KECE471 Computer Vision

Edge Detection

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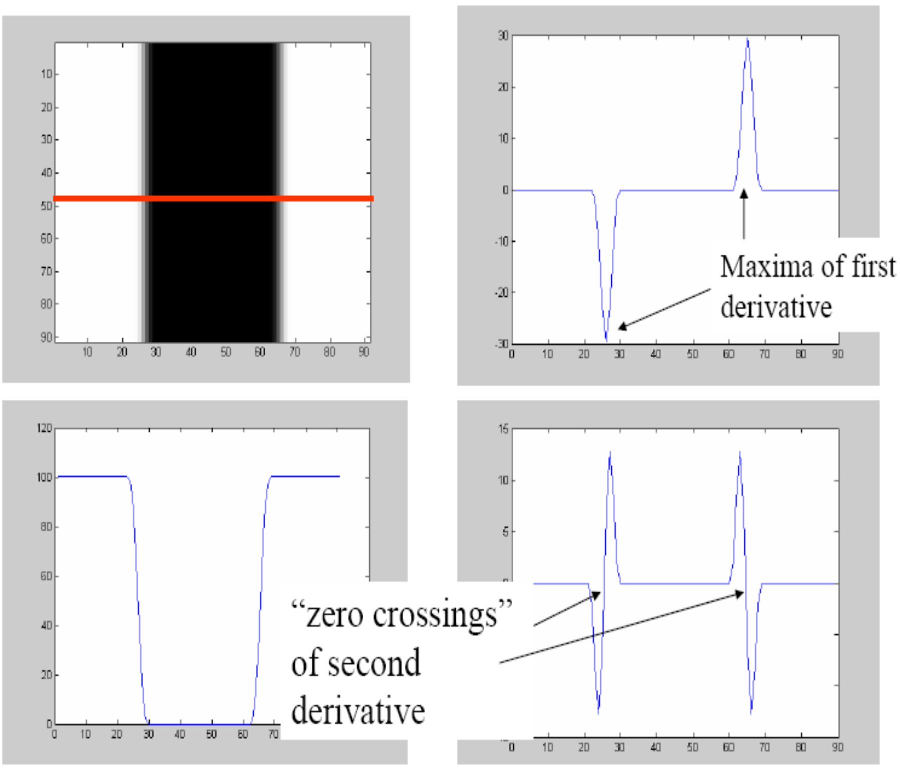
Chapter 8, Computer Vision by Forsyth and Ponce
Note: Many contents were extracted from the lecture notes of Prof. Kyoung Mu Lee.

Edge Detection

Edges



- Where the image values exhibit sharp variations
- Edges can be measured by
  - 1st order derivatives
    - Determine the gradients
    - Perform non-maximal suppression
    - Threshold
  - 2nd order derivatives
    - Find zero crossings in 2nd derivatives using Laplacian

First-order derivative filters (1D)

- Sharp changes correspond to peaks of the first-derivative of the input signal

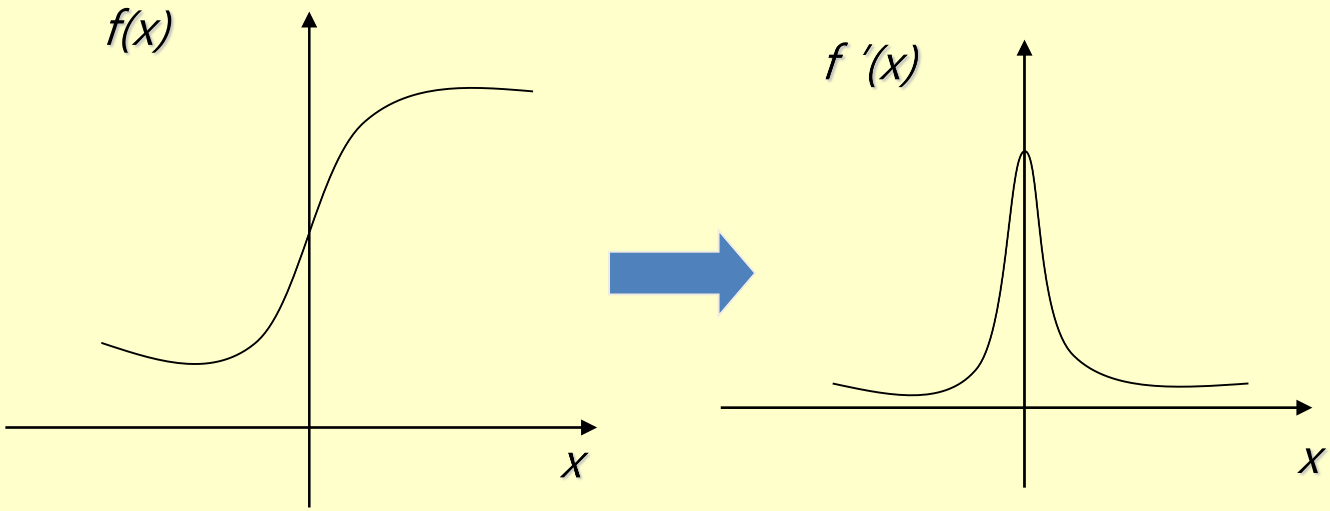


Image gradient

- 2D gradient of an image:

$$\nabla I = (I_x, I_y) = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

- The gradient magnitude (edge strength):

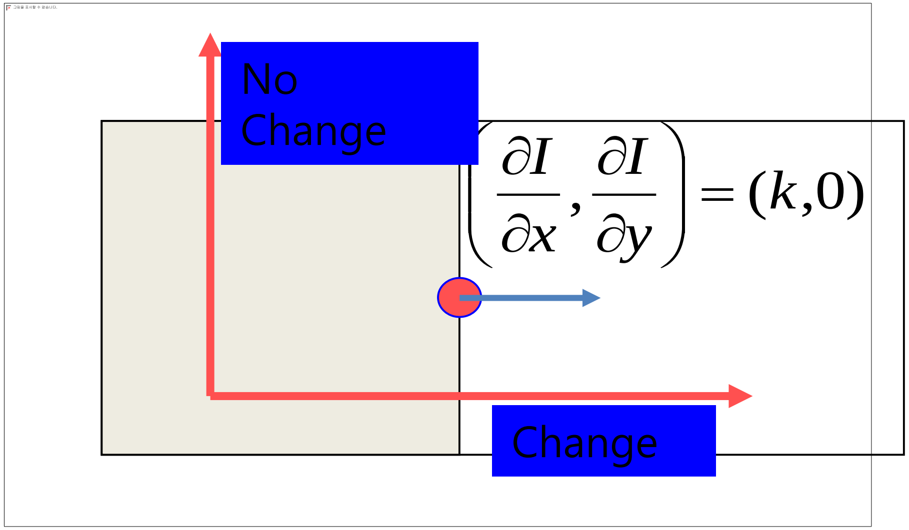
$$\|\nabla I\| = \sqrt{I_x^2 + I_y^2}$$

- The gradient direction:

$$\theta = \tan^{-1}\left(\frac{I_y}{I_x} \right)$$

Image gradient

- Horizontal change:



- Vertical change:

