Assignment 5. Page 1.

1.2 : 7, 22 // 1.3 : 1, 10, 21, 25 // 1.4 : 11, 14, 15, 40 // 1.5 : 3, 7, 14, 19, 28, 34.
1.6 : 2, 4, 7, 14, 37, 41, 50 // 2.1 : 2, 8, 19, 26 // 2.2 : 1, 5, 18, 33, 44, 56, 64 //
2.3 : 1, 7, 13, 25, 31, 40 // 2.4 : 2, 6, 13, 29, 40 // 2.~~5~~6 : 5, 9, 20.

1.2.

7. $$\left.\begin{array}{l} -(u+v+w)+(u+2v+3w)-(v+2w) = 0 \\ -2 \quad + 1 \quad - \underbrace{0}_{\to -1} = -1 \end{array}\right] \; 0=-1 \Rightarrow 0=1$$

$$\therefore \left.\begin{array}{l} u+v+w=2 \\ u+2v+3w=1 \\ v+2w=-1 \end{array}\right)$$

22. $y=1.\ x+z=1. \Rightarrow (1,1,0),\ (\tfrac{1}{2},1,\tfrac{1}{2}),\ (0,1,1).$

1.3.

1. $$\begin{array}{l} 3x+2y=10 \\ 6x+4y=a \end{array} \Rightarrow \begin{bmatrix} 3 & 2 & 10 \\ 6 & 4 & a \end{bmatrix} \to \begin{bmatrix} 3 & 2 & 10 \\ 0 & 0 & a-20 \end{bmatrix}.$$

$a \neq 20$: 해가 존재하지 않음, $a=20$: 무수히 많은해, $(x,y)=(0,5),(2,2)$.

10. $$\begin{array}{l} x+by=0 \\ x-2y-z=0 \\ y+z=0 \end{array} \Rightarrow \begin{bmatrix} 1 & b & 0 & 0 \\ 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & b & 0 & 0 \\ 0 & -2-b & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & b & 0 & 0 \\ 0 & -2-b & -1 & 0 \\ 0 & 1-k(-2-b) & 1+k & 0 \end{bmatrix}.$$

행교환을 유도 : $-2-b=0 \Rightarrow b=-2$. 중쇠가 제거 $\left.\begin{array}{l} 1-k(-2-b)=0 \\ 1+k=0 \end{array}\right).$

$\Rightarrow k=-1,\ b=-1.$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (x,y,z)=(1,1,-1)$$

21. foward elimination.

Pivot

$$\begin{bmatrix} 2 & 3 & 0 & 0 \\ 4 & 5 & 1 & 3 \\ 2 & -1 & -3 & 5 \end{bmatrix} \to \begin{bmatrix} 2 & 3 & 0 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & -4 & -3 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 3 & 0 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & -7 & -7 \end{bmatrix}.$$

back-substitution.

$w=1. \Rightarrow -v+w=3 \Rightarrow v=-2 \Rightarrow 2u+3v=0 \Rightarrow u=3.$

25. $$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & -1 & 0 \\ 0 & 0 & 0 & \frac{5}{4} & 5 \end{bmatrix}$$

$\Rightarrow \frac{5}{4}z=5 \Rightarrow z=4 \Rightarrow \frac{4}{3}w-z=0 \Rightarrow w=3 \Rightarrow \frac{3}{2}v-w=0 \Rightarrow v=2 \Rightarrow 2u-v=0$
$\Rightarrow u=1. \qquad (u,v,w,z)=(1,2,3,4).$

Assignment 5. Page2.

1.4. Nine coefficients, Three unknowns, Three righthand sides.

11. (1). $B = [\, b_1 \;\; b_2 \;\; b_3 \,] \Rightarrow AB = [\, Ab_1 \;\; Ab_2 \;\; Ab_3 \,] \Rightarrow$ TRUE.

(2) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix} \Rightarrow$ FALSE.

(3). $A = \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} \Rightarrow AB = \begin{bmatrix} a_1^T B \\ a_2^T B \\ a_3^T B \end{bmatrix} \Rightarrow$ TRUE.

(4). $(AB)^2 = A^2B^2$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $B^2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $AB = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

$A^2B^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$, $(AB)^2 = 0$. $\Rightarrow$ FALSE.

