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5.1.

$$y[n] = \begin{cases} 1, & 0 \le n \le 10, \\ 0, & \text{otherwise} \end{cases}$$

Therefore,

$$Y(e^{j\omega}) = e^{-j5\omega} \frac{\sin \frac{11}{2}\omega}{\sin \frac{\omega}{2}}$$

This $Y(e^{j\omega})$ is full band. Therefore, since $Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$, the only possible $x[n]$ and ω_c that could produce $y[n]$ is $x[n] = y[n]$ and $\omega_c = \pi$.

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5.4. (a)

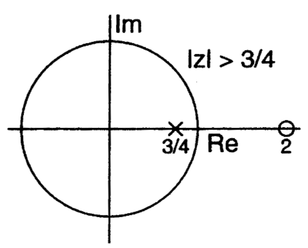
$$x[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[-n-1]$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{z}{z-2}, \qquad \frac{1}{2} < |z| < 2$$

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]$$

$$Y(z) = \frac{6}{1-\frac{1}{2}z^{-1}} - \frac{6}{1-\frac{3}{4}z^{-1}}, \qquad \frac{3}{4} < |z|$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})} \cdot \frac{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}{-\frac{3}{2}z^{-1}} = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}, \qquad |z| > \frac{3}{4}$$



(b)

$$H(z) = \frac{1}{1-\frac{3}{4}z^{-1}} - \frac{2z^{-1}}{1-\frac{3}{4}z^{-1}}, \qquad |z| > \frac{3}{4}$$

$$h[n] = \left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{3}{4}\right)^{n-1} u[n-1]$$

(c)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$$

$$Y(z) - \frac{3}{4}z^{-1}Y(z) = X(z) - 2z^{-1}X(z)$$

$$y[n] - \frac{3}{4}y[n-1] = x[n] - 2x[n-1]$$

(d) **The ROC is outside $|z| = \frac{3}{4}$, which includes the unit circle. Therefore the system is stable. The $h[n]$ we found in part (b) tells us the system is also causal.**

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5.6. (a)

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}(2)^n u[-n-1]$$

$$X(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{4}{3}}{1-2z^{-1}} = \frac{1}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}, \quad \frac{1}{2} < |z| < 2$$

(b)

$$Y(z) = \frac{1-z^{-2}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

This has the same poles as the input, therefore the ROC is still $\frac{1}{2} < |z| < 2$.

(c)

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-2} \Leftrightarrow h[n] = \delta[n] - \delta[n-2]$$

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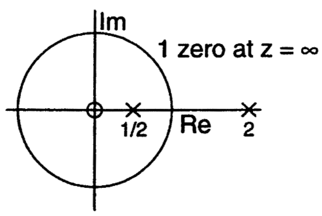
5.9.

$$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n]$$

$$z^{-1}Y(z) - \frac{5}{2}Y(z) + zY(z) = X(z)$$

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} \\ &= \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}} \\ &= \frac{z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})} \\ &= \frac{\frac{2}{3}}{1-2z^{-1}} - \frac{\frac{2}{3}}{1-\frac{1}{2}z^{-1}} \end{aligned}$$



Three regions of convergence:

(a) $|z| < \frac{1}{2}$:

$$h[n] = -\frac{2}{3}(2)^n u[-n-1] + \frac{2}{3}\left(\frac{1}{2}\right)^n u[-n-1]$$

(b) $\frac{1}{2} < |z| < 2$:

$$h[n] = -\frac{2}{3}(2)^n u[-n-1] - \frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$$

Includes $|z| = 1$, so this is stable.

(c) $|z| > 2$:

$$h[n] = \frac{2}{3}(2)^n u[n] - \frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$$

ROC outside of largest pole, so this is causal.

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5.12. (a) *Yes.* The poles $z = \pm j(0.9)$ are inside the unit circle so the system is stable.

(b) First, factor $H(z)$ into two parts. The first should be minimum phase and therefore have all its poles and zeros inside the unit circle. The second part should contain the remaining poles and zeros.

$$H(z) = \underbrace{\frac{1+0.2z^{-1}}{1+0.81z^{-2}}}_{\text{minimum phase}} \qquad \underbrace{\frac{1-9z^{-2}}{1}}_{\substack{\text{poles \& zeros}\\ \text{outside unit circle}}}$$

Allpass systems have poles and zeros that occur in conjugate reciprocal pairs. If we include the factor $(1-\frac{1}{9}z^{-2})$ in both parts of the equation above the first part will remain minimum phase and the second will become allpass.

$$\begin{aligned} H(z) &= \frac{(1+0.2z^{-1})(1-\frac{1}{9}z^{-2})}{1+0.81z^{-2}} \cdot \frac{1-9z^{-2}}{1-\frac{1}{9}z^{-2}} \\ &= H_1(z)H_{ap}(z) \end{aligned}$$

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5.13. *An aside*: Technically, this problem is not well defined, since a pole/zero plot does not uniquely determine a system. That is, many system functions can have the same pole/zero plot. For example, consider the systems

$$H_1(z) = z^{-1}$$
$$H_2(z) = 2z^{-1}$$

Both of these systems have the same pole/zero plot, namely a pole at zero and a zero at infinity. Clearly, the system $H_1(z)$ is allpass, as it passes all frequencies with unity gain (it is simply a unit delay). However, one could ask whether $H_2(z)$ is allpass. Looking at the standard definition of an allpass system provided in this chapter, the answer would be no, since the system does not pass all frequencies with *unity* gain.

A broader definition of an allpass system would be a system for which the system magnitude response $|H(e^{j\omega})| = a$, where a is a real constant. Such a system would pass all frequencies, and scale the output by a constant factor a. In a practical setting, this definition of an allpass system is satisfactory. Under this definition, both systems $H_1(z)$ and $H_2(z)$ would be considered allpass.

For this problem, it is assumed that none of the poles or zeros shown in the pole/zero plots are scaled, so this issue of using the proper definition of an allpass system does not apply. The standard definition of an allpass system is used.

Solution:

(a) Yes, the system is allpass, since it is of the appropriate form.

(b) No, the system is not allpass, since the zero does not occur at the conjugate reciprocal location of the pole.

(c) Yes, the system is allpass, since it is of the appropriate form.

(d) Yes, the system is allpass. This system consists of an allpass system in cascade with a pole at zero. The pole at zero is simply a delay, and does not change the magnitude spectrum.

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5.17. A minimum phase system is one which has all its poles and zeros inside the unit circle. It is causal, stable, and has a causal and stable inverse.

(a) $H_1(z)$ has a zero outside the unit circle at $z = 2$ so it is not minimum phase.

(b) $H_2(z)$ is minimum phase since its poles and zeros are inside the unit circle.

(c) $H_3(z)$ is minimum phase since its poles and zeros are inside the unit circle.

(d) $H_4(z)$ has a zero outside the unit circle at $z = \infty$ so it is not minimum phase. Moreover, the inverse system would not be causal due to the pole at infinity.

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5.20. (a) *Yes.* The system function could be a generalized linear phase system implemented by a linear constant-coefficient differential equation (LCCDE) with real coefficients. The zeros come in a set of four: a zero, its conjugate, and the two conjugate reciprocals. The pole-zero plot could correspond to a Type I FIR linear phase system.

(b) *No.* This system function could not be a generalized linear phase system implemented by a LCCDE with real coefficients. Since the LCCDE has real coefficients, its poles and zeros must come in conjugate pairs. However, the zeros in this pole-zero plot do not have corresponding conjugate zeros.