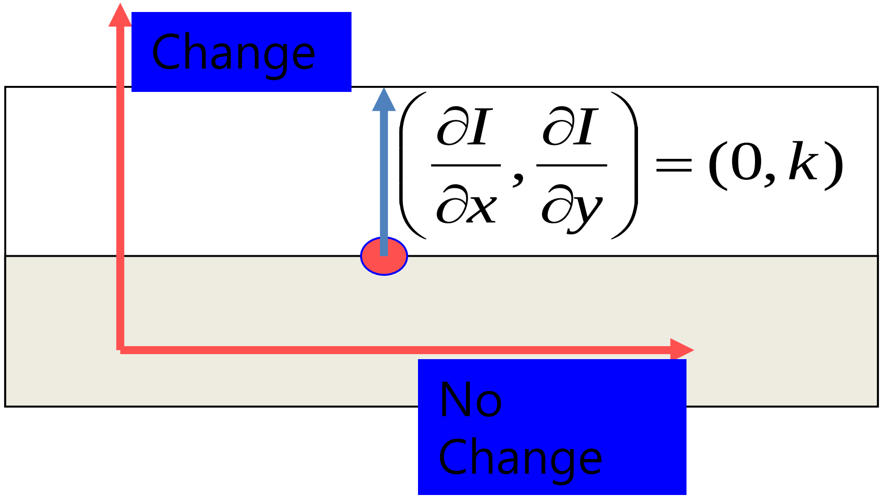
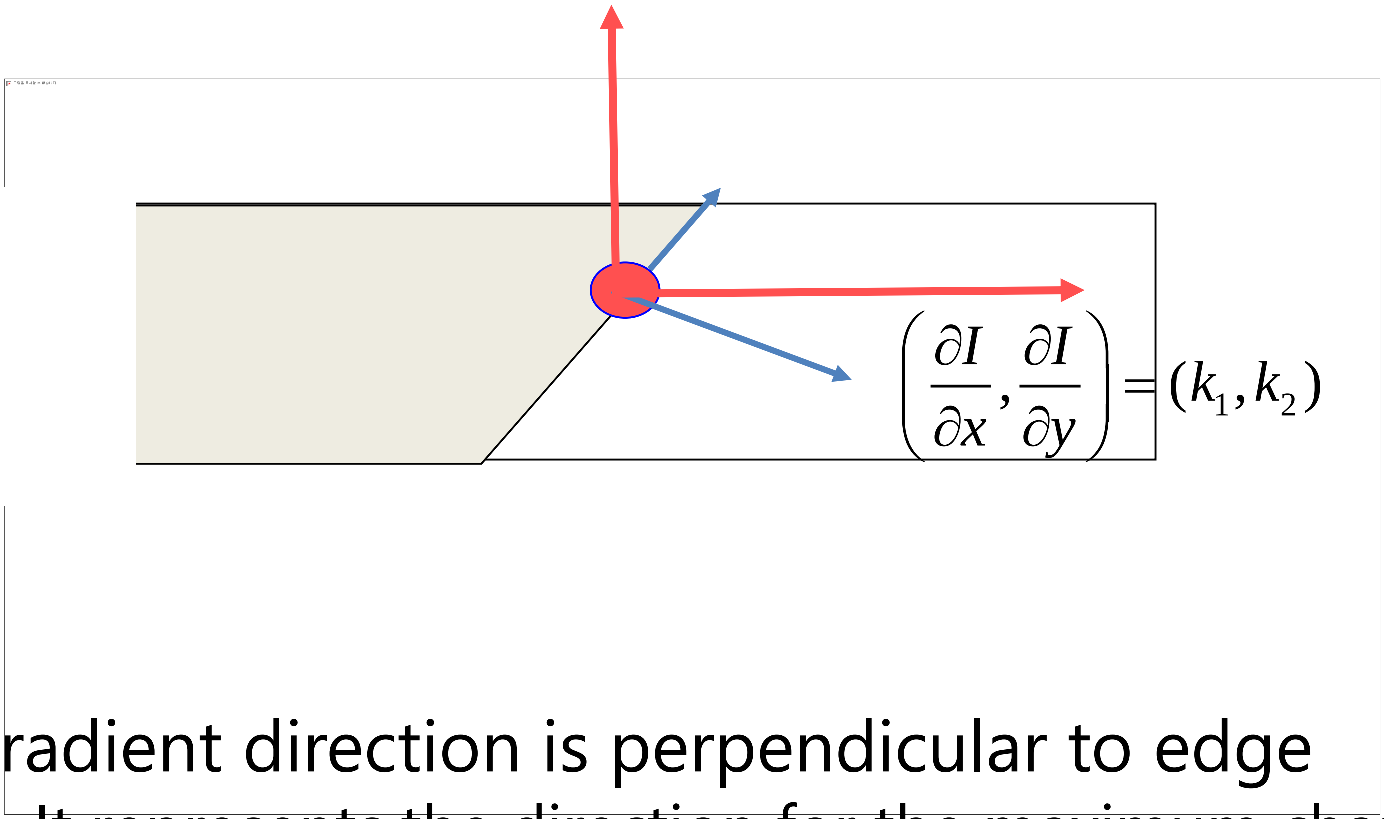


Image gradient

- General directions:



$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) = (k_1, k_2)$$

- Gradient direction is perpendicular to edge
  - It represents the direction for the maximum change
- Gradient magnitude measures edge strength.

Discrete approximation of derivatives

- 1D derivative

$$\frac{df(x)}{dx} = \begin{cases} \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x} & : \text{forward} \\ \lim_{\Delta x \to 0} \dfrac{f(x) - f(x-\Delta x)}{\Delta x} & : \text{backward} \\ \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} & : \text{central} \end{cases}$$

- Discrete approximations

$$\frac{df(x)}{dx} \cong \begin{cases} f(x+1) - f(x) \\ f(x) - f(x-1) \\ \dfrac{f(x+1) - f(x-1)}{2} \end{cases}$$

-1	1

-1	1

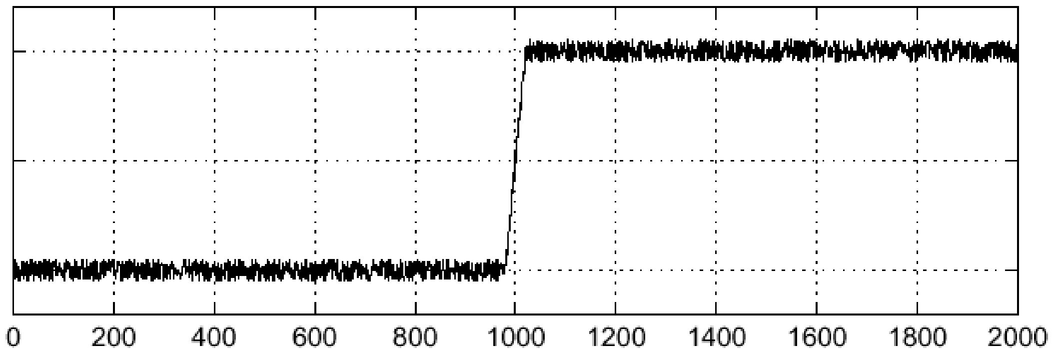
-1	0	1

symmetric

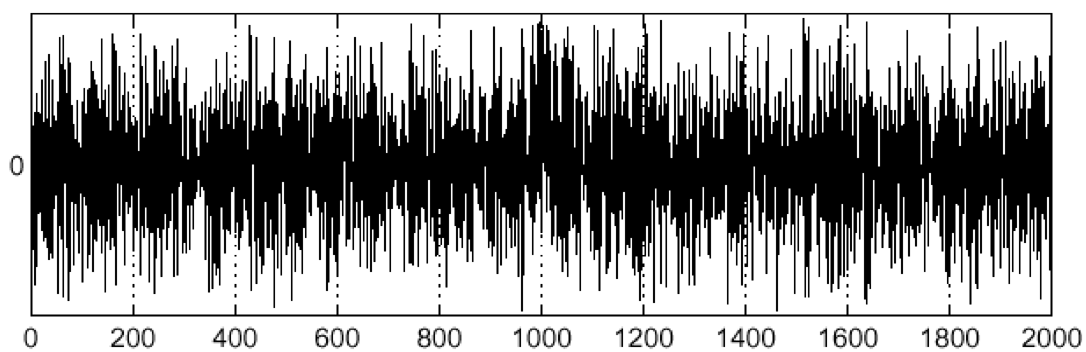
Effects of noises

- Consider an 1-D signal

$I(x)$

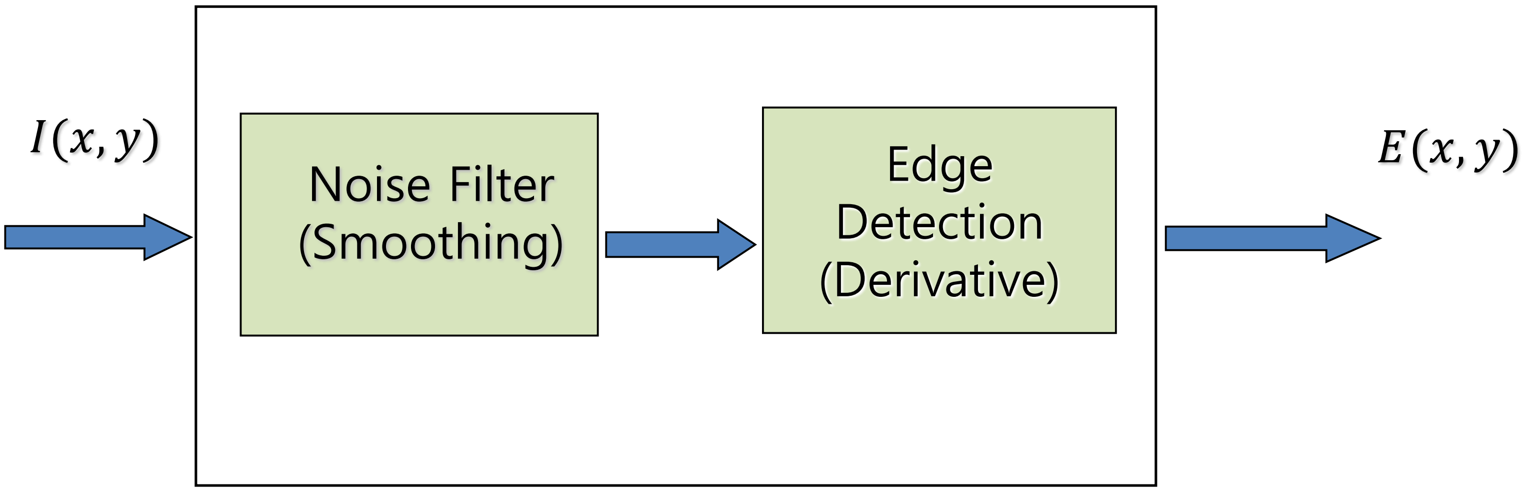


$\frac{dI(x)}{dx}$



- Can you detect the edge?

