzero terms.

2.32

$$y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$$

A. Impulse response:

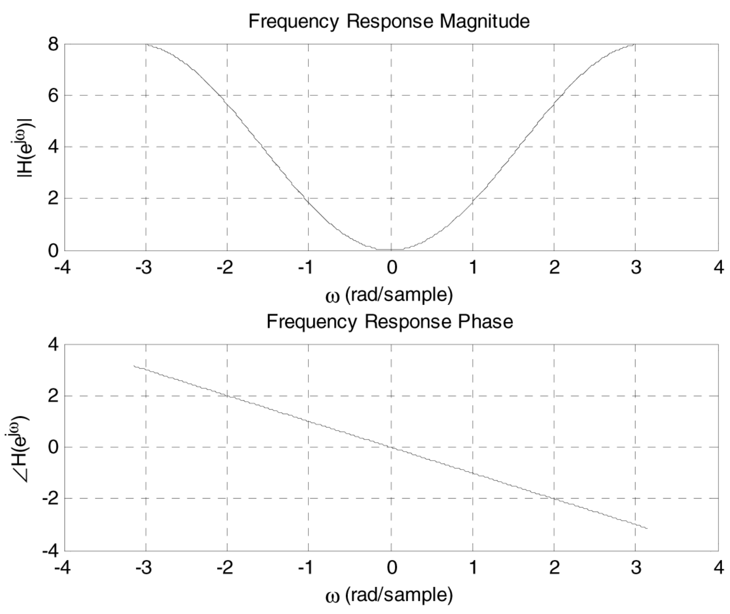
$$h[n] = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$$

B.

$$\begin{aligned} H\left(e^{j\omega}\right) &= -2 + 4e^{-j\omega} - 2e^{-j2\omega} \\ &= -2e^{-j\omega}\left(e^{j\omega} + e^{-j\omega} - 2\right) \\ &= -2e^{-j\omega}\left(2\cos(\omega) - 2\right) \\ &= 4e^{-j\omega}\left(1 - \cos(\omega)\right) \\ &= 4e^{-j\omega}\left(2\sin^2(\omega/2)\right) \\ &= 8\sin^2(\omega/2)e^{-j\omega}. \end{aligned}$$

The delay is $n_d = 1$.

C.



D. If

$$x_1[n] = 1 + e^{j0.5\pi n} = e^{j0n} + e^{j\frac{\pi}{2}n}$$

then

$$\begin{aligned} y_1[n] &= H\left(e^{j0}\right)e^{j0n} + H\left(e^{j\frac{\pi}{2}n}\right)e^{j\frac{\pi}{2}n} \\ &= 0 \times e^{j0} + 8\sin^2(\pi/4)e^{-j\frac{\pi}{2}}e^{j\frac{\pi}{2}n} \\ &= 8 \times \frac{1}{2}e^{j\frac{\pi}{2}(n-1)}, \quad -\infty < n < \infty. \end{aligned}$$

E. Using the convolution sum,

$$\begin{aligned} y_2[n] &= \sum_{k=-\infty}^{\infty} h[k]x_2[n-k] \\ &= \sum_{k=-\infty}^{\infty} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right)u[n-k] \\ &= \sum_{k=-\infty}^{n} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right) \end{aligned}$$

$$y_2[n] = \begin{cases} 0, & n < 0 \text{ (as the system is causal)} \\ \sum_{k=0}^{n} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right), & n \geq 0 \end{cases}$$

Consider $n \geq 0$,

$$\begin{aligned} y_2[n] &= \left(\sum_{k=0}^{\infty} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right)\right) - \left(\sum_{k=n+1}^{\infty} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right)\right) \\ &= \sum_{k=0}^{\infty} h[k] + \left(\sum_{k=0}^{\infty} h[k]e^{-j\frac{\pi}{2}k}\right)e^{j\frac{\pi}{2}n} - \left(\sum_{k=n+1}^{\infty} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right)\right) \\ &= H\left(e^{j0}\right) + H\left(e^{j\frac{\pi}{2}}\right)e^{j\frac{\pi}{2}n} - \left(\sum_{k=n+1}^{\infty} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right)\right). \end{aligned}$$

Now $\left(\sum_{k=n+1}^{\infty} h[k]\left(1 + e^{j\frac{\pi}{2}(n-k)}\right)\right)$ becomes zero for $n \geq 2$ since $h[n] = 0$ for $n > 2$. Thus $y_2[n] = y_1[n]$ for all $n \geq 2$.