(c) *Yes.* The system function could be a generalized linear phase system implemented by a LCCDE with real coefficients. The pole-zero plot could correspond to a Type II FIR linear phase system.

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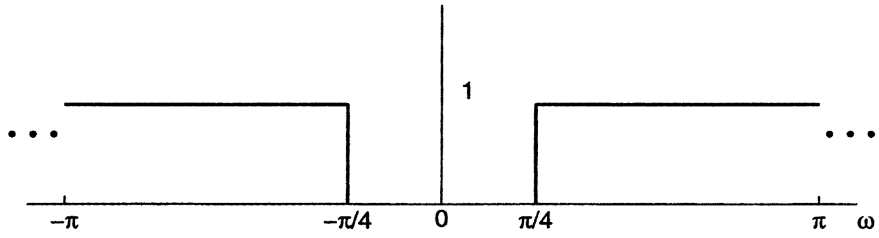
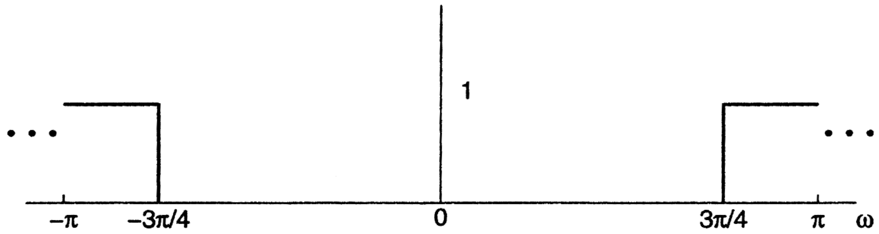
5.21. $h_{lp}[n]$ is an ideal lowpass filter with $\omega_c = \frac{\pi}{4}$

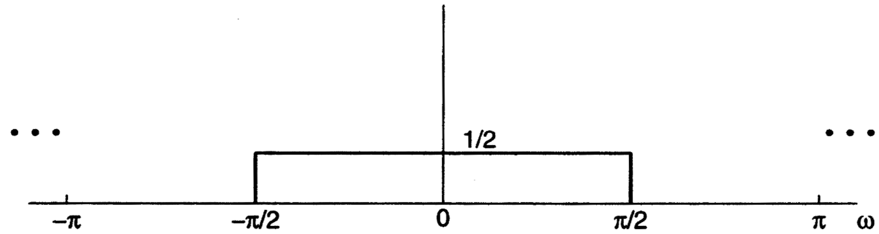
(a) $y[n] = x[n] - x[n] * h_{lp}[n] \Rightarrow H(e^{j\omega}) = 1 - H_{lp}(e^{j\omega})$
This is a highpass filter.



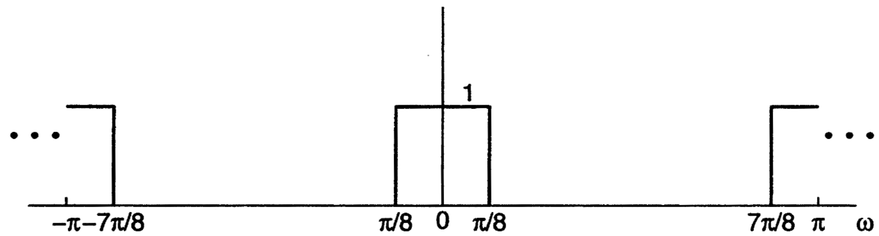
(b) $x[n]$ is first modulated by π, lowpass filtered, and demodulated by π. Therefore, $H_{lp}(e^{j\omega})$ filters the high frequency components of $X(e^{j\omega})$.
This is a highpass filter.



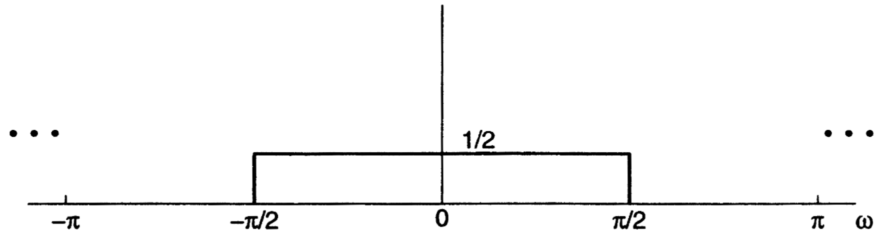
(c) $h_{lp}[2n]$ is a downsampled version of the filter. Therefore, the frequency response will be "spread out" by a factor of two, with a gain of $\frac{1}{2}$.
This is a lowpass filter.



(d) This system upsamples $h_{lp}[n]$ by a factor of two. Therefore, the frequency axis will be compressed by a factor of two.
This is a bandstop filter.



(e) This system upsamples the input before passing it through $h_{lp}[n]$. This effectively doubles the frequency bandwidth of $H_{lp}(e^{j\omega})$.
This is a lowpass filter.



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5.23.

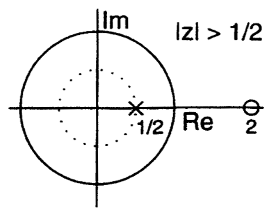
$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}} = \frac{Y(z)}{X(z)}, \qquad \text{causal, so ROC is } |z| > a$$

(a) Cross multiplying and taking the inverse transform

$$y[n] - ay[n-1] = x[n] - \frac{1}{a}x[n-1]$$

(b) Since $H(z)$ is causal, we know that the ROC is $|z| > a$. For stability, the ROC must include the unit circle. So, $H(z)$ is stable for $|a| < 1$.

(c) $a = \frac{1}{2}$



(d)

$$H(z) = \frac{1}{1 - az^{-1}} - \frac{a^{-1}z^{-1}}{1 - az^{-1}}, \qquad |z| > a$$

$$h[n] = (a)^n u[n] - \frac{1}{a}(a)^{n-1}u[n-1]$$

(e)

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = \frac{1 - a^{-1}e^{-j\omega}}{1 - ae^{-j\omega}}$$

$$|H(e^{j\omega})|^2 = H(e^{j\omega})H^*(e^{j\omega}) = \frac{1 - a^{-1}e^{-j\omega}}{1 - ae^{-j\omega}} \cdot \frac{1 - a^{-1}e^{j\omega}}{1 - ae^{j\omega}}$$

$$\begin{aligned} |H(e^{j\omega})| &= \left(\frac{1 + \frac{1}{a^2} - \frac{2}{a}\cos\omega}{1 + a^2 - 2a\cos\omega}\right)^{\frac{1}{2}} \\ &= \frac{1}{a}\left(\frac{a^2 + 1 - 2a\cos\omega}{1 + a^2 - 2a\cos\omega}\right)^{\frac{1}{2}} \\ &= \frac{1}{a} \end{aligned}$$

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5.29

A.