Noise suppression: pre-smoothing



$$
\begin{aligned}
E(x,y) &= D(x,y) * (S(x,y) * I(x,y)) \\
&= (D(x,y) * S(x,y)) * I(x,y) \\
&= S(x,y) * (D(x,y) * I(x,y))
\end{aligned}
$$

Noise smoothing and edge detection

- Prewitt edge detector:
  – Vertical mask

$$\frac{\partial}{\partial x}$$

-1	0	1
-1	0	1
-1	0	1

Noise Smoothing

Vertical Edge Detection

Noise smoothing and edge detection

- Prewitt edge detector:
  - Horizontal mask

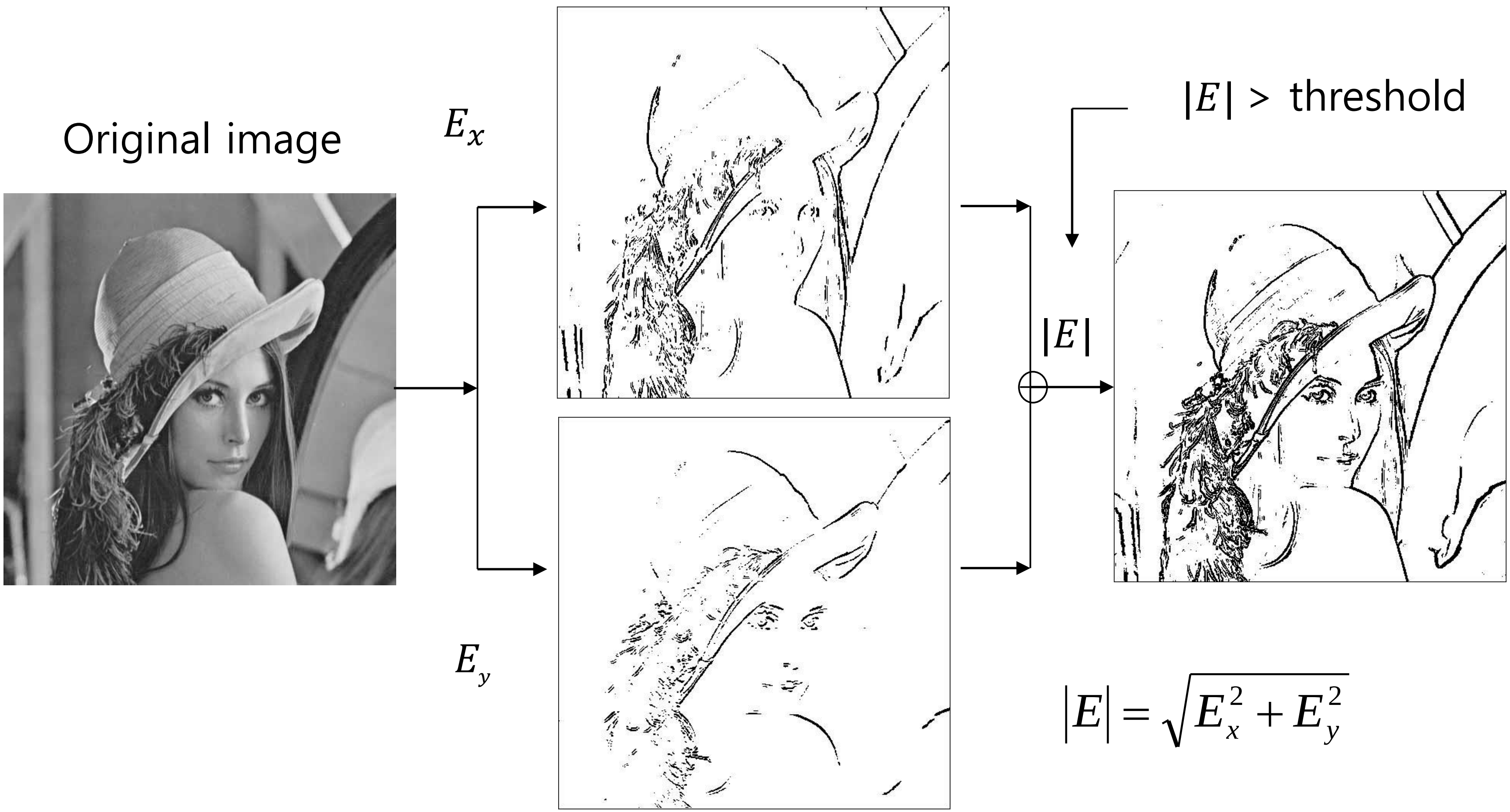
$$\frac{\partial}{\partial y}$$

-1	-1	-1
0	0	0
1	1	1

Noise Smoothing

Horizontal Edge Detection

Prewitt Edge Detector



$$|E| = \sqrt{E_x^2 + E_y^2}$$

Result of Prewitt operator (threshold = 100)

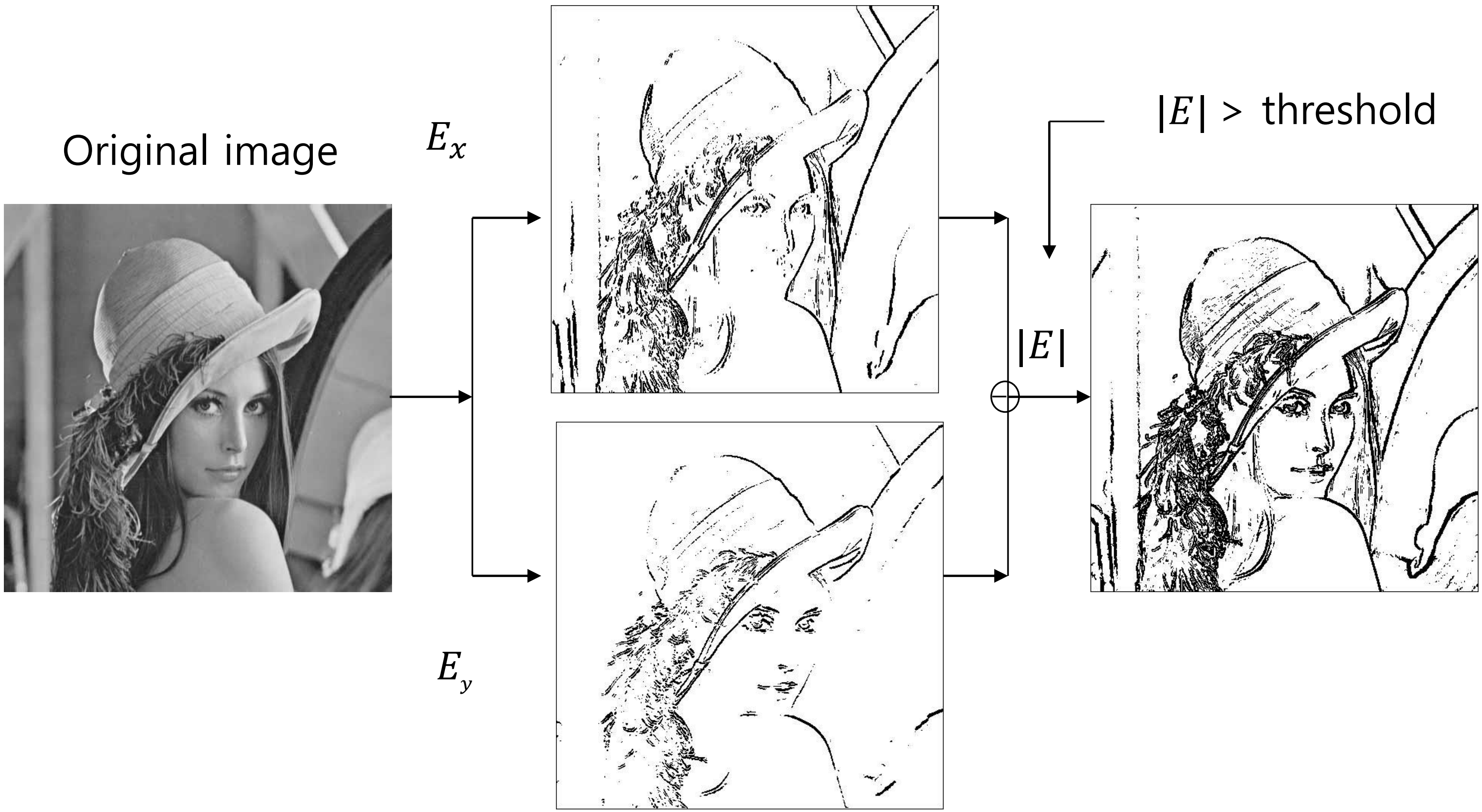
Sobel Edge Detector

- Sobel Masks:
  - Gives more weight to the 4-neighbors

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

Sobel Edge Detector



Result of Sobel operator (threshold = 100)