2.33. Recall that an eigenfunction of a system is an input signal which appears at the output of the system scaled by a complex constant.

(a) $x[n] = 5^n u[n]$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} h[k]5^{(n-k)}u[n-k] \\ &= 5^n \sum_{k=-\infty}^{n} h[k]5^{-k} \end{aligned}$$

Becuase the summation depends on n, $x[n]$ is NOT AN EIGENFUNCTION.

(b) $x[n] = e^{j2\omega n}$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]e^{j2\omega(n-k)} \\ &= e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j2\omega k} \\ &= e^{j2\omega n} \cdot H(e^{j2\omega}) \end{aligned}$$

YES, EIGENFUNCTION.

(c) $e^{j\omega n} + e^{j2\omega n}$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)} + \sum_{k=-\infty}^{\infty} h[k]e^{j2\omega(n-k)} \\ &= e^{j\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k} + e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j2\omega k} \\ &= e^{j\omega n} \cdot H(e^{j\omega}) + e^{j2\omega n} \cdot H(e^{j2\omega}) \end{aligned}$$

Since the input cannot be extracted from the above expression, the sum of complex exponentials is NOT AN EIGENFUNCTION. (Although, separately the inputs are eigenfunctions. In general, complex exponential signals are always eigenfunctions of LTI systems.)

(d) $x[n] = 5^n$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]5^{(n-k)} \\ &= 5^n \sum_{k=-\infty}^{\infty} h[k]5^{-k} \end{aligned}$$

YES, EIGENFUNCTION.

(e) $x[n] = 5^n e^{j2\omega n}$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]5^{(n-k)}e^{j2\omega(n-k)} \\ &= 5^n e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k]5^{-k}e^{-j2\omega k} \end{aligned}$$

YES, EIGENFUNCTION.

2.35. We first re-write the system function $H(e^{j\omega})$:

$$\begin{aligned} H(e^{j\omega}) &= e^{j\pi/4} \cdot e^{-j\omega} \left(\frac{1 + e^{-j2\omega} + 4e^{-j4\omega}}{1 + \frac{1}{2}e^{-j2\omega}} \right) \\ &= e^{j\pi/4} G(e^{j\omega}) \end{aligned}$$

Let $y_1[n] = x[n] * g[n]$, then

$$\begin{aligned} x[n] &= \cos(\frac{\pi n}{2}) = \frac{e^{j\pi n/2} + e^{-j\pi n/2}}{2} \\ y_1[n] &= \frac{G(e^{j\pi/2})e^{j\pi n/2} + G(e^{-j\pi/2})e^{-j\pi n/2}}{2} \end{aligned}$$

Evaluating the frequency response at $\omega = \pm\pi/2$:

$$\begin{aligned} G(e^{j\frac{\pi}{2}}) &= e^{-j\frac{\pi}{2}} \left(\frac{1 + e^{-j\pi} + 4e^{-j2\pi}}{1 + \frac{1}{2}e^{-j\pi}} \right) = 8e^{-j\pi/2} \\ G(e^{-j\frac{\pi}{2}}) &= 8e^{j\pi/2} \end{aligned}$$