14. a). a_{11}

b) $l_{i1} = + \frac{a_{i1}}{a_{11}}$

c). $a_{ij} \to a_{ij} - l_{i1} a_{1j} = a_{ij} - \frac{a_{i1} a_{1j}}{a_{11}}$

d). $l_{12} = \frac{a_{21}}{a_{11}}$, $a_{22} \to a_{22} - l_{21} a_{12} = a_{22} - \frac{a_{21} a_{12}}{a_{11}}$

15. a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ c) $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. d) $E = F = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

40. a). True. b). False. $A: 2\times 3$, $B: 3\times 2$ ... c). True.

d). False. $B = 0$.

1.5. U: upper triangular matrix. L: lower triangular matrix.

3. $GFEA = U \Rightarrow \underbrace{E^{-1}F^{-1}G^{-1}}_{L} U = A$. $L = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ 1 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ 1 & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & 4 & 1 \end{bmatrix}$

$l_{32} = 4$.

7. $Ax = b \Rightarrow Lc = b$, $Ux = c$.

$U = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}$ $L = \begin{bmatrix} 1 & & \\ & 1 & \\ 3 & & 1 \end{bmatrix}$, $Lc = b \Rightarrow \begin{matrix} c_1 = 2 \\ c_2 = 2 \\ 3c_1 + c_3 = 5 \end{matrix} \Rightarrow c = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.

$Ux = c \Rightarrow \begin{matrix} 2x_1 + 3x_2 + 3x_3 = 2 \\ 5x_2 + 7x_3 = 2 \\ -x_3 = -1 \end{matrix} \Rightarrow x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

Assignment 5. Page3

1.5.

14. $L^{-1}A = \begin{bmatrix}1&&\\&1&\\-2&&1\end{bmatrix}\begin{bmatrix}1&&\\-1&1&\\&&1\end{bmatrix}\begin{bmatrix}1&1&1\\1&1&3\\2&5&8\end{bmatrix}. \Rightarrow L = \begin{bmatrix}1&&\\1&1&\\&&1\end{bmatrix}\begin{bmatrix}1&&\\&1&\\2&&1\end{bmatrix} = \begin{bmatrix}1&0&0\\1&1&0\\2&0&1\end{bmatrix}$

19. (1) $P_{12}A = \begin{bmatrix}1&0&1\\0&1&1\\2&3&4\end{bmatrix}$, $L = \begin{bmatrix}1&&\\0&1&\\2&3&1\end{bmatrix}$, $DU = \begin{bmatrix}1&0&1\\0&1&1\\0&0&-1\end{bmatrix} = \begin{bmatrix}1&&\\&1&\\&&-1\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&1\\0&0&1\end{bmatrix}$

$PA = LDU \Rightarrow \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&1&1\\1&0&1\\2&3&4\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\2&3&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&1\\0&0&1\end{bmatrix}.$

(2). $P_{23}A = \begin{bmatrix}1&2&1\\1&1&1\\2&4&2\end{bmatrix}$, $L = \begin{bmatrix}1&&\\1&1&\\2&0&1\end{bmatrix}$, $DU = \begin{bmatrix}1&2&1\\0&-1&0\\0&0&0\end{bmatrix} \longrightarrow$ 특이행렬.

28. (1). $\underset{A}{\begin{bmatrix}1&1&0\\1&2&1\\0&1&2\end{bmatrix}} = \underset{L}{\begin{bmatrix}1&0&0\\1&1&0\\0&1&1\end{bmatrix}}\underset{U}{\begin{bmatrix}1&1&0\\0&1&1\\0&0&1\end{bmatrix}}.$

(2). $\begin{bmatrix}a&a&0\\a&a+b&b\\0&b&b+c\end{bmatrix} = \begin{bmatrix}1&&\\1&1&\\0&1&1\end{bmatrix}\begin{bmatrix}a&a&0\\0&b&b\\0&0&c\end{bmatrix}.$

$= \underset{L}{\begin{bmatrix}1&0&0\\1&1&0\\0&1&1\end{bmatrix}}\underset{D}{\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}}\underset{V}{\begin{bmatrix}1&1&0\\0&1&1\\0&0&1\end{bmatrix}}.$

34. $c=6 \Rightarrow A=LU$가 불가. (P가 필요).

else. $A = \begin{bmatrix}1&&\\3&1&\\0&\frac{1}{c-6}&1\end{bmatrix}\begin{bmatrix}1&2&0\\0&c-6&1\\0&0&\boxed{1-\frac{1}{c-6}}\end{bmatrix} \longrightarrow \neq 0.$

$1-\frac{1}{c-6} = \frac{c-7}{c-6}$

$\therefore c \neq 7$ 일때

3개의 중심가 존재.