Gaussian Smoothing

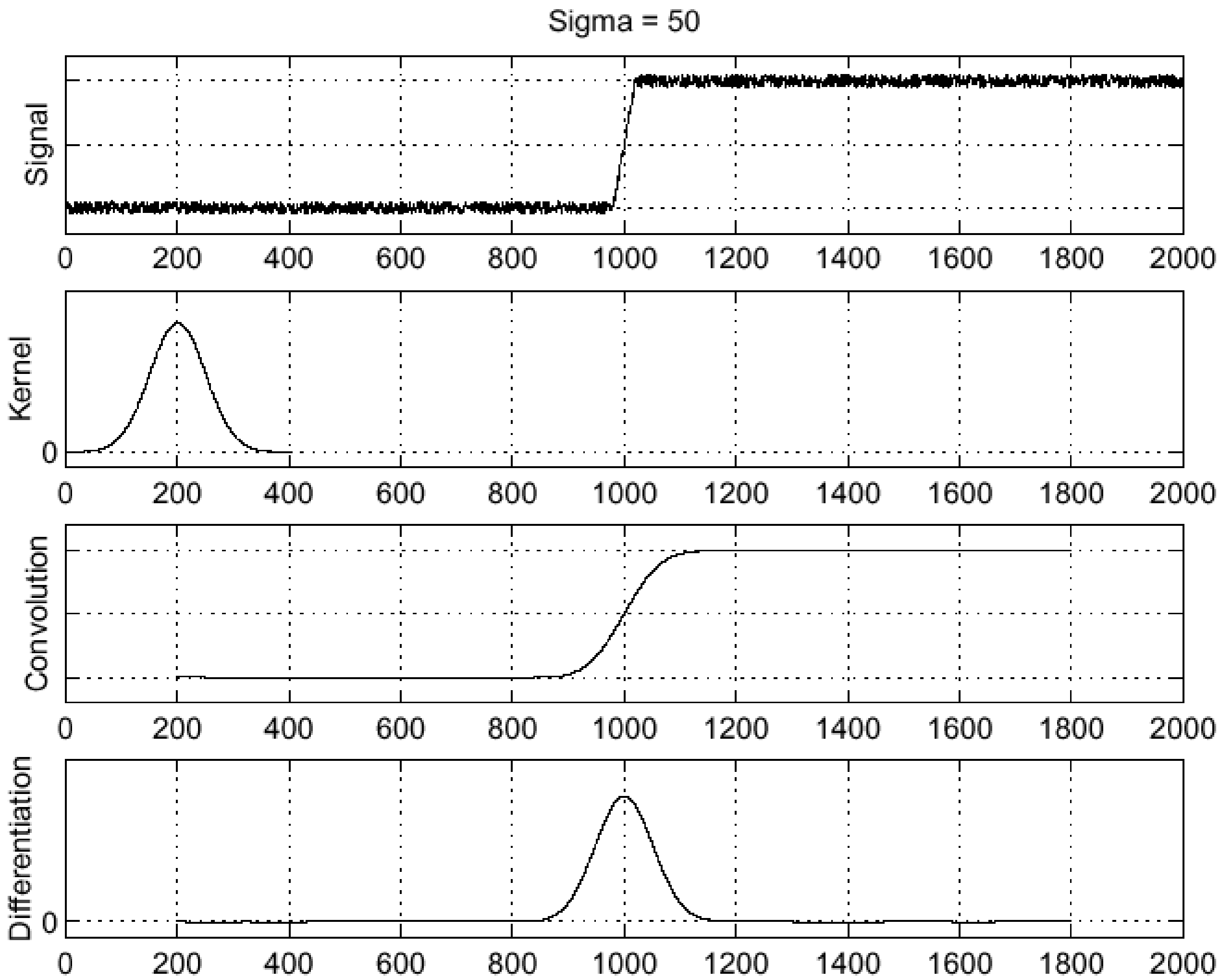
- Consider smoothing with Gaussian kernel

$$I$$

$$G(x) = e^{-\frac{x^2}{2\sigma^2}}$$

$$G * I$$

$$\frac{d}{dx}(G * I)$$



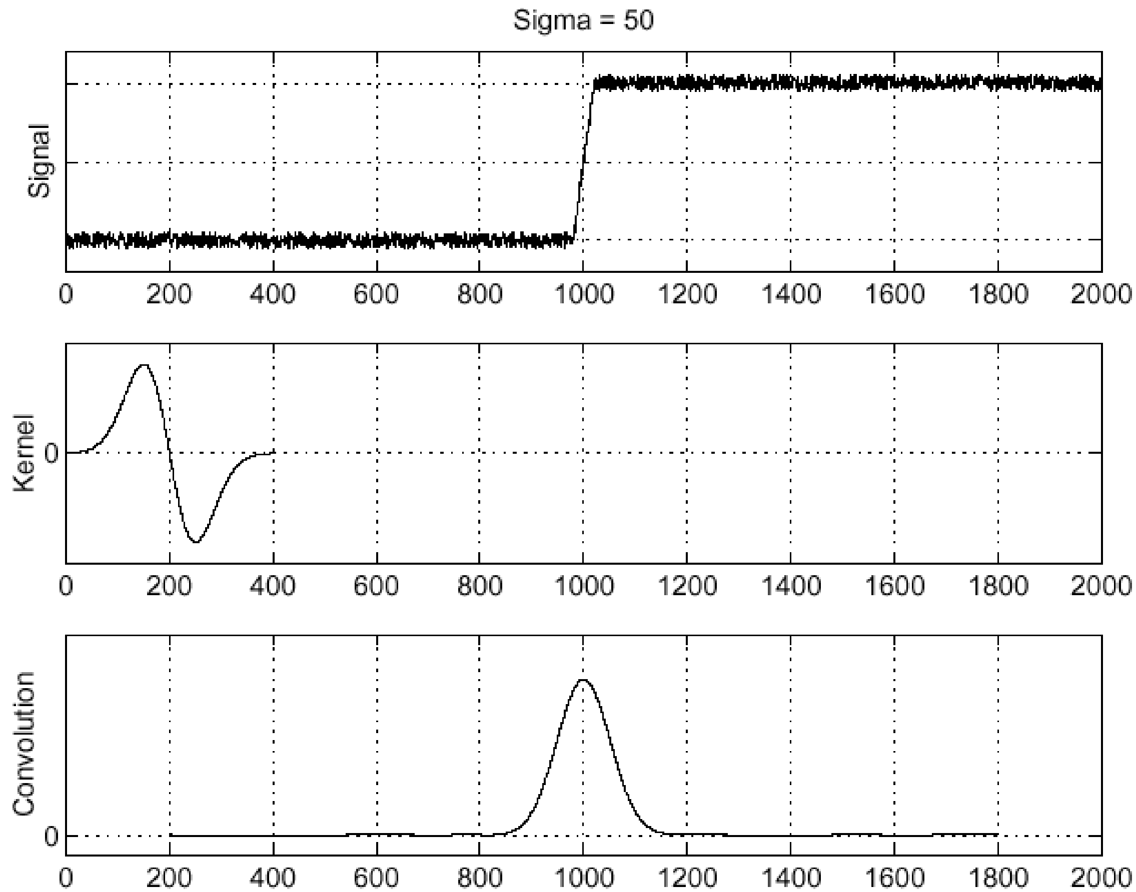
Derivative of Gaussian

- Note that $\frac{d}{dx}(G * I) = \left(\frac{d}{dx}G\right) * I$ and $G'(x) = -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$
- This saves us one step

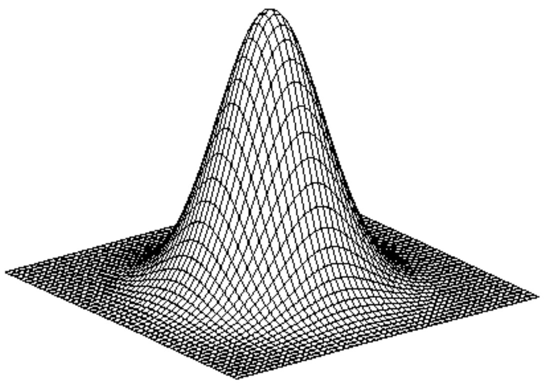
$$I$$

$$\frac{d}{dx}G$$

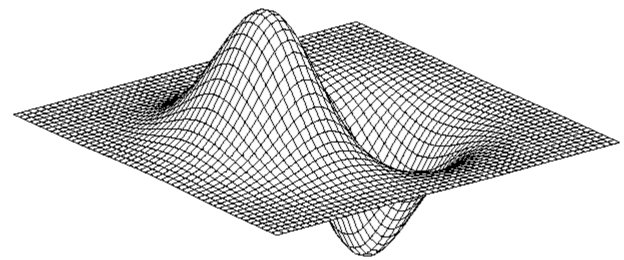
$$\left(\frac{d}{dx}G\right) * I$$



2D edge detection filters



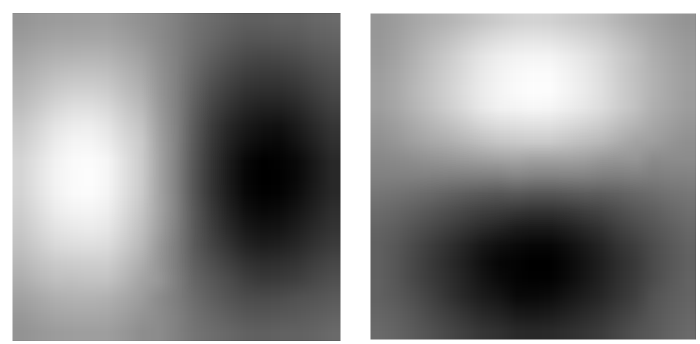
Gaussian



derivative of Gaussian (DOG)