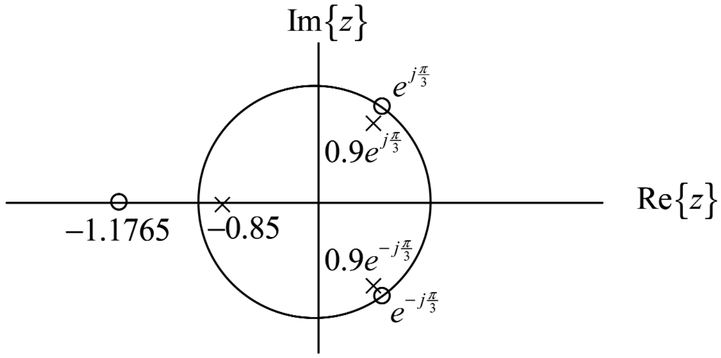
$$H(z)=\frac{\left(1-e^{j\frac{\pi}{3}}z^{-1}\right)\left(1-e^{-j\frac{\pi}{3}}z^{-1}\right)\left(1+1.1765z^{-1}\right)}{\left(1-0.9e^{j\frac{\pi}{3}}z^{-1}\right)\left(1-0.9e^{-j\frac{\pi}{3}}z^{-1}\right)\left(1+0.85z^{-1}\right)}$$

$$=\frac{1+0.1765z^{-1}-0.1765z^{-2}+1.1765z^{-3}}{1-0.05z^{-1}+0.045z^{-2}+0.6885z^{-3}}$$

$$=\frac{Y(z)}{X(z)}.$$

$$y[n]=0.05y[n-1]-0.45y[n-2]-0.6885y[n-3]$$
$$+x[n]+0.1765x[n-1]-0.1765x[n-2]+1.1765x[n-3].$$

B.



Since the system is causal, the ROC is the region outside the outermost pole.
$|z|>0.9$.

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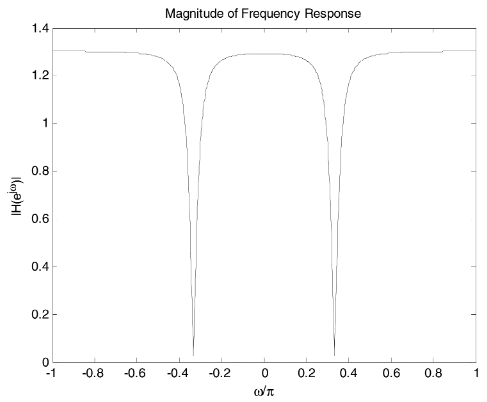
C.



The zeros on the unit circle null the frequency response at $\omega = \pm\pi/3$. The sharpness of the nulls depend on how close the nearby poles are to the zeros. The factor $\frac{1+1.1765z^{-1}}{1+0.85z^{-1}} = 1.1765\frac{z^{-1}+0.85}{1+0.85z^{-1}}$ is allpass and does not affect the magnitude response.

D. 1. True. The system is stable because the ROC contains the unit circle.

2. False. The impulse response must approach zero for large n because the system is stable.

3. False. The system function has a zero on the unit circle at $\omega = \pi/3$. This negates the effect of the pole, and since the pole is not on the unit circle, the pole does not cancel the zero. Instead, the sharpness of the notch depends on how close the pole is to the zero.

4. False. There is a zero outside the unit circle.

5. False. The system is not a minimum-phase system so it does not have a causal and stable inverse.

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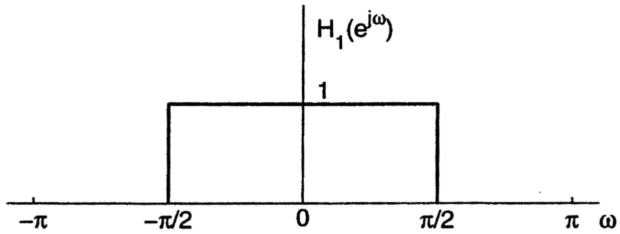
5.30. Making use of some DTFT properties can aide in the solution of this problem. First, note that

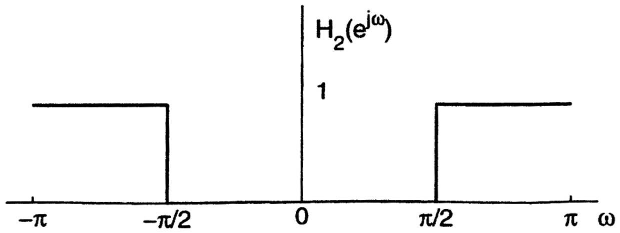
$$\begin{aligned} h_2[n] &= (-1)^n h_1[n] \\ h_2[n] &= e^{-j\pi n} h_1[n] \end{aligned}$$

Using the DTFT property that states that modulation in the time domain corresponds to a shift in the frequency domain,

$$H_2(e^{j\omega}) = H_1(e^{j(\omega+\pi)})$$

Consequently, $H_2(e^{j\omega})$ is simply $H_1(e^{j\omega})$ shifted by π. The ideal low pass filter has now become the ideal high pass filter, as shown below.





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5.34. Appears in: Fall05 PS1, Spring05 PS1, Fall04 PS1, Fall02 PS1, Spring01 PS2. Note: Spring01 PS2 uses different plots than Fall04 and Fall02. The problem statement in Spring01 has also been modified for Fall02 and Fall04. The Spring01 version of the problem is included after the Fall04 and Fall02 version.

Problem

Filter A is a discrete-time LTI system with input $x[n]$ and output $y[n]$.

The frequency response magnitude and group delay functions for Filter A are shown in Figure 1. The signal $x[n]$, also shown in Figure 1, is the sum of three narrowband pulses. In particular, Figure 1 contains the following plots:

- $x[n]$.
- $|X(e^{j\omega})|$, the Fourier transform magnitude of $x[n]$.
- Frequency response magnitude plot for filter A.
- Group delay plot for filter A.

In Figure 2 you are given 4 possible output signals, $y_i[n]$ $i = 1, 2, \ldots, 4$. Determine which one of the possible output signals is the output of filter A when the input is $x[n]$. Provide a justification for your choice.

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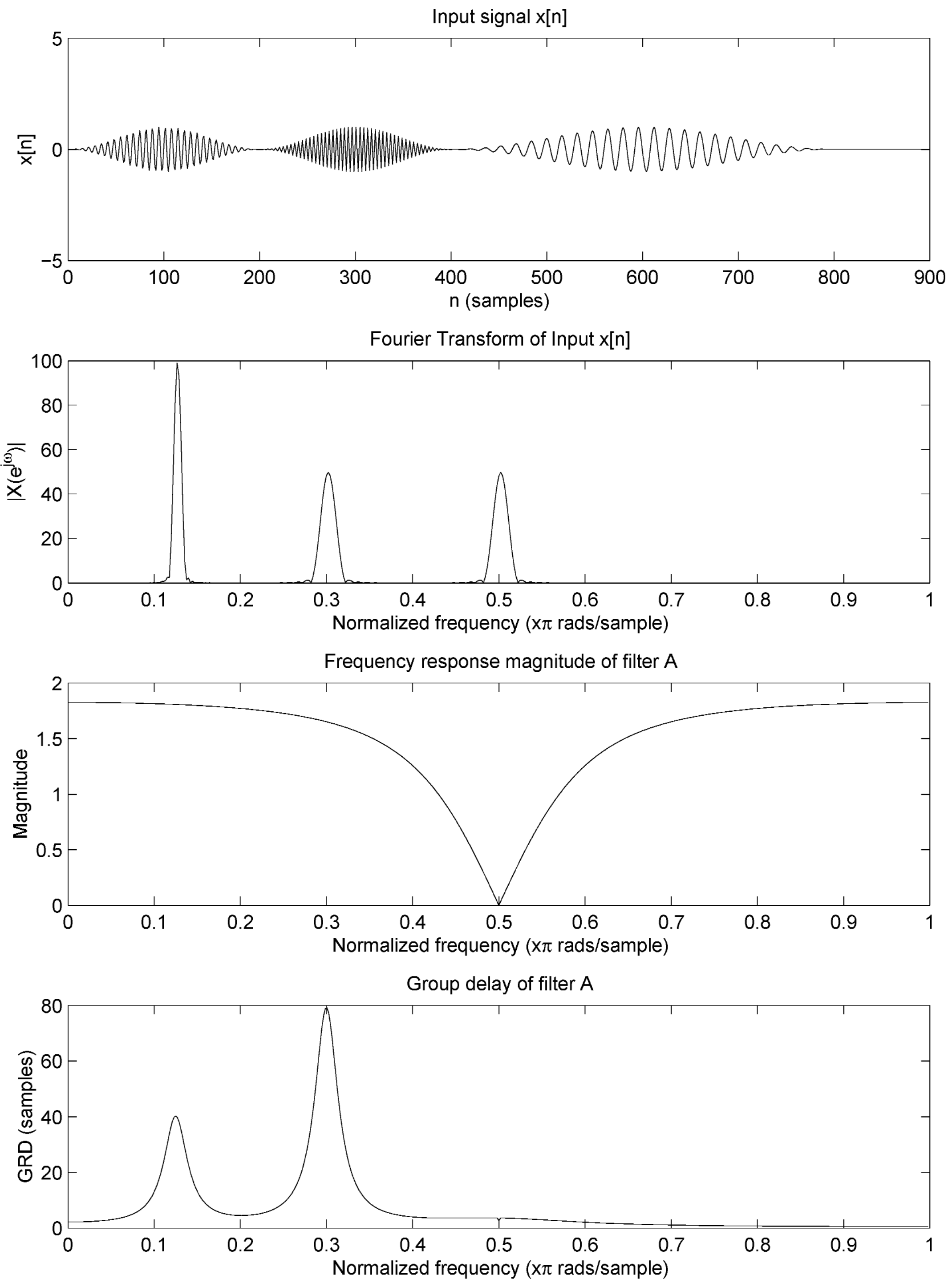