Therefore,

$$y_1[n] = (8e^{j(\pi n/2 - \pi/2)} + 8e^{j(-\pi n/2 + \pi/2)})/2 = 8\cos(\frac{\pi}{2}n - \frac{\pi}{2})$$

and

$$y[n] = e^{j\pi/4} y_1[n] = 8e^{j\pi/4} \cos(\frac{\pi}{2}n - \frac{\pi}{2})$$

2.37

A. The impulse response $h_2[n] = 2\frac{\sin(0.5\pi n)}{\pi n}$ corresponds to a frequency response of

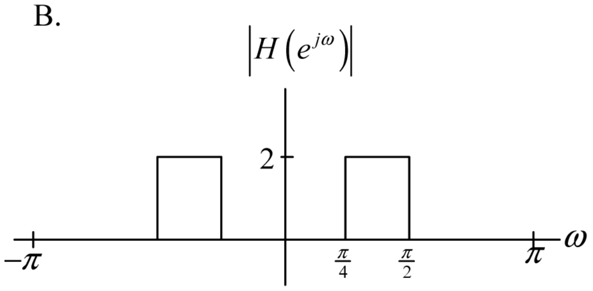
$$H_2(e^{j\omega}) = \begin{cases} 2, & \omega < \pi/2 \\ 0, & \pi/2 < \omega \le \pi \end{cases}$$

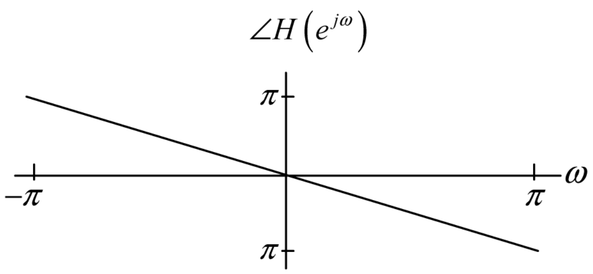
Then

$$H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})$$

$$= e^{-j\omega} \times \begin{cases} 0, & |\omega| < \pi/4 \\ 2, & \pi/4 < |\omega| < \pi/2 \\ 0, & \pi/2 < |\omega| \le \pi. \end{cases}$$

B.





C. Method 1 (Easiest):
The overall cascade system can be viewed as the difference of two lowpass filters with a one-sample delay.

$$h[n] = 2\frac{\sin\left(\frac{\pi}{2}(n-1)\right)}{\pi(n-1)} - 2\frac{\sin\left(\frac{\pi}{4}(n-1)\right)}{\pi(n-1)}$$

Method 2 (Harder):
The overall cascade system can be viewed as having a lowpass response modulated up to frequency $3\pi/8$.

$$h[n] = 4\frac{\sin\left(\frac{\pi}{8}(n-1)\right)}{\pi(n-1)}\cos\left(\frac{3\pi}{8}(n-1)\right)$$

Method 3 (Direct):
Just evaluate

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega$$

2.40. $x[n]$ is periodic with period N if $x[n] = x[n+N]$ for some integer N.

(a) $x[n]$ is periodic with period 5:

$$e^{j(\frac{2\pi}{5}n)} = e^{j(\frac{2\pi}{5})(n+N)} = e^{j(\frac{2\pi}{5}n+2\pi k)}$$

$$\Longrightarrow 2\pi k = \frac{2\pi}{5}N, \text{for integers } k, N$$

Making $k = 1$ and $N = 5$ shows that $x[n]$ has period 5.

(b) $x[n]$ is periodic with period 38. Since the sin function has period of 2π:

$$x[n+38] = \sin(\pi(n+38)/19) = \sin(\pi n/19 + 2\pi) = x[n]$$

(c) This is not periodic because the linear term n is not periodic.

(d) This is again not periodic. $e^{j\omega}$ is periodic over period 2π, so we have to find k, N such that

$$x[n+N] = e^{j(n+N)} = e^{j(n+2\pi k)}$$

Since we can make k and N integers at the same time, $x[n]$ is not periodic.