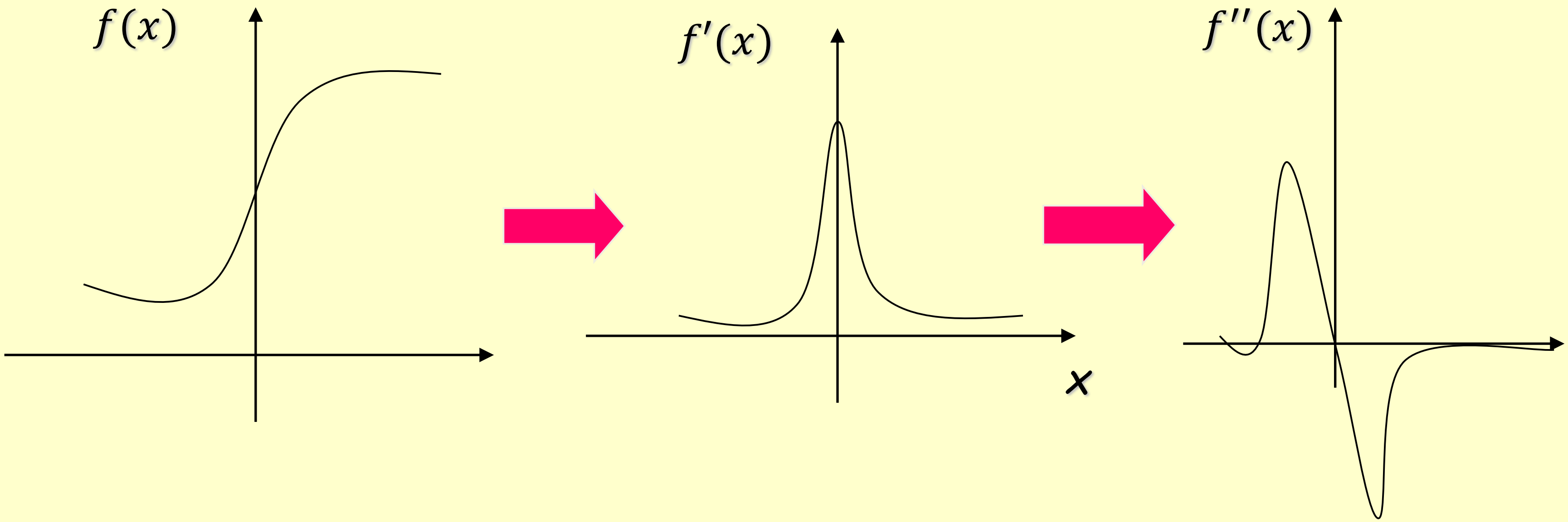
$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\nabla G(x,y) = \left(G_x, G_y\right)$$



Second-order derivative filters (1D)

- Peaks of the first-derivative of the input signal correspond to "zero-crossings" of the second-derivative.



Second-order derivative filters (1D)

- The condition: $f''(x) = 0$ is not enough for edgeness
  - $f(x) = c$ has $f''(x) = 0$, but there is no edge
- We need check whether $|f'(x)|$ is big enough

2D Laplacian Operator

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

- Negative definition

$$\begin{aligned}
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}, \\
\frac{\partial^2 f}{\partial x^2} &= f(x+1, y) + f(x-1, y) - 2f(x, y), \\
\frac{\partial^2 f}{\partial y^2} &= f(x, y+1) + f(x, y-1) - 2f(x, y), \\
\nabla^2 f &= f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y).
\end{aligned}$$

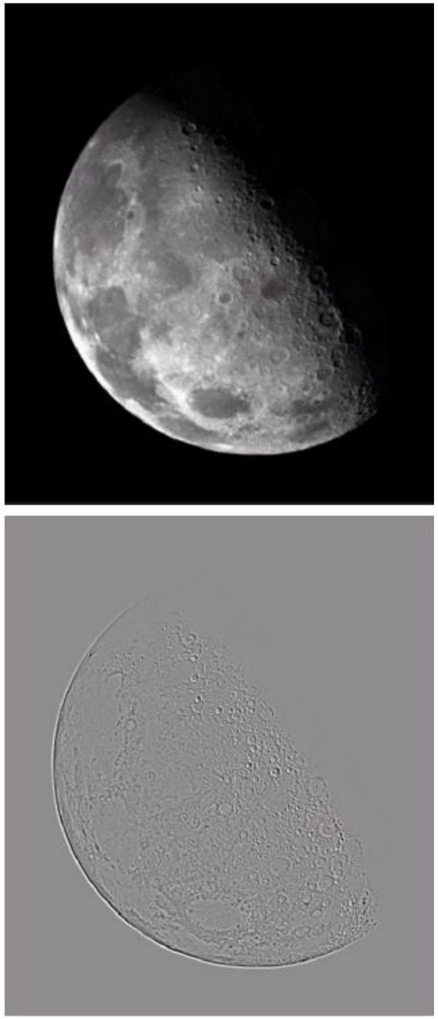
- Positive definition

$$\nabla^2 f = -[f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] + 4f(x, y).$$

- Diagonal derivatives also can be included.

2D Laplacian Operator

$f(x, y)$



$\nabla^2 f(x, y)$

- $\nabla^2 I(x, y)$ is a scalar (isotropic)
  - Pros: It can be found using a SINGLE mask
  - Cons: The orientation information is lost
- $\nabla^2 I(x, y)$ is the sum of second-order derivatives
  - But taking derivatives increases noises
  - Very sensitive to noises
- It is always **combined with a smoothing (Gaussian) operation**

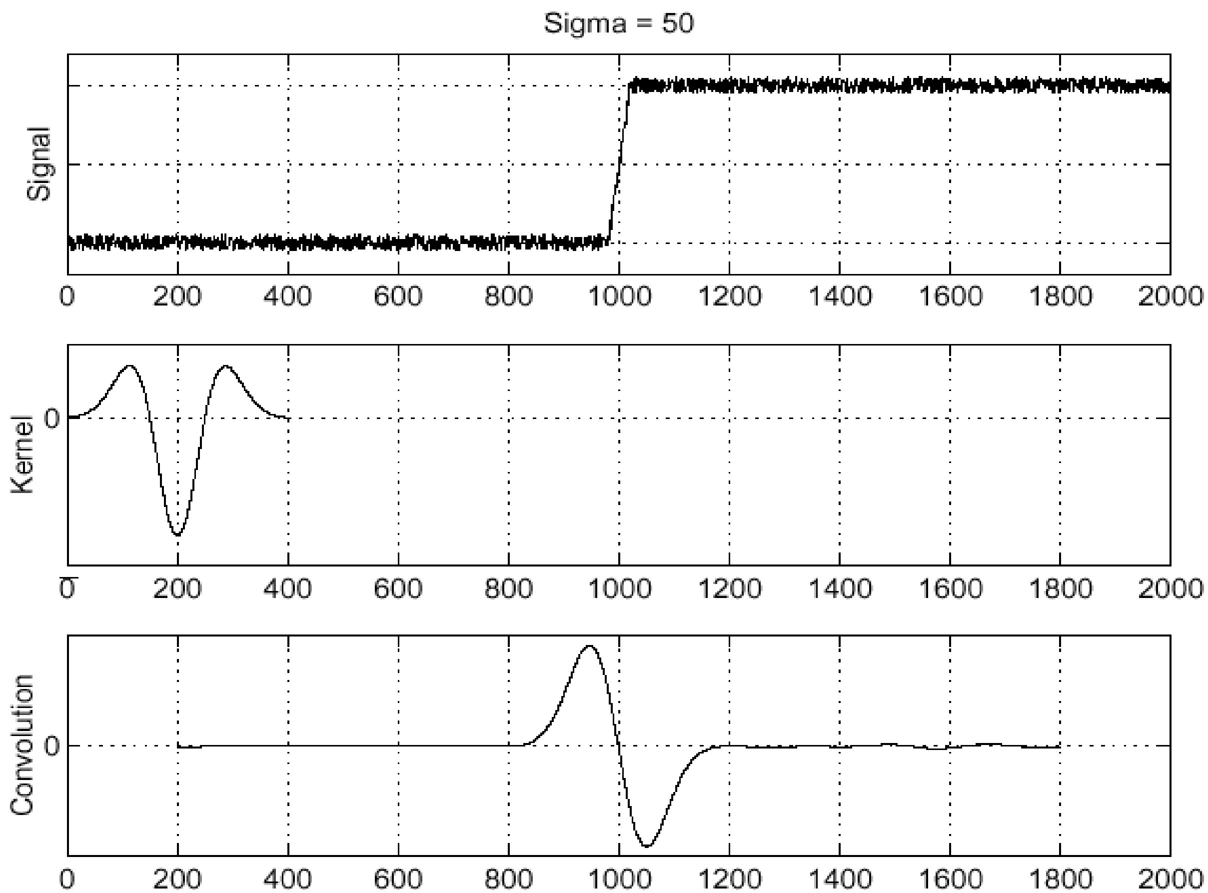
Laplacian of Gaussian (LOG)

- In 1D, consider $\frac{\partial^2}{\partial x^2}(G * I) = \left(\frac{\partial^2}{\partial x^2} G\right) * I$

$$I$$

$$\frac{\partial^2}{\partial x^2} G$$

$$\left(\frac{\partial^2}{\partial x^2} G\right) * I$$



- Edge is the zero-crossing of the bottom graph

Laplacian of Gaussian (LOG)

- $O(x,y) = \nabla^2(I(x,y) * G(x,y))$
  1. Smoothing with a Gaussian filter
  2. Finding zero-crossings with a Laplacian filter

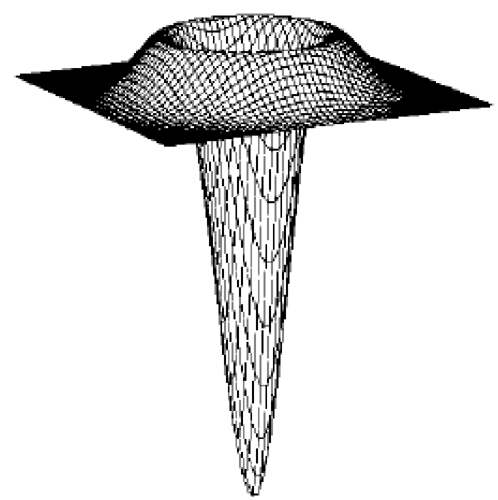
- Using linearity:
  - $O(x,y) = \nabla^2 G(x,y) * I(x,y)$
  - The combined filter is called LOG

$$G(x,y) = \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$\nabla^2 G(x,y) = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}$$

$$= \left(\frac{x^2 + y^2}{\sigma^4} - \frac{2}{\sigma^2}\right)\exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$= \left(\frac{r^2}{\sigma^4} - \frac{2}{\sigma^2}\right)\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