1.6.

2. $(A^2)^{-1} = B$. // $(A^2)^{-1} = (AA)^{-1} = A^{-1}A^{-1}$.

$\Rightarrow A(A^{-1}A^{-1}) = AB. \Rightarrow A(A^{-1}A^{-1}) = (AA^{-1})A^{-1} = A^{-1}.$

$\therefore A^{-1} = AB.$

4. $A_1^{-1} = \begin{bmatrix}0&0&0&\frac{1}{4}\\0&0&\frac{1}{3}&0\\0&\frac{1}{2}&0&0\\1&0&0&0\end{bmatrix}$. $A_2^{-1} = \begin{bmatrix}1&0&0&0\\\frac{1}{2}&1&0&0\\\frac{1}{3}&\frac{2}{3}&1&0\\\frac{1}{4}&\frac{2}{4}&\frac{3}{4}&0\end{bmatrix}$ $A_3^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d&-b&0&0\\-c&a&0&0\\0&0&d&-b\\0&0&-c&a\end{bmatrix}$

Assignment 5. Page 4.

1.6

7. (a). $P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. $\Rightarrow P_1^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow P_2^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(b). $(P^T)^{-1} = (P^{-1})^T$. $\Rightarrow (PP^T)^{-1} = (P^T)^{-1} P^{-1} = (P^{-1})^T P^{-1}$

$$P^T P = P^T \cdot \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} = \begin{bmatrix} P^T P_1 & P^T P_2 & P^T P_3 \end{bmatrix} = \begin{bmatrix} P_1^T P_1 & P_1^T P_2 & P_1^T P_3 \\ P_2^T P_1 & P_2^T P_2 & P_2^T P_3 \\ P_3^T P_1 & P_3^T P_2 & P_3^T P_3 \end{bmatrix} = E.$$

$\Rightarrow P^T = P^{-1}$, $PP^T = E$.

14. $A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/4 & 1 & 0 & 0 \\ -1/4 & -1/3 & 1 & 0 \\ -1/4 & -1/3 & -1/2 & 1 \end{bmatrix}$.

37. $[A \ I] = \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$. $\Rightarrow \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1/3 & -\frac{2}{3} & 1 \end{bmatrix} = [U \ L^{-1}]$. $\Rightarrow \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 0 & -\frac{3}{4} & \frac{3}{2} & -\frac{3}{4} \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{bmatrix}$. $\Rightarrow \begin{bmatrix} 2 & 0 & 0 & \frac{3}{2} & -1 & \frac{1}{2} \\ 0 & \frac{3}{2} & 0 & -\frac{3}{4} & \frac{3}{2} & -\frac{3}{4} \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix} = [I \ A^{-1}]$. $\Rightarrow A^{-1} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$.

41. a) True. b) False c) True. d) True.

50. $A_1 = \begin{bmatrix} 1 & 0 \\ 9 & 3 \end{bmatrix}$. $A_1^T = \begin{bmatrix} 1 & 9 \\ 0 & 3 \end{bmatrix}$ $A_1^{-1} = \begin{bmatrix} \frac{1}{3} & 0 \\ -3 & 1 \end{bmatrix}$. $(A_1^{-1})^T = (A_1^T)^{-1} = \begin{bmatrix} \frac{1}{3} & -3 \\ 0 & 1 \end{bmatrix}$.

$A_2 = \begin{bmatrix} 1 & c \\ c & 0 \end{bmatrix}$ $A_2^T = \begin{bmatrix} 1 & c \\ c & 0 \end{bmatrix}$ $A_2^{-1} = \begin{bmatrix} 0 & \frac{1}{c} \\ \frac{1}{c} & -\frac{1}{c^2} \end{bmatrix}$. $(A_2^{-1})^T = (A_2^T)^{-1} = \begin{bmatrix} 0 & \frac{1}{c} \\ \frac{1}{c} & -\frac{1}{c^2} \end{bmatrix}$.

Assignment 5. page 5.

2.1

2. (a), (d), (e).