2.43

1. The Fourier transform of $x[n]$ is given by $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$. Then

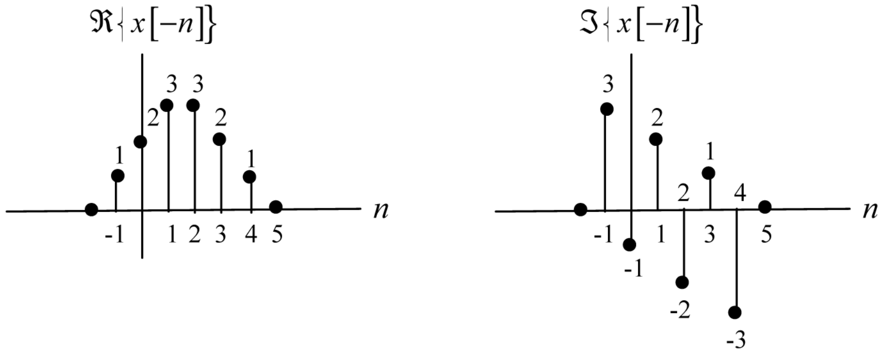
$$X(e^{j\omega})\Big|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n] = 12.$$

2. $X(e^{j\omega})\Big|_{\omega=\pi} = \sum_{n=-\infty} x[n]e^{-j\pi n} = \sum_{n=-\infty} (-1)^n x[n] = -j12.$

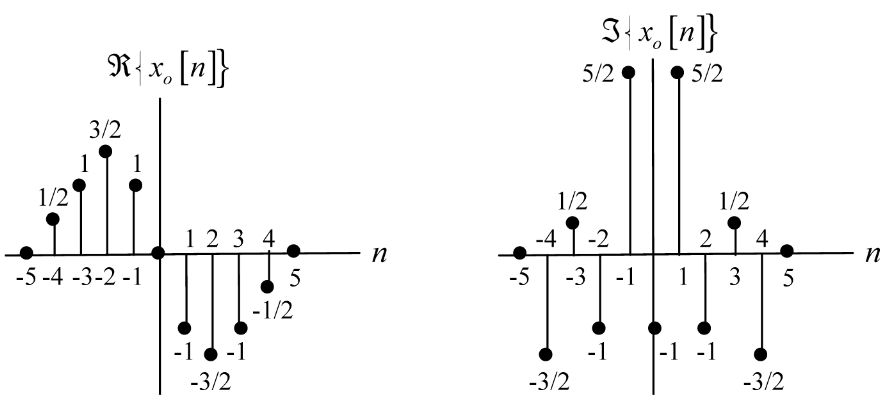
3. The inverse Fourier transform is given by $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega = x[n]$. Then

$$\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega = 2\pi x[n]$$
$$\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega 0}d\omega = 2\pi x[0]$$
$$\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 2\pi(2-j).$$

4. If $x[n] \stackrel{F}{\longleftrightarrow} X(e^{j\omega})$, then $x[-n] \stackrel{F}{\longleftrightarrow} X(e^{-j\omega})$.



5. If $x[n] \stackrel{F}{\longleftrightarrow} X(e^{j\omega})$, then $x_o[n] \stackrel{F}{\longleftrightarrow} j\Im\{X(e^{j\omega})\}$, where $x_o[n] = \frac{1}{2}(x[n] - x^*[-n])$.



2.46

$$x[n] = w[n]\cos(\omega_0 n)$$

A. Fourier transforming gives

$$\begin{aligned} X(e^{j\omega}) &= \frac{1}{2\pi} W(e^{j\omega}) * \{\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)\} \\ &= \frac{1}{2\pi}\left\{\pi W\left(e^{j(\omega-\omega_0)}\right) + \pi W\left(e^{j(\omega+\omega_0)}\right)\right\} \\ &= \tfrac{1}{2} W\left(e^{j(\omega-\omega_0)}\right) + \tfrac{1}{2} W\left(e^{j(\omega+\omega_0)}\right), \end{aligned}$$

for $-\pi < \omega \le \pi$.