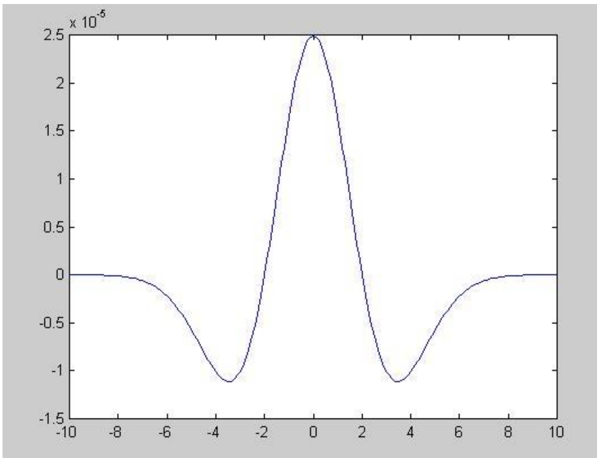


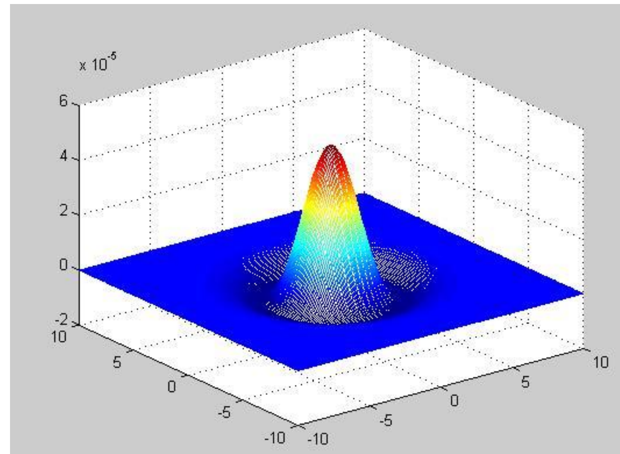
LOG Filter

- Mexican hat operator (inverted LoG)

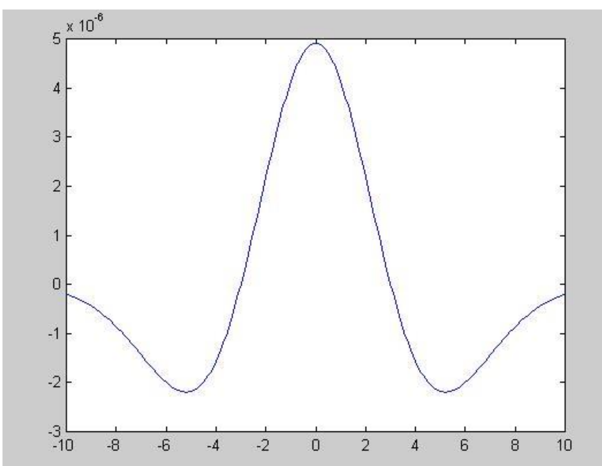
1-D

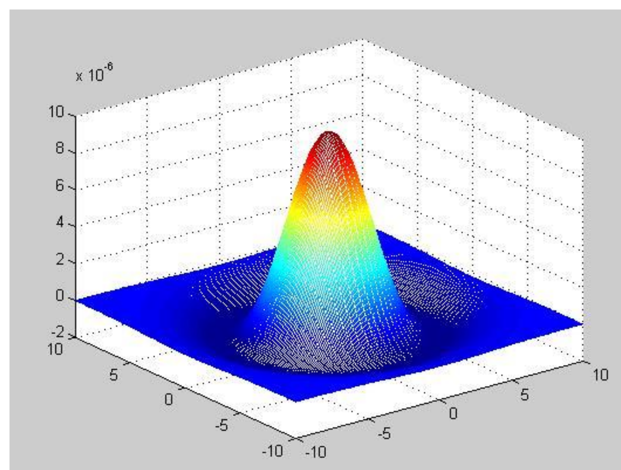
2-D





$\sigma = 2$

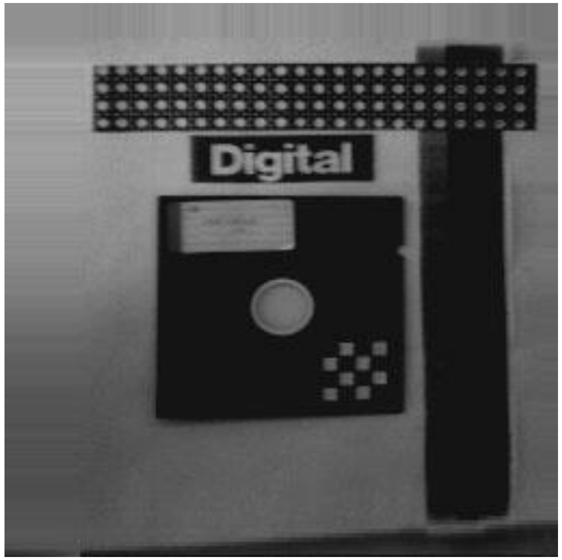


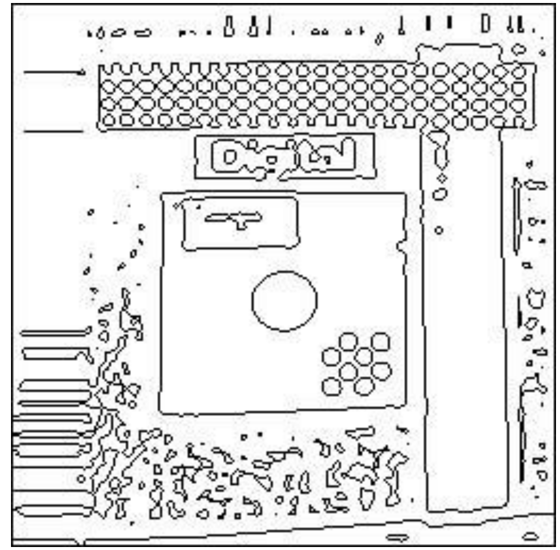


$\sigma = 3$

LOG Filter

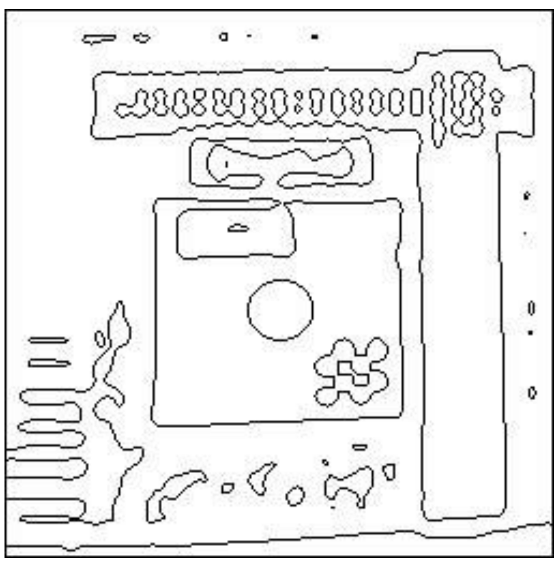
Original image

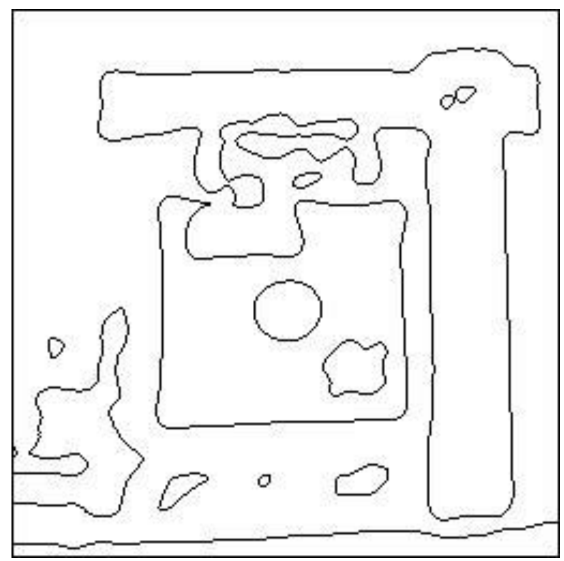




$\sigma = 2.0$

$\sigma = 4.0$





$\sigma = 6.0$

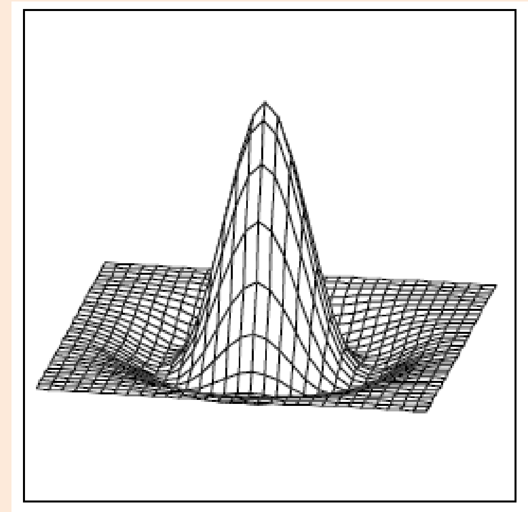
Second-Order Edge Detectors

- The Marr-Hildreth Operator
  1. Laplacian of Gaussian (LoG)
  2. Finding zero-crossing points