Figure 1: The signal and the filter for Problem

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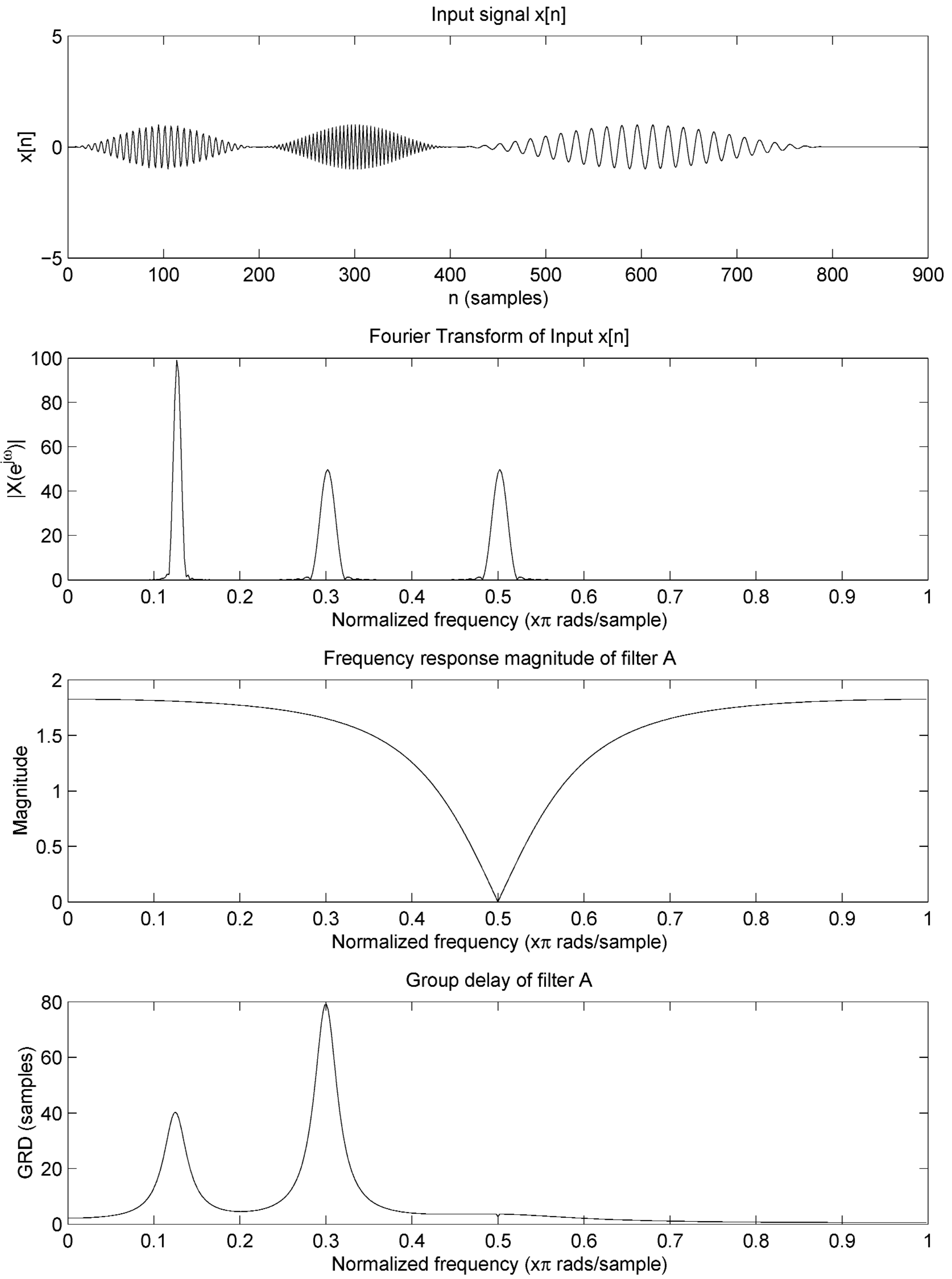

