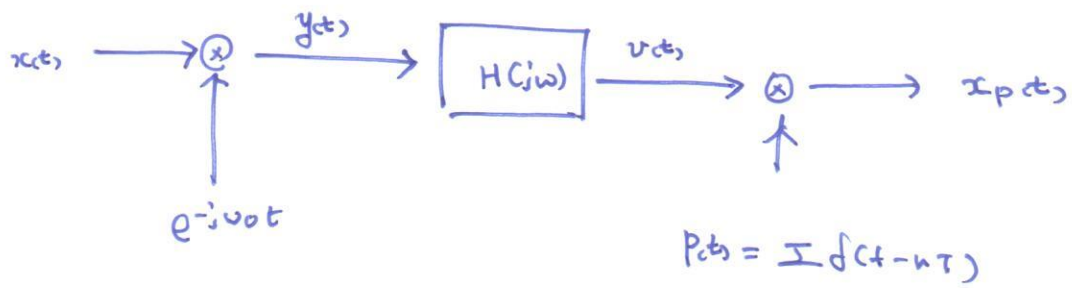
7. 27

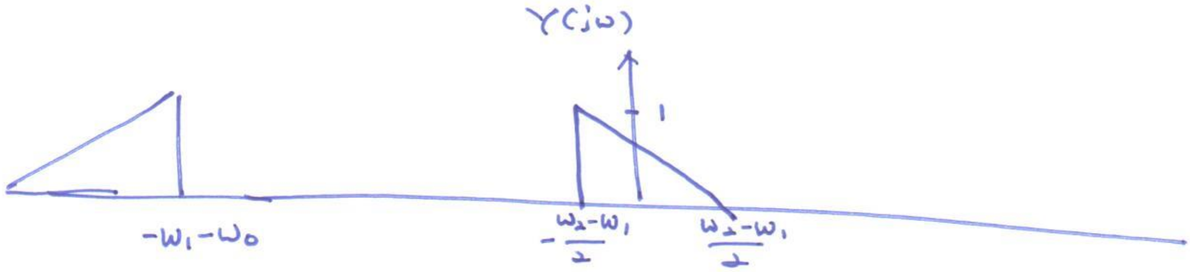


(a)

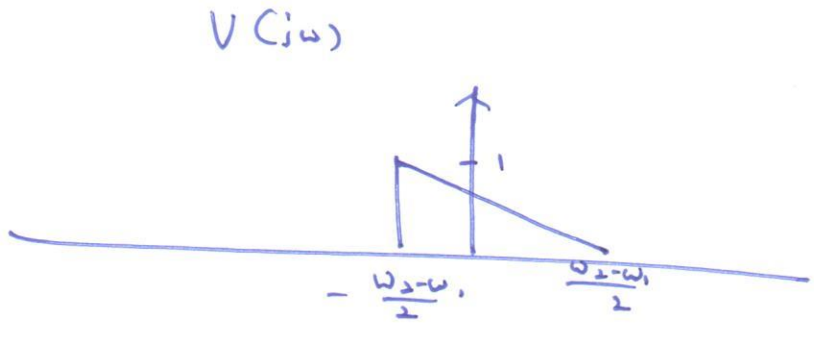
$$y(t) = x(t)\, e^{-j\omega_0 t}$$

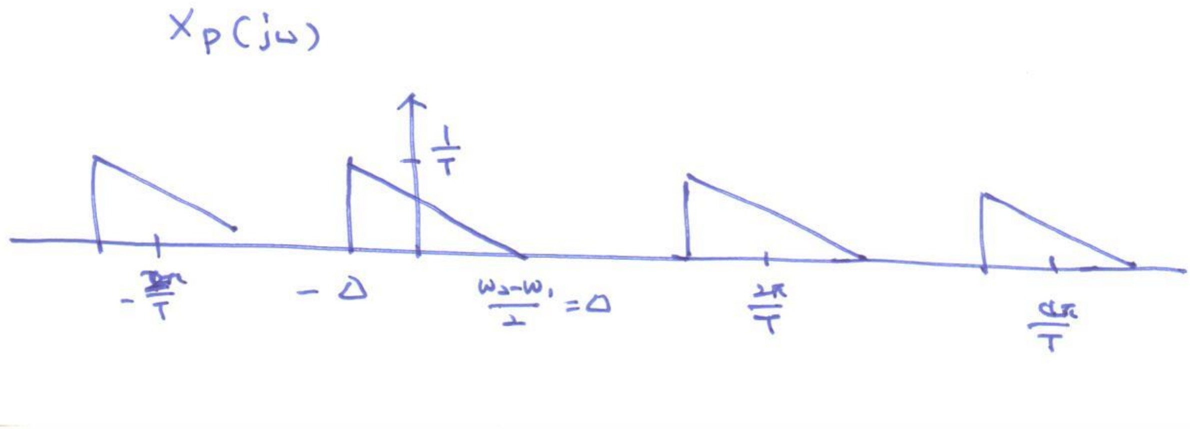
$$Y(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 t}\, e^{-j\omega t}\, dt$$