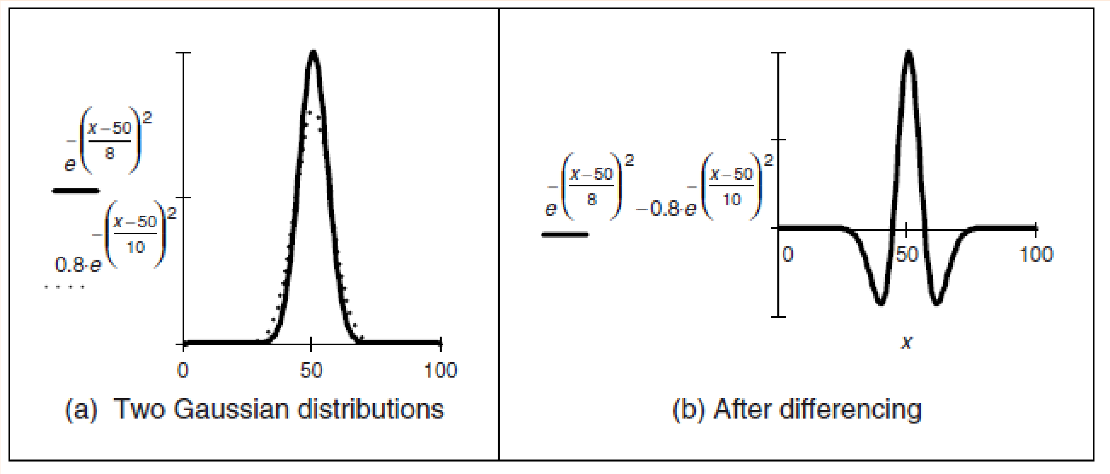
Second-Order Edge Detectors

- Laplacian of Gaussian (LoG)

$$\nabla^2(g(x,y) * \mathbf{P}) = \nabla^2(g(x,y)) * \mathbf{P}$$



  - $\nabla^2(g(x,y)) = \frac{1}{\sigma^2}\left(\frac{(x^2+y^2)}{\sigma^2} - 2\right)e^{-\frac{(x^2+y^2)}{2\sigma^2}}$
  - Maxican hat operator
  - It is similar to the difference of Gaussian

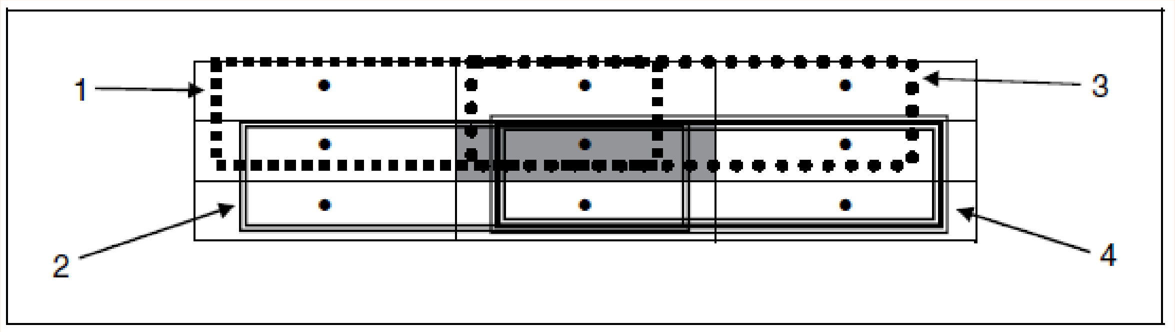


(a) Two Gaussian distributions

(b) After differencing

Second-Order Edge Detectors

- Finding zero-crossing points



  - Find the averages of the four quadrants
  - If the max average is positive and the min average is negative, then the center point is detected

Second-Order Edge Detectors

- Result of the Marr-Mildreth operator

(a) Face image | (b) 11×11 LoG | (c) 15×15 LoG

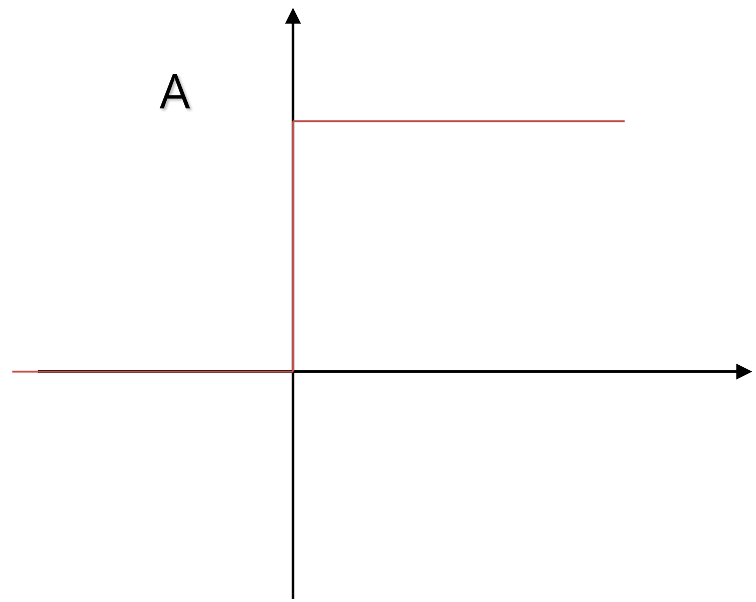
Canny Edge Detector

- Canny Edge Detector
  - Uses a mathematical model of the edge and noises
  - Sets a performance criterion
  - Synthesizes the optimal filter
- Experiments consistently show that it performs very well
- Widely used by C.V. practitioners for 30 years

- **J. Canny,** *"A Computational Approach to Edge Detection"*, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol 8, No. 6, Nov 1986.

Edge & Noise Model (1D)

- An ideal edge can be modeled as an step



- Additive, white Gaussian noise

Performance Criteria

- Good detection
  - The filter must have a strong response at the edge location ($x = 0$)
- Good localization
  - The filter response must be maximum very close to $x = 0$
- Low false positives
  - There should be only one maximum in a reasonable neighborhood of $x = 0$

Optimal Filter

- Canny found a linear, continuous filter that maximized the three given criteria

- There is no close-form solution for the optimal filter

- However, it looks very similar to the derivative of Gaussian (DoG)

Canny Edge Detector

- Three procedures
  - Gradient computation
  - Nonmaximum suppression
  - Thresholding

Procedure: Gradient Computation

- Given an input image I and a zero mean Gaussian filter G (std = σ)
  1. $J = I * G$ (smoothing)
  2. For each pixel (i, j) (Gradient computation)
     - Compute the image gradient
       $$J(i,j) = (J_x(i,j), J_y(i,j))$$
     - Estimate edge strength
       $$E_s(i,j) = \left(J_x^2(i,j) + J_y^2(i,j)\right)^{1/2}$$
     - Estimate edge orientation
       $$E_{o(i,j)} = \arctan\left(\frac{J_y(i,j)}{J_x(i,j)}\right)$$
- The output are images E_s and E_o

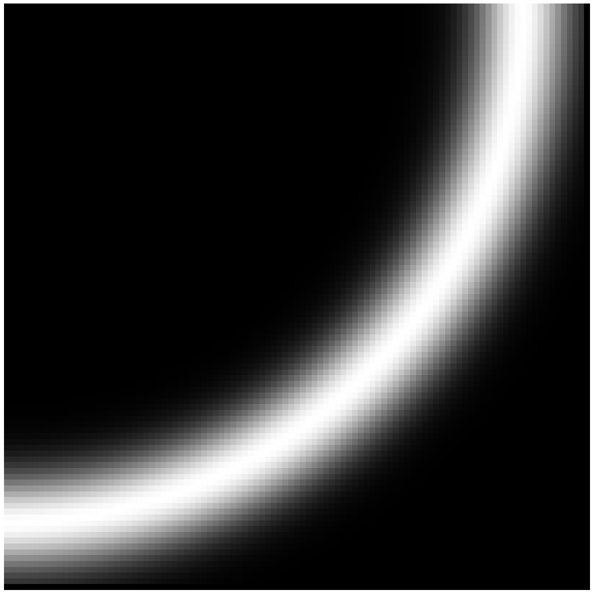
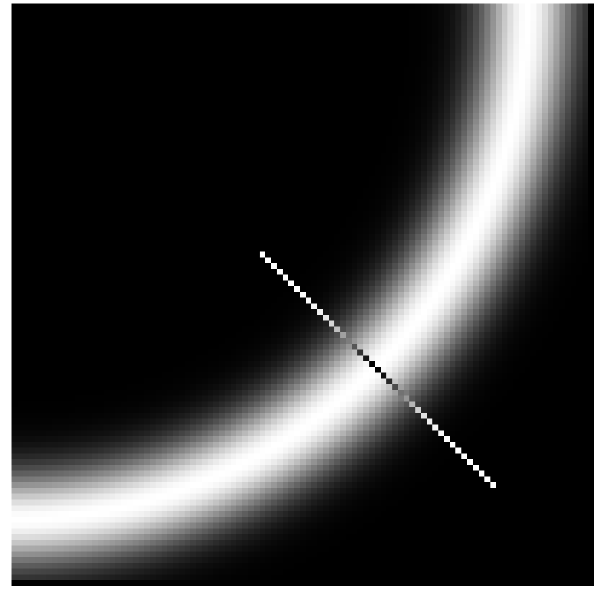
Nonmaximum Suppression

- E_s has the magnitudes of the smoothed gradient.
  - σ determines the amount of smoothing