Figure 1: The signal and the filter for Problem

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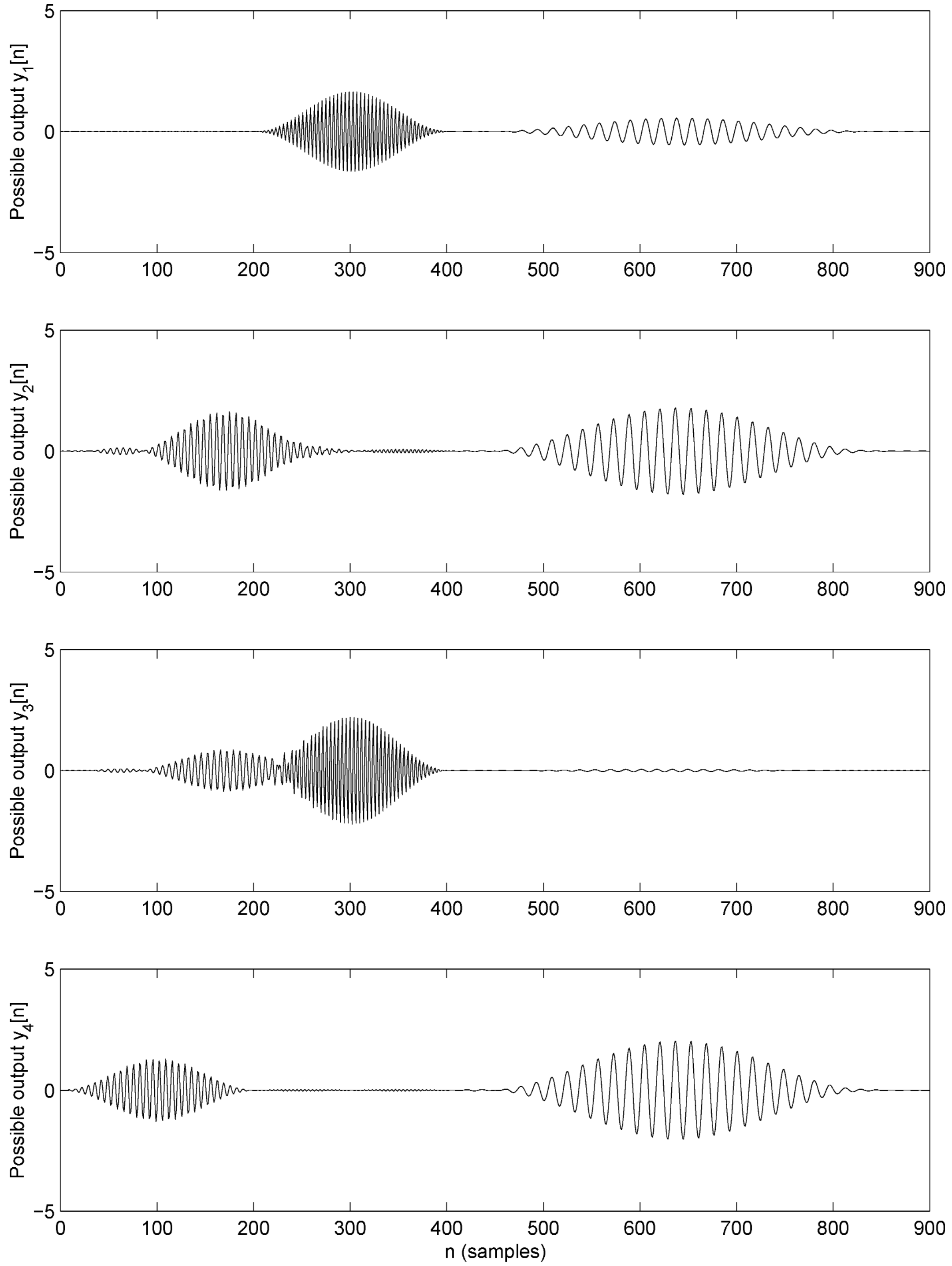

Figure 2: Possible output signals for Problem.

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Spring01 Version of Problem Filter A is a discrete-time LTI system. Its frequency response magnitude and group delay functions are shown in Figure 2.6a. A signal, $x[n]$, also shown in Figure 2.6a, is the sum of three narrowband pulses which do not overlap in time. In Figures 2.6b and 2.6c you are given 8 possible output signals, $y_i[n]$ $i = 1, 2, \ldots, 8$. Determine which of the possible output signals is the output of filter A when the input is $x[n]$. **Clearly state your reasoning.**

Figure 2.6a contains the following plots:

- $x[n]$.
- $|X(e^{j\omega})|$, the Fourier transform magnitude of $x[n]$.
- Group delay plot for filter A.
- Frequency response magnitude plot for filter A.