B. We know from tables that if

$$y[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise,} \end{cases}$$

then the DTFT $Y(e^{j\omega})$ is

$$Y(e^{j\omega}) = \frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)} e^{-j\omega M/2}$$

Let $M = 2L$. Then we have

$$y[n] = \begin{cases} 1, & 0 \le n \le 2L \\ 0, & \text{otherwise,} \end{cases}$$

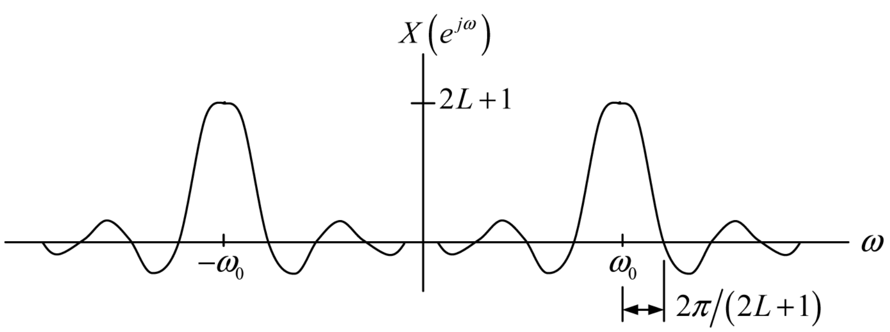
with DTFT

$$Y(e^{j\omega}) = \frac{\sin(\omega(2L+1)/2)}{\sin(\omega/2)} e^{-j\omega L}$$

Now $w[n] = y[n+L]$, which implies $W(e^{j\omega}) = Y(e^{j\omega})e^{j\omega L}$. That is,

$$W(e^{j\omega}) = \frac{\sin(\omega(2L+1)/2)}{\sin(\omega/2)}$$

C. $X(e^{j\omega}) = \tfrac{1}{2} W\left(e^{j(\omega-\omega_0)}\right) + \tfrac{1}{2} W\left(e^{j(\omega+\omega_0)}\right)$



As ω_0 gets closer to $\omega = 0$, the two peaks merge into a single peak. We will have two distinct peaks if $\omega_0 \ge \dfrac{2\pi}{2L+1}$.

2.48. (a) Suppose we form the impulse:

$$\delta[n] = \frac{1}{2}x_1[n] - \frac{1}{2}x_2[n] + x_3[n]$$

Since the system is linear,

$$L\{\delta[n]\} = \frac{1}{2}y_1[n] - \frac{1}{2}y_2[n] + y_3[n]$$

A shifted impulse results when:

$$\delta[n-1] = -\frac{1}{2}x_1[n] + \frac{1}{2}x_2[n]$$

The response to the shifted impulse

$$L\{\delta[n-1]\} = -\frac{1}{2}y_1[n] + \frac{1}{2}y_2[n]$$

Since,

$$L\{\delta[n]\} \neq L\{\delta[n-1]\}$$

The system is NOT TIME INVARIANT.

(b) An impulse may be formed:

$$\delta[n] = \frac{1}{2}x_1[n] - \frac{1}{2}x_2[n] + x_3[n]$$

since the system is linear,

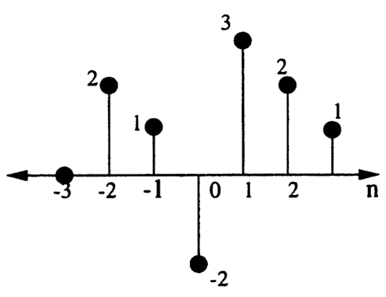
$$\begin{aligned} L\{\delta[n]\} &= \frac{1}{2}y_1[n] - \frac{1}{2}y_2[n] + y_3[n] \\ &= h[n] \end{aligned}$$

from the figure,

$$y_1[n] = -\delta[n+1] + 3\delta[n] + 3\delta[n-1] + \delta[n-3]$$
$$y_2[n] = -\delta[n+1] + \delta[n] - 3\delta[n-1] - \delta[n-3]$$
$$y_3[n] = 2\delta[n+2] + \delta[n+1] - 3\delta[n] + 2\delta[n-2]$$

Combining:

$$\begin{aligned} h[n] &= 2\delta[n+2] + \delta[n+1] - 2\delta[n] + 3\delta[n-1] \\ &\quad +2\delta[n-2] + \delta[n-3] \end{aligned}$$



2.50. (a) For $x_1[n] = \delta[n]$,

$$\begin{aligned} y_1[0] &= 1 \\ y_1[1] &= ay[0] = a \end{aligned}$$

For $x_2[n] = \delta[n-1]$,

$$\begin{aligned} y_2[0] &= 1 \\ y_2[1] &= ay[0] + x_2[1] = a + 1 \neq y_1[0] \end{aligned}$$

Even though $x_2[n] = x_1[n-1]$, $y_2[n] \neq y_2[n-1]$. Hence the system is NOT TIME INVARIANT.