8. (b), (d), (e).

19. (a) True. (b) False. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

(c) True.

26. A. $(x, 0, 0)$ C. $(x, 2x, 0)$.

B. $(x, y, 0)$.

2.2.

1. $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & 3 & -1 & 5 \\ 3 & 4 & 1 & c \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & 1 & -5 & 1 \\ 0 & 1 & -5 & c-6 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & 1 & -5 & 1 \\ 0 & 0 & 0 & c-7 \end{array}\right]$.

$c=7$.

5. $A \to \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$. $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \to x_3, x_4$: 자유변수.

$\to R = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $Rx = 0$, $x_2 = -x_3$, $x_1 = 2x_3 - x_4$.

$$x = x_n + x_p = x_3\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$B \to U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} \to x_3$: 자유변수. $\to R = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$.

$x_1 = x_3$, $x_2 = -2x_3$. $\therefore x = x_n + x_p = x_3\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

18. $A_1 \to \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$, $A_2 \to [1]$. $A_3 \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

33. (a) $Ax=b \Rightarrow Ux=b'$: $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix} x = \begin{bmatrix} b_1 \\ b_2-2b_1 \\ b_3-b_2+2b_1 \end{bmatrix}$ 모든 벡터 b가 포함된다.

(b) $Ax=b \Rightarrow Ux=b'$: $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} b_1 \\ b_2-b_1 \\ b_3-2b_2 \end{bmatrix}$. $b_2 = \frac{1}{2}b_3$. $\therefore b = b_3\begin{bmatrix} 0 \\ 1/2 \\ 1 \end{bmatrix} + b_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Assignment 5. Page 6.

2.2.

44. $A=\begin{bmatrix}6&4&2\\-3&-2&-1\\9&6&q\end{bmatrix}$, $B=\begin{bmatrix}3&1&3\\q&2&q\end{bmatrix}$.

(a). $A\to\begin{bmatrix}6&4&2\\0&0&0\\0&0&q-3\end{bmatrix}$. $B\to\begin{bmatrix}3&1&3\\0&2-\frac{q}{3}&0\end{bmatrix}$.

$\hookrightarrow q=3$. $\hookrightarrow q=6$.

(b) $q\neq3,\ q\neq6$. (C). 존재하지 않는다.

56. $Ax=b\to Ux=c:\begin{bmatrix}1&0&2&3\\0&3&0&-3\\0&0&0&3\end{bmatrix}x=\begin{bmatrix}2\\3\\6\end{bmatrix}$.

$\to Rx=d:\begin{bmatrix}1&0&2&0\\0&1&0&0\\0&0&0&1\end{bmatrix}x=\begin{bmatrix}-4\\3\\2\end{bmatrix}$. 자유변수: x_3.

$x_4=2,\ x_2=3,\ x_1+2x_3=-4\Rightarrow x_1=-2x_3-4$.

$x=x_n+x_p=x_3\begin{bmatrix}-2\\0\\1\\0\end{bmatrix}+\begin{bmatrix}-4\\3\\0\\2\end{bmatrix}$.

64. (a) $A=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}$ (b) $A=\begin{bmatrix}1&1&1\\2&2&3\\3&3&4\end{bmatrix}$ c) $A=\begin{bmatrix}1&1&1\\2&2&3\\3&3&4\end{bmatrix}$.

2.3.

1. U: $\begin{bmatrix}2\\0\\0\\0\end{bmatrix}\begin{bmatrix}3\\6\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\9\\0\end{bmatrix}$ or $\begin{bmatrix}2\\0\\0\\0\end{bmatrix},\begin{bmatrix}4\\7\\0\\0\end{bmatrix},\begin{bmatrix}1\\0\\9\\0\end{bmatrix}\Rightarrow\mathbb{R}^3$.

A: $\begin{bmatrix}2\\0\\0\\4\end{bmatrix}\begin{bmatrix}3\\6\\0\\6\end{bmatrix}\begin{bmatrix}1\\0\\9\\2\end{bmatrix}$ or $\begin{bmatrix}2\\0\\0\\4\end{bmatrix}\begin{bmatrix}4\\7\\0\\8\end{bmatrix},\begin{bmatrix}1\\0\\9\\2\end{bmatrix}\Rightarrow\mathbb{R}^3$.

7. dimension, 4.

