Normal Distributions

- $P(\mathbf{x}) \sim N(\boldsymbol{\mu}, \Sigma)$

$$P(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{l}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x}-\boldsymbol{\mu})\right)$$

  where $\boldsymbol{\mu} = E[\mathbf{x}]$ and $\Sigma = E[(\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^T]$

- Σ is symmetric and positive definite, and thus its eigenvalue decomposition

$$\Sigma = \mathrm{Q}\Lambda\mathrm{Q}^T$$

  is possible

- A contour line of equal probability density

$$(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x}-\boldsymbol{\mu}) = 1$$

  - It is a hyper-ellipsoid
  - Its principal axes are given by the eigenvectors $\mathbf{v}_1, \ldots, \mathbf{v}_l$ of Σ
  - Its axes have lengths $\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_l}$
  - The main axis with length $\sqrt{\lambda_1}$ is in the direction of $\mathbf{v}_1$