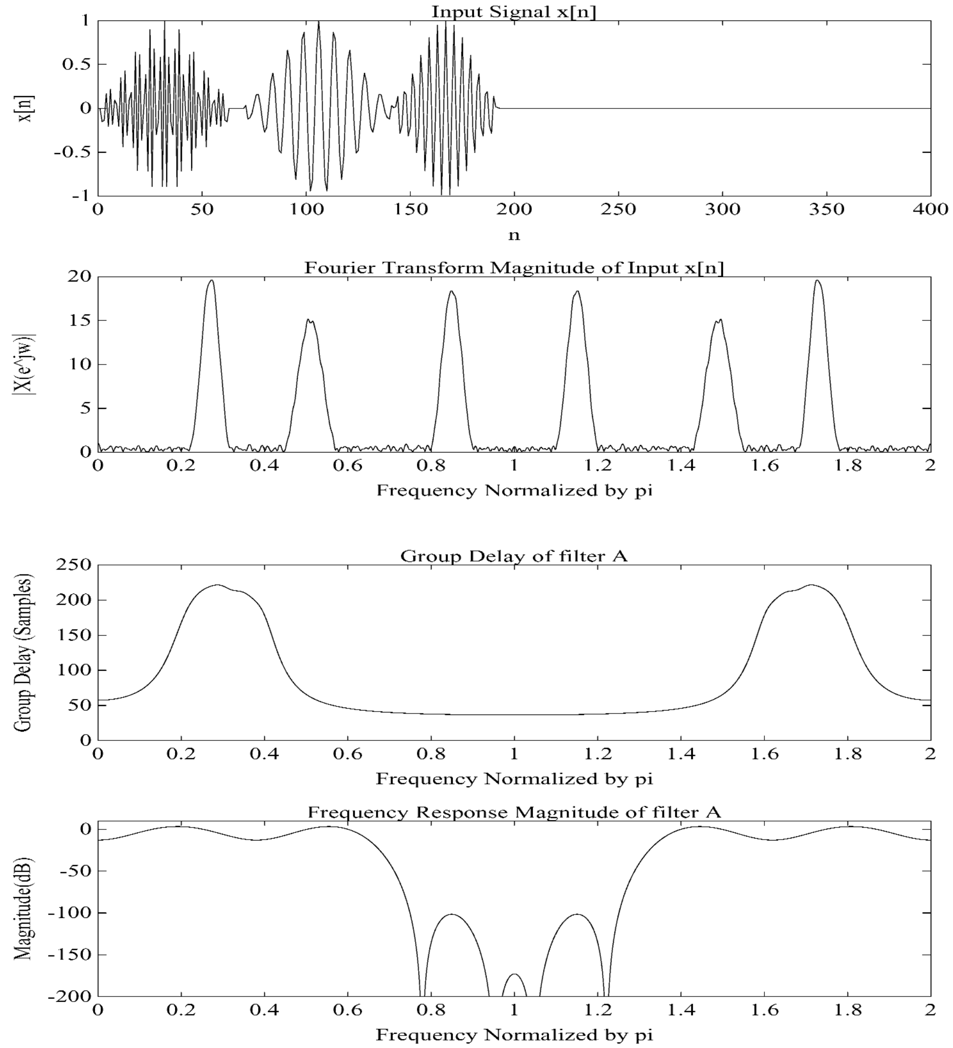

Figure 2.6a

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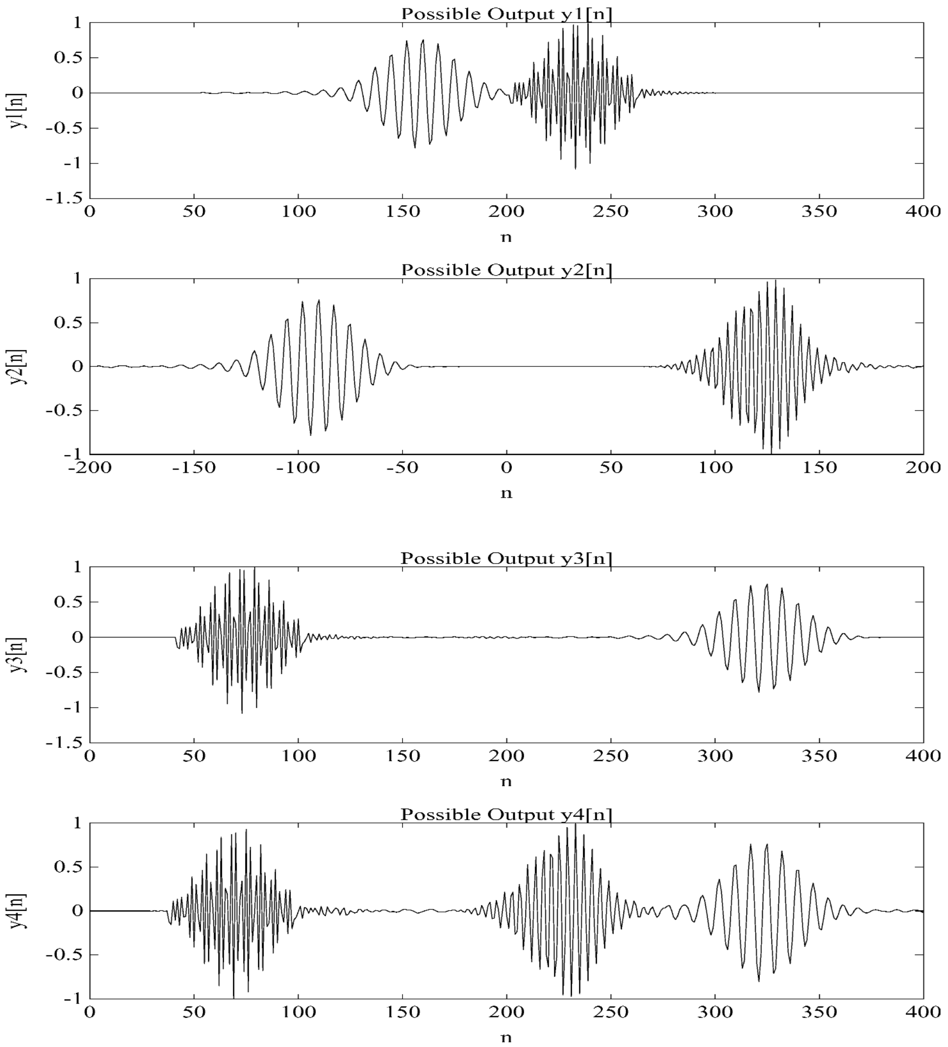

Figure 2.6b

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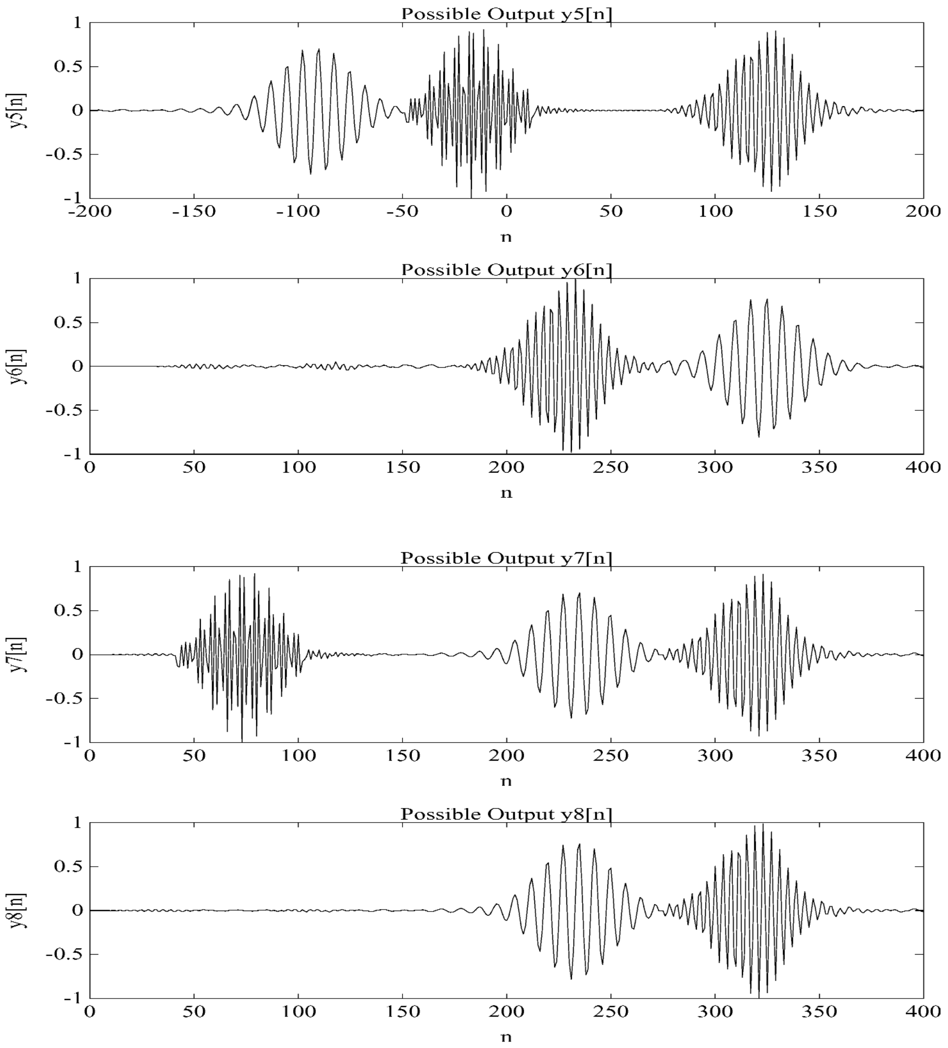

Figure 2.6c

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Solution from Fall05 PS1

$y[n] = y_2[n]$

Justification:

The input signal $x[n]$ is made up of three narrow-band pulses: pulse-1 is a low-frequency pulse (whose peak is around 0.12π radians), pulse-2 is a higher-frequency pulse (0.3π radians), and pulse-3 is the highest-frequency pulse (0.5π radians).

Let $H(e^{j\omega})$ be the frequency response of Filter A. We read off the following values from the frequency response magnitude and group delay plots:

$$\begin{aligned}
|H(e^{j(0.12\pi)})| &\approx 1.8 \\
|H(e^{j(0.3\pi)})| &\approx 1.7 \\
|H(e^{j(0.5\pi)})| &\approx 0 \\
\tau_g(0.12\pi) &\approx 40 \text{ samples} \\
\tau_g(0.3\pi) &\approx 80 \text{ samples}
\end{aligned}$$

From these values, we would expect pulse-3 to be totally absent from the output signal $y[n]$. Pulse-1 will be scaled up by a factor of 1.8 and its envelope delayed by about 40 samples. Pulse-2 will be scaled up by a factor of 1.7 and its envelope delayed by about 80 samples. The correct output is thus $y_2[n]$.

Solution from Spring05 PS1

$y[n] = y_2[n]$

Justification:

We see that the input signal $x[n]$ is made up of three narrow-band pulses; pulse-1 is a low-frequency pulse (whose peak is at $.12\pi$ radians) pulse-2 is a higher-frequency pulse (whose peak is at $.3\pi$ radians), and pulse-3 is the highest-frequency pulse (whose peak is at $.5\pi$ radians).

From the given figure, we can also read off the following values of the filters frequency response magnitude and group delay. Call $H(e^{j\omega})$ the frequency response magnitude of Filter A. Then

$$\begin{aligned}
|H(e^{j(.12\pi)})| &\approx 1.8 \\
|H(e^{j(.3\pi)})| &\approx 1.75 \\
|H(e^{j(.5\pi)})| &\approx 0 \\
\tau_g(.12\pi) &\approx 40 \text{ samples} \\
\tau_g(.3\pi) &\approx 80 \text{ samples}
\end{aligned}$$

From these values, we would expect pulse-3 to be totally absent from the output signal $y[n]$. We would expect pulse-1 to be scaled up by a factor of 1.8, and its envelope delayed by about 40 samples. Pulse-2 will be scaled up by a factor of 1.75, with its envelope delayed by about 80 samples. The output which corresponds to this is $y_2[n]$.