13. $\left.\begin{array}{l}c_1+c_2=0\\c_1+c_4=0\\c_2+c_3=0\\c_3+c_4=0\end{array}\right)$ $\begin{array}{l}c_2=-c_1,\\c_4=-c_1\\c_3=c_1\end{array}\Rightarrow c_1\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix}$이 해가된다. $\Rightarrow$ 일차독립이 아님.

$c_3+c_4=0 \updownarrow 1$.

$\Rightarrow$. $\begin{array}{l}c_3=1-c_4.\\c_2=-1+c_4.\\c_1=-c_4\\c_1=1-c_4.\end{array}$ $\Rightarrow$ 모순. $\Rightarrow\mathbb{R}^4$를 생성하지 못한다.

Assignment 5. Page 7.

2.3.

25. (a) $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$. (b) $\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\-1\end{bmatrix}$ (c) $\begin{bmatrix}1\\-1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\-1\end{bmatrix}$. (d). $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$.

$\begin{bmatrix}1\\0\\-1\\0\\0\end{bmatrix} \begin{bmatrix}1\\0\\0\\0\\-1\end{bmatrix} \begin{bmatrix}0\\1\\0\\-1\\0\end{bmatrix}$.

31. A. $c=0, d=2$.

B. $c \neq d$.

40. (C).

2.4.

2. $C(A)$: $r=2$. $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\2\end{bmatrix}$. $N(A)$ $n-r=2$. $\begin{bmatrix}2\\-1\\1\\0\end{bmatrix} \begin{bmatrix}-1\\0\\0\\1\end{bmatrix}$.

$C(A^T)$: $r=2$, $\begin{bmatrix}1\\2\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}$. $N(A^T)$ $m-r=1$, $\begin{bmatrix}-1\\0\\1\end{bmatrix}$.

$C(U)$: $\begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix}$. $N(U)$: $\begin{bmatrix}+2\\-1\\1\\0\end{bmatrix} \begin{bmatrix}-1\\0\\0\\0\end{bmatrix}$. $C(U^T)$: $\begin{bmatrix}1\\2\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}$. $N(U^T)$ $\begin{bmatrix}0\\0\\1\end{bmatrix}$.

$r=2$. $n-r=2$. $r=2$ $m-r=1$.

6. A_1. $r=1$. $\begin{bmatrix}1\\0\\2\end{bmatrix} \begin{bmatrix}1 & 0 & 0 & 3\end{bmatrix}$.

A_2 $r=1$. $\begin{bmatrix}1\\3\end{bmatrix} \begin{bmatrix}2 & -2\end{bmatrix}$

13. $C(A)$: $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}3\\4\\1\end{bmatrix}$. $N(A)$. $\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\-2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\\-2\\1\end{bmatrix}$.

$C(A^T)$ $\begin{bmatrix}0\\1\\2\\3\\4\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix}$. $N(A^T)$ $\begin{bmatrix}1\\-1\\1\end{bmatrix}$.

29. $C(A)$. 2, $\begin{bmatrix}3\\0\\1\end{bmatrix} \begin{bmatrix}3\\0\\0\end{bmatrix}$. $N(A)$ 2, $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$. $\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}$. $C(A^T)$ 2, $\begin{bmatrix}0\\3\\3\\3\end{bmatrix}, \begin{bmatrix}0\\1\\0\\1\end{bmatrix}$.

$N(A^T)$ 1, $\begin{bmatrix}0\\1\\0\end{bmatrix}$, $C(B)$, 1, $\begin{bmatrix}1\\4\\5\end{bmatrix}$. $N(B)$ 1, $\begin{bmatrix}1\\-1\end{bmatrix}$.

$C(B^T)$ 1, $\begin{bmatrix}1\\1\end{bmatrix}$. $N(B^T)$ 2, $\begin{bmatrix}-4\\1\\0\end{bmatrix}, \begin{bmatrix}-5\\0\\1\end{bmatrix}$.

40. $AB=0 \Rightarrow \underset{\substack{\| \\ r_B.}}{\dim C(B)} \leq \underset{\substack{\| \\ n-r_A.}}{\dim N(A)}$. $\Rightarrow r_A + r_B \leq n$.

Assignment 5. page 8.

2. 6.

5. (a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$. (c) $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(d). $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. (e). $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

9. $\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$, 0.

20. (a), (b), (c).