(b) A linear system has the property that

$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\}\}$$

Hence, if the input is doubled, the output must also double at each value of n.
Because $y[0] = 1$, always, the system is NOT LINEAR.

(c) Let $x_3 = \alpha x_1[n] + \beta x_2[n]$.
For $n \geq 0$:

$$\begin{aligned} y_3[n] &= x_3[n] + ay_3[n-1] \\ &= \alpha x_1[n] + \beta x_2[n] + a(x_3[n-1] + y_3[n-2]) \\ &= \alpha \sum_{k=0}^{n-1} a^k x_1[n-k] + \beta \sum_{k=0}^{n-1} a^k x_2[n-k] \\ &= \alpha(h[n] * x_1[n]) + \beta(h[n] * x_2[n]) \\ &= \alpha y_1[n] + \beta y_2[n]. \end{aligned}$$

For $n < 0$:

$$\begin{aligned} y_3[n] &= a^{-1}(y_3[n+1] - x_3[n]) \\ &= -\alpha \sum_{k=-1}^{n} a^k x_1[n-k] - \beta \sum_{k=-1}^{n} a^k x_2[n-k] \\ &= \alpha y_1[n] + \beta y_2[n]. \end{aligned}$$

For $n = 0$:

$$y_3[n] = y_1[n] = y_2[n] = 0.$$

Conclude,

$$y_3[n] = \alpha y_1[n] + \beta y_2[n], \text{ for all } n.$$

Therefore, the system is LINEAR. The system is still NOT TIME INVARIANT.

2.62.

$$\begin{aligned} x[n] &= \cos(\frac{15\pi n}{4} - \frac{\pi}{3}) \\ &= \cos(-\frac{\pi n}{4} - \frac{\pi}{3}) \\ &= \cos(\frac{\pi n}{4} + \frac{\pi}{3}) \\ &= \frac{e^{j\frac{\pi}{3}} e^{j\frac{\pi n}{4}}}{2} + \frac{e^{-j\frac{\pi}{3}} e^{-j\frac{\pi n}{4}}}{2}. \end{aligned}$$

Using the fact that complex exponentials are eigenfunctions of LTI systems, we get:

$$\begin{aligned} y[n] &= e^{-j\frac{3\pi}{8}} \frac{e^{j\frac{\pi}{3}} e^{j\frac{\pi n}{4}}}{2} + e^{-j\frac{\pi}{8}} \frac{e^{-j\frac{\pi}{3}} e^{-j\frac{\pi n}{4}}}{2} \\ &= \frac{e^{-j\frac{\pi}{24}} e^{j\frac{\pi n}{4}}}{2} + \frac{e^{-j\frac{11\pi}{24}} e^{-j\frac{\pi n}{4}}}{2} \\ &= e^{-j\frac{\pi}{4}} (\frac{e^{j\frac{5\pi}{24}} e^{j\frac{\pi n}{4}}}{2} + \frac{e^{-j\frac{5\pi}{24}} e^{-j\frac{\pi n}{4}}}{2}) \\ &= e^{-j\frac{\pi}{4}} \cos(\frac{\pi n}{4} + \frac{5\pi}{24}). \end{aligned}$$

2.67. **(a)** **Note that** $x_2[n] = -\sum_{k=0}^{k=4} x[n-k]$. **Since the system is LTI, we have:**

$$y_2[n] = -\sum_{k=0}^{k=4} y[n-k].$$

(b) **By carrying out the convolution, we get:**

$$h[n] = \begin{cases} -1 & , \quad n = 0, n = 2 \\ -2 & , \quad n = 1 \\ 0 & , \quad \text{o.w.} \end{cases}$$