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Solution from Fall04 PS1

$y[n] = y_2[n]$

Justification:

We see that the input signal $x[n]$ is made up of three narrow-band pulses; pulse-1 is a low-frequency pulse of frequency $.12\pi$ radians, pulse-2 is a higher-frequency pulse of frequency $.3\pi$ radians, and pulse-3 is the highest-frequency pulse of frequency $.5\pi$ radians.

From the given figure, we can also read off the following values of the filters frequency response magnitude and group delay. Call $H(e^{j\omega})$ the frequency response magnitude of Filter A. Then

$$\begin{aligned}
|H(e^{j(.12\pi)})| &\approx 1.8 \\
|H(e^{j(.3\pi)})| &\approx 1.75 \\
|H(e^{j(.5\pi)})| &\approx 0 \\
\tau_g(.12\pi) &\approx 40 \text{ samples} \\
\tau_g(.3\pi) &\approx 80 \text{ samples}
\end{aligned}$$

From these values, we would expect pulse-3 to be totally absent from the output signal $y[n]$. We would expect pulse-1 to be scaled up by a factor of 1.8, and its envelope delayed by about 40 samples. Pulse-2 will be scaled up by a factor of 1.75, with its envelope delayed by about 80 samples. The output which corresponds to this is $y_2[n]$.

Solution from Fall02 PS1

$y[n] = y_2[n]$

Justification:

We see that the input signal $x[n]$ is made up of three narrow-band pulses; pulse-1 is a low-frequency pulse of frequency $.12\pi$ radians, pulse-2 is a higher-frequency pulse of frequency $.3\pi$ radians, and pulse-3 is the highest-frequency pulse of frequency $.5\pi$ radians.

From the given figure, we can also read off the following values of the filters frequency response magnitude and group delay. Call $H(e^{j\omega})$ the frequency response magnitude of Filter A. Then

$$\begin{aligned}
|H(e^{j(.12\pi)})| &\approx 1.8 \\
|H(e^{j(.3\pi)})| &\approx 1.75 \\
|H(e^{j(.5\pi)})| &\approx 0 \\
\tau_g(.12\pi) &\approx 40 \text{ samples} \\
\tau_g(.3\pi) &\approx 80 \text{ samples}
\end{aligned}$$

From these values, we would expect pulse-3 to be totally absent from the output signal $y[n]$. We would expect pulse-1 to be scaled up by a factor of 1.8, and its envelope delayed by about 40 samples. Pulse-2 will be scaled up by a factor of 1.75, with its envelope delayed by about 80 samples. The output which corresponds to this is $y_2[n]$.

Solution from Spring01 PS2

N/A

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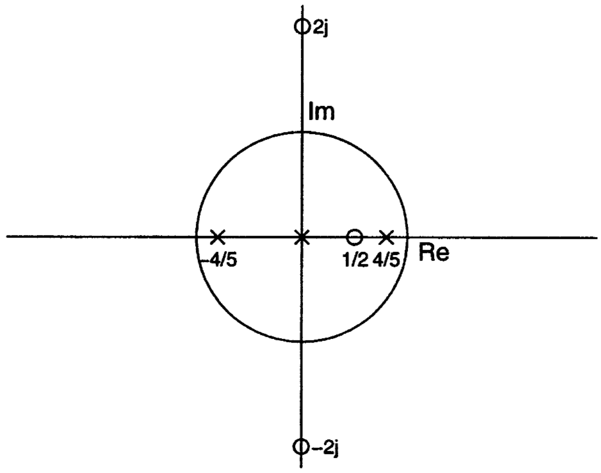
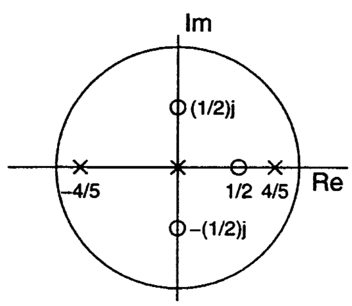
5.38.

$$H(z) = \frac{(1-0.5z^{-1})(1+2jz^{-1})(1-2jz^{-1})}{(1-0.8z^{-1})(1+0.8z^{-1})}$$

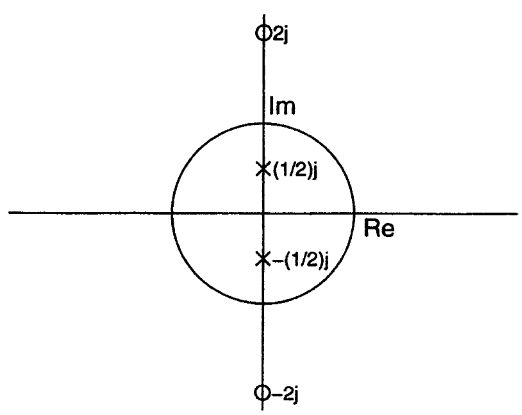


(a) A minimum phase system has all poles and zeros inside $|z| = 1$

$$H_1(z) = \frac{(1-0.5z^{-1})(1+\frac{1}{4}z^{-2})}{(1-0.64z^{-2})}$$



$$H_{ap}(z) = \frac{(1+4z^{-2})}{(1+\frac{1}{4}z^{-2})}$$



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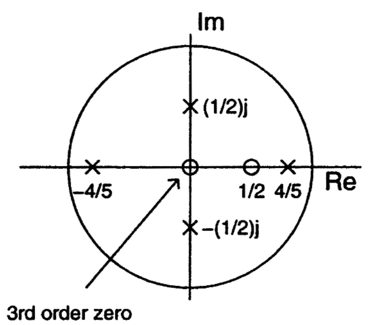
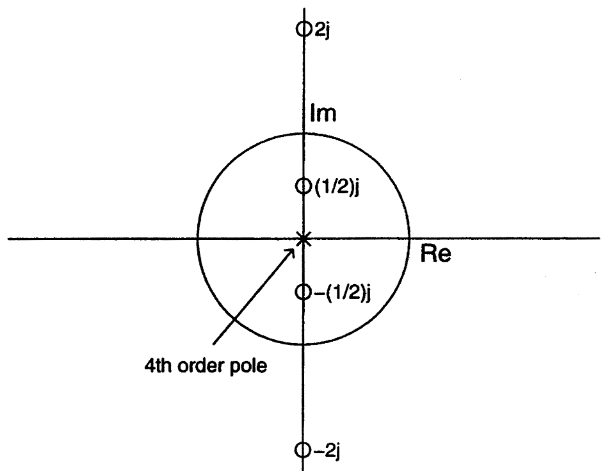
(b) A generalized linear phase system has zeros and poles at $z = 1, -1, 0$ or ∞ or in conjugate reciprocal pairs.

$$H_2(z) = \frac{(1 - 0.5z^{-1})}{(1 - 0.64z^{-2})(1 + \frac{1}{4}z^{-2})}$$



$$H_{lin}(z) = (1 + \tfrac{1}{4}z^{-2})(1 + 4z^{-2})$$



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5.41. (a)

Property	Applies?	Comments
Stable	No	For a stable, causal system, all poles must be inside the unit circle.
IIR	Yes	The system has poles at locations other than $z = 0$ or $z = \infty$.
FIR	No	FIR systems can only have poles at $z = 0$ or $z = \infty$.
Minimum Phase	No	Minimum phase systems have all poles and zeros located inside the unit circle.
Allpass	No	Allpass systems have poles and zeros in conjugate reciprocal pairs.
Generalized Linear Phase	No	The causal generalized linear phase systems presented in this chapter are FIR.
Positive Group Delay for all w	No	This system is not in the appropriate form.