$$= X\left(j(\omega+\omega_0)\right)$$



After the lowpass filtering



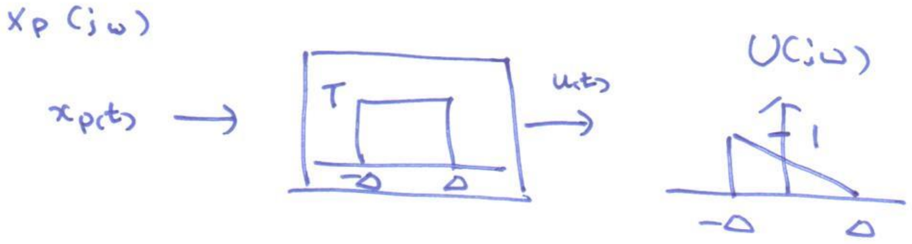


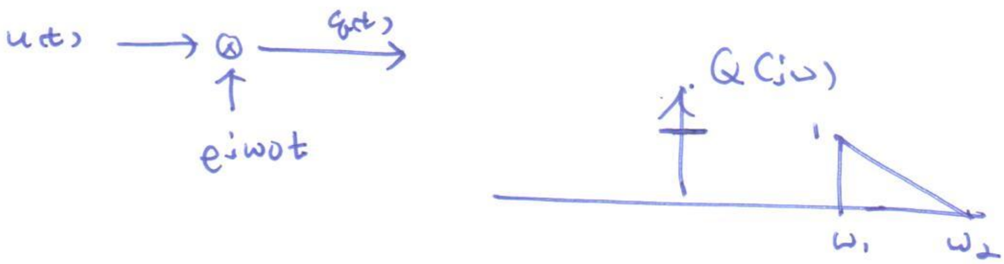
(b) There should be no aliasing

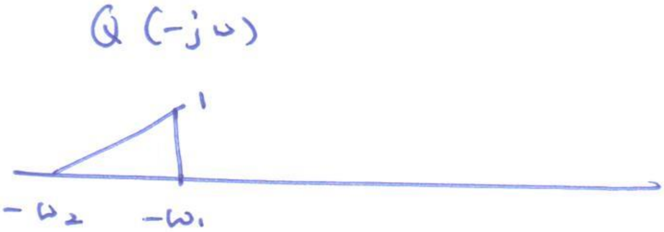
$$\therefore \quad \frac{2\pi}{T} > 2\Delta = \omega_2 - \omega_1$$

$$\therefore \quad T < \frac{2\pi}{\omega_2 - \omega_1}$$

(c)







$$r(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Q(-j\omega)\, e^{j\omega t}\, d\omega$$

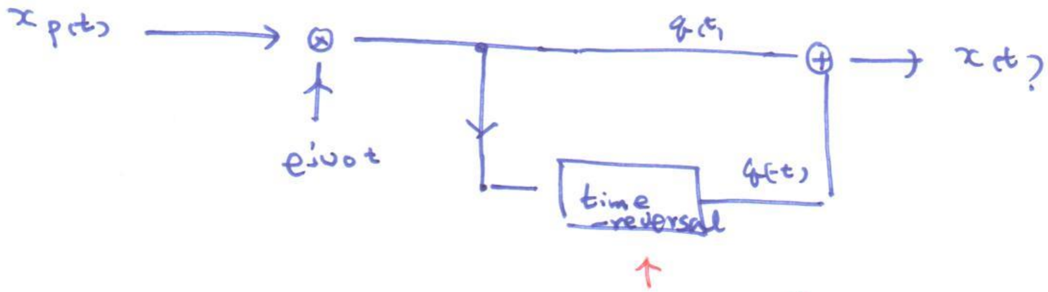
$$-\omega = \rho$$

$$= \frac{1}{2\pi}\int_{+\infty}^{-\infty} Q(j\rho)\, e^{-j\rho t} - d\rho$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} Q(j\omega)\, e^{-j\omega t}\, d\omega$$

$$= q(-t)$$

∴



non causal system.