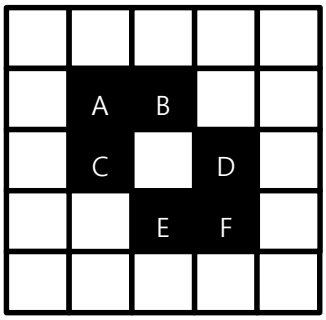
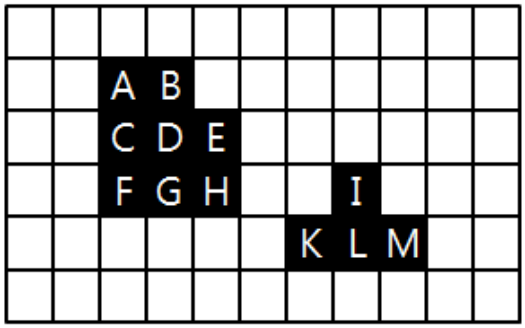
Computer Vision 2015: Assignment #2

Due 8 April 2015

1. Answer the questions in Yes or No.
   (a) In the 4-neighborhood definition, are A and F connected?
   (b) In the 4-neighborhood definition, is {A, B, C} a connected component?
   (c) In the 8-neighborhood definition, are A and F connected?
   (d) In the 8-neighborhood definition, is {A, B, C} a connected component?



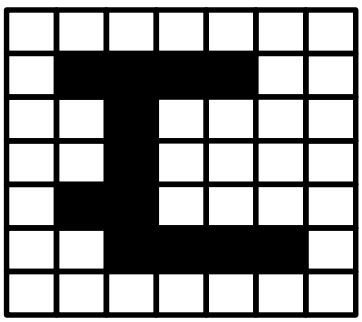
2. An image **B** and a structuring element **S** is given below. The origin of the structuring element is denoted by the letter 'O.'



Binary Image **B** Structuring Element **S**

   (a) In the 8-neighborhood sense, are 'A' and 'K' connected?
   (b) In the 8-neighborhood sense, is (A, B, D, H) a path?
   (c) In the 8-neighborhood sense, is (A, D, H) a path?
   (d) Let us define the length of a path $(A_0, A_1, \ldots, A_n)$ as
   $$n = \text{the number of elements in the sequence} - 1.$$
   In the 4-neighborhood sense, what is the shortest length of a path between 'A' and 'H'?
   (e) Plot the dilation result $\mathbf{B} \oplus \mathbf{S}$.
   (f) In the dilation result $\mathbf{B} \oplus \mathbf{S}$, in the 8-neighborhood sense, are 'A' and 'K' connected?

3. Close the image B with the structuring element S. The origin of the structuring element is its center.



B

S

4. We learned the concepts of entropy and mutual information.
   (a) Let X be a random variable with $p(1) = \frac{1}{2}, p(2) = \frac{1}{4}, p(3) = \frac{1}{8}, p(4) = \frac{1}{8}$. What is the entropy H(X)?
   (b) Let Y be another random variable. Given the random variable X in (a), what is the maximum possible value of I(X; Y)? When is the maximum achieved?

5. Let X and Y be random variables with symbols {1, 2, 3} and joint distribution p(x,y), which is given by

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

   where the element in x-th row and y-th column is p(x,y). Determine H(X) and I(X;Y)

6. A method to design a decision tree is to use the concept of the mutual information. Specifically, we find the feature X, which maximizes the mutual information $I(X; C)$ with the class C, and use it in the root node. Suppose that you find the optimal feature X. A friend of yours told you that she made a better feature Y from X using the relation $Y = 1 - 2X$ and improved the classification performance. Did she tell you a lie or a truth?

7. An apple farmer needs to classify her apples into green ones or red ones. Suppose that she has a camera that captures the reddishness (x) of an apple. The probability distribution function (PDF) of the reddishness of an red apple is given by $p(x)$ and the PDF of the reddishness of a green apple is given by $q(x)$.

$$p(x) = \begin{cases} \frac{1}{3}, & 2 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases} \quad q(x) = \begin{cases} x - 1, & 1 \le x \le 2, \\ 3 - x, & 2 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

   For the last decade, she harvested 2 million green apples and 1 million red apples.
   (a) Plot $p(x)$ and $q(x)$.
   (b) Given the reddishness $(1 \le x \le 5)$ of an apple, what is the Bayesian or maximum a posteriori (MAP) classification rule?
   (c) In (b), what is the classification error rate? You may provide only the equation without the exact computation.
   (d) What is the maximum likelihood (ML) classification rule?