(b)

Property	Applies?	Comments
Stable	Yes	The ROC for this system function, $\|z\| > 0$, contains the unit circle. (Note there is 7th order pole at $z = 0$).
IIR	No	The system has poles only at $z = 0$.
FIR	Yes	The system has poles only at $z = 0$.
Minimum Phase	No	By definition, a minimum phase system must have all its poles and zeros located *inside* the unit circle.
Allpass	No	Note that the zeros on the unit circle will cause the magnitude spectrum to drop zero at certain frequencies. Clearly, this system is not allpass.
Generalized Linear Phase	Yes	This is the pole/zero plot of a type II FIR linear phase system.
Positive Group Delay for all w	Yes	This system is causal and linear phase. Consequently, its group delay is a positive constant.

(c)

Property	Applies?	Comments
Stable	Yes	All poles are inside the unit circle. Since the system is causal, the ROC includes the unit circle.
IIR	Yes	The system has poles at locations other than $z = 0$ or $z = \infty$.
FIR	No	FIR systems can only have poles at $z = 0$ or $z = \infty$.
Minimum Phase	No	Minimum phase systems have all poles and zeros located inside the unit circle.
Allpass	Yes	The poles inside the unit circle have corresponding zeros located at conjugate reciprocal locations.
Generalized Linear Phase	No	The causal generalized linear phase systems presented in this chapter are FIR.
Positive Group Delay for all w	Yes	Stable allpass systems have positive group delay for all w.

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5.43. (a) To be rational, X(z) must be of the form

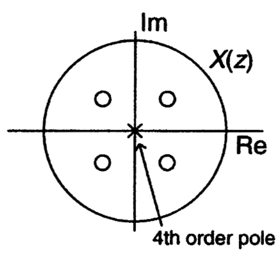
$$X(z) = \frac{b_0}{a_0} \frac{\prod_{k=1}^{M}(1 - c_k z^{-1})}{\prod_{k=1}^{N}(1 - d_k z^{-1})}$$

Because x[n] is real, its zeros must appear in conjugate pairs. Consequently, there are two more zeros, at $z = \frac{1}{2}e^{-j\pi/4}$, and $z = \frac{1}{2}e^{-j3\pi/4}$. Since x[n] is zero outside $0 \le n \le 4$, there are only four zeros (and poles) in the system function. Therefore, the system function can be written as

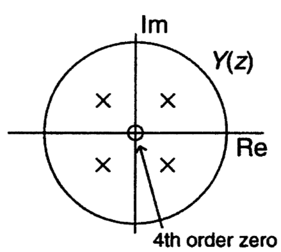
$$X(z) = \left(1 - \frac{1}{2}e^{j\pi/4}z^{-1}\right)\left(1 - \frac{1}{2}e^{j3\pi/4}z^{-1}\right)\left(1 - \frac{1}{2}e^{-j\pi/4}z^{-1}\right)\left(1 - \frac{1}{2}e^{-j3\pi/4}z^{-1}\right)$$

Clearly, X(z) is rational.

(b) A sketch of the pole-zero plot for $X(z)$ is shown below. Note that the ROC for X(z) is $|z| > 0$.



(c) A sketch of the pole-zero plot for $Y(z)$ is shown below. Note that the ROC for Y(z) is $|z| > \frac{1}{2}$.



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5.52.

$$X(z) = \frac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1-\frac{1}{5}z)}{(1-\frac{1}{6}z)} = \frac{6}{5}\frac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1-5z^{-1})}{(1-6z^{-1})}$$

$$\alpha^n x[n] \Leftrightarrow X(\alpha^{-1}z) = \frac{6}{5}\frac{(1-\frac{1}{2}\alpha z^{-1})(1-\frac{1}{4}\alpha z^{-1})(1-5\alpha z^{-1})}{(1-6\alpha z^{-1})}$$

A minimum phase sequence has all poles and zeros inside the unit circle.

$$|\alpha/2| < 1 \quad \Rightarrow \quad |\alpha| < 2$$
$$|\alpha/4| < 1 \quad \Rightarrow \quad |\alpha| < 4$$
$$|5\alpha| < 1 \quad \Rightarrow \quad |\alpha| < \frac{1}{5}$$
$$|6\alpha| < 1 \quad \Rightarrow \quad |\alpha| < \frac{1}{6}$$

Therefore, $\alpha^n x[n]$ is real and minimum phase iff α is real and $|\alpha| < \frac{1}{6}$.

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5.54. All zeros inside the unit circle means the sequence is minimum phase. Since

$$\sum_{n=0}^{M} |h_{min}[n]|^2 \geq \sum_{n=0}^{M} |h[n]|^2$$

is true for all M, we can use $M = 0$ and just compute $h^2[0]$. The largest result will be the minimum phase sequence.

A	B	C	D	E	F	G	H
44.5	28.4	1.8	2.8	1.8	177.7	113.8	7.1

The answer is F.

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5.59. (a) Minimum phase systems have all poles and zeros inside $|z| = 1$. Allpass systems have pole-zero pairs at conjugate reciprocal locations. Generalized linear phase systems have pole pairs and zero pairs in conjugate reciprocal locations and at $z = 0, 1, -1$ and ∞. This implies that all the poles and zeros of $H_{min}(z)$ are second-order. When the allpass filter flips a pole or zero outside the unit circle, one is left in the conjugate reciprocal location, giving us linear phase.

(b) We know that $h[n]$ is length 8 and therefore has 7 zeros. Since it is an even length generalized linear phase filter with real coefficients and odd symmetry it must be a Type IV filter. It therefore has the property that its zeros come in conjugate reciprocal pairs stated mathematically as $H(z) = H(1/z^*)$. The zero at $z = -2$ implies a zero at $z = -\frac{1}{2}$, while the zero at $z = 0.8e^{j(\pi/4)}$ implies zeros at $z = 0.8e^{-j(\pi/4)}$, $z = 1.25e^{j(\pi/4)}$ and $z = 1.25e^{-j(\pi/4)}$. Because it is a IV filter, it also must have a zero at $z = 1$. Putting all this together gives us

$$\begin{aligned} H(z) &= (1 + 2z^{-1})(1 + 0.5z^{-1})(1 - 0.8e^{j(\pi/4)}z^{-1})(1 - 0.8e^{-j(\pi/4)}z^{-1}) \\ &\quad (1 - 1.25e^{j(\pi/4)}z^{-1})(1 - 1.25e^{-j(\pi/4)}z^{-1})(1 - z^{-1}) \end{aligned}$$

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5.60. *False.* Let $h[n]$ equal

$$h[n] = \frac{\sin \omega_c (n - 4.3)}{\pi (n - 4.3)} \longleftrightarrow H(e^{jw}) = \begin{cases} e^{-4.3\omega}, & |\omega| < \omega_c \\ 0, & \text{otherwise} \end{cases}$$

Proof: Although the group delay is constant (grd $\left[H(e^{jw})\right] = 4.3$) the resulting M is not an integer.

$$\begin{aligned} h[n] &= \pm h[M - n] \\ H(e^{j\omega}) &= \pm e^{jM\omega} H(e^{-j\omega}) \\ e^{-j4.3\omega} &= \pm e^{j(M+4.3)\omega}, \qquad |\omega| < \omega_c \\ M &= -8.6 \end{aligned}$$