- E_s has large values at edges

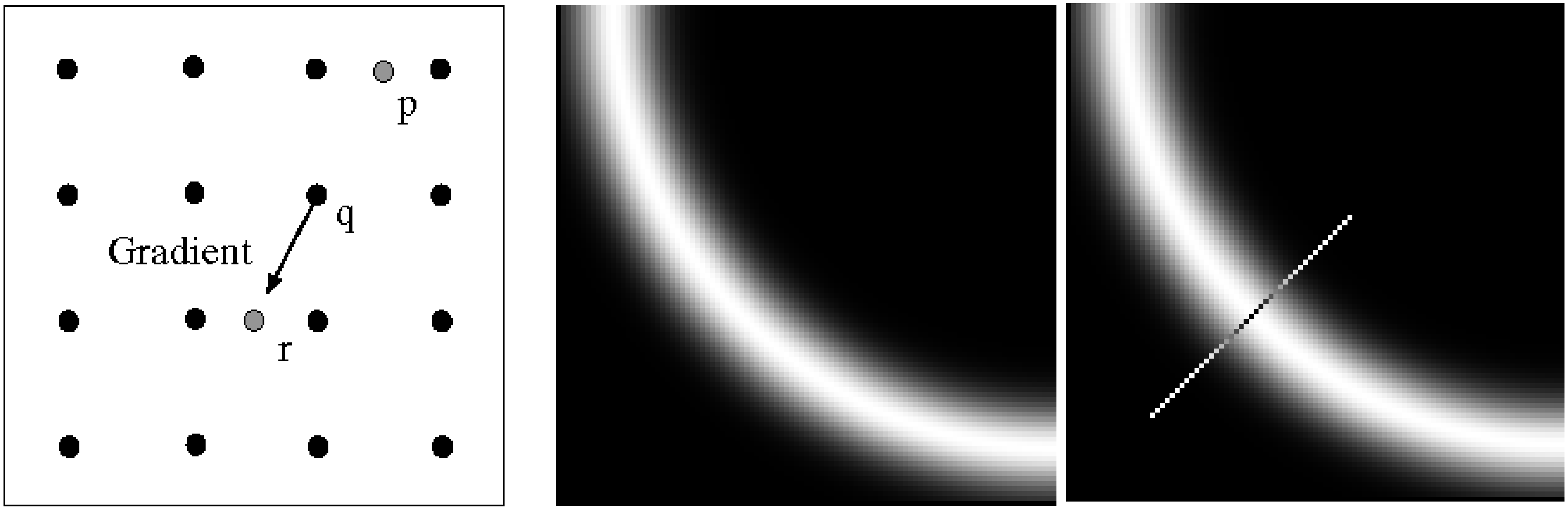
- However, E_s is large along thick trail. how do we identify the significant points?

NONMAXIMUM SUPRESSION

- We wish to mark points along the curve where the magnitude is biggest.
- We can do this by looking for the maximum along a slice normal to the curve (nonmaximum suppression).

NONMAXIMUM SUPRESSION



- Non-maximum suppression:
  - ✓ At *q*, we have a maximum if the value is larger than those at both *p* and at *r*.
  - ✓ Interpolate to get these value

Procedure: Nonmaximum Suppression

- The inputs are E_s & E_o
- Consider 4 directions $D = \{0°, 45°, 90°, 135°\}$
- For each pixel (i, j) do:
  1. Find the direction $d \in D$ s.t. $d \cong E_o(i, j)$ (normal to the edge)
  2. If $E_s(i, j)$ is smaller than at least one of its neighbor along d

     $$I_N(i, j) = 0$$

     Otherwise,

     $$I_N(i, j) = E_s(i, j)$$
- The output is the thinned edge image I_N

Procedure: Thresholding

- Edges are found by thresholding the output of NONMAX_SUPRESSION
- If the threshold is too high:
  - Very few (none) edges
    - Many false negatives, many gaps
- If the threshold is too low:
  - Too many (all pixels) edges
    - Many false positives, many extra edges

Results

original image

Gradients

Nonmaximum suppression and thresholding

Results

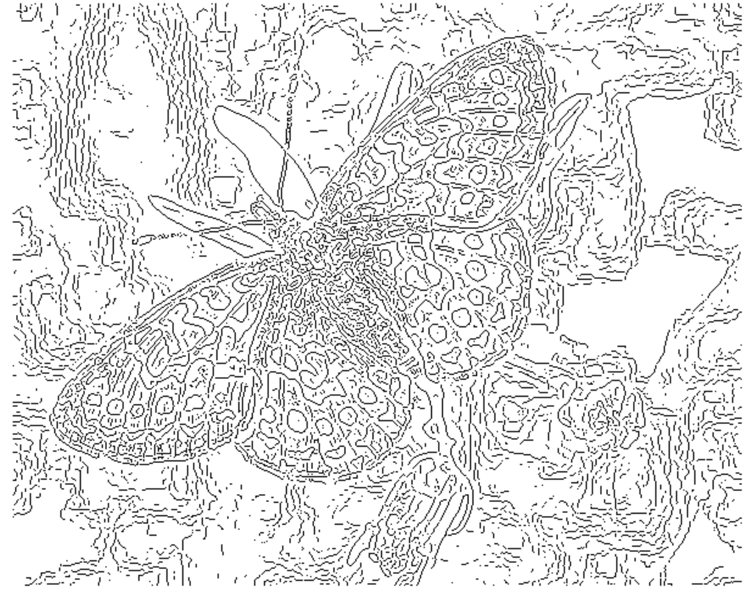
original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

- σ controls the scale of the features
  - ✓ large σ detects large scale edges only
  - ✓ small σ detects fine features as well

fine scale, high threshold

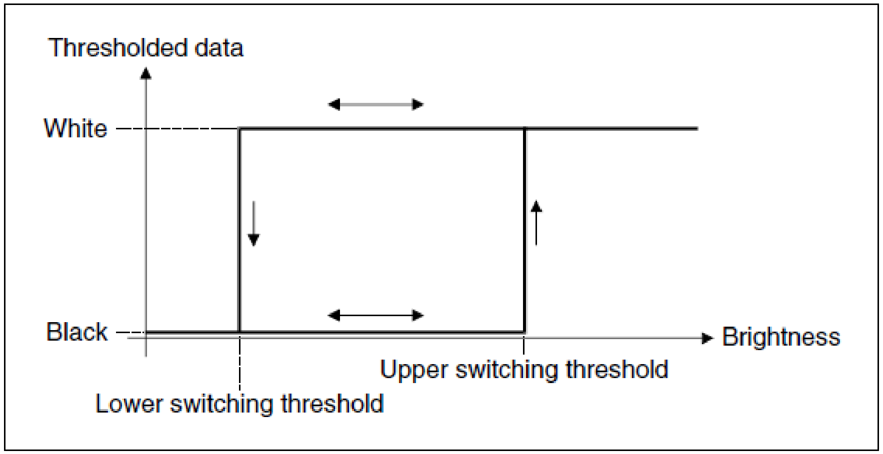


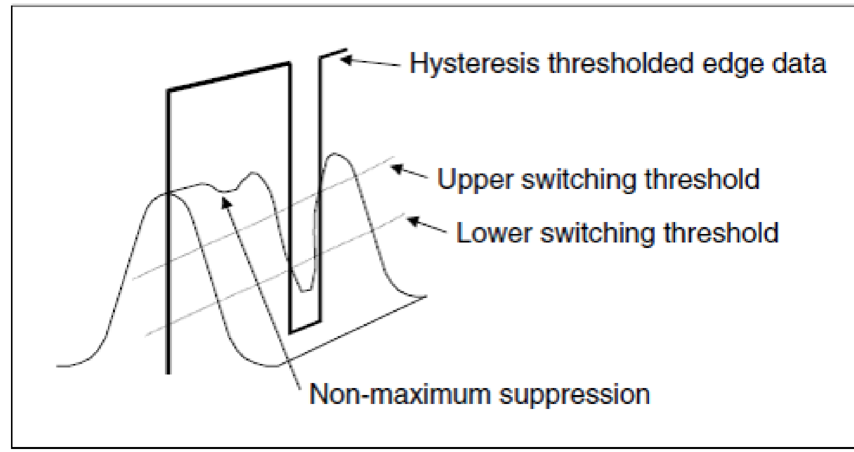
coarse scale, high threshold

coarse scale, low threshold

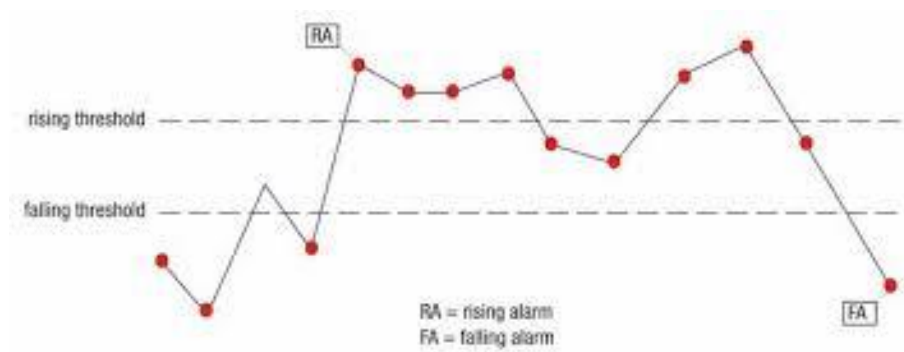
Canny Edge Detector

- Hysteresis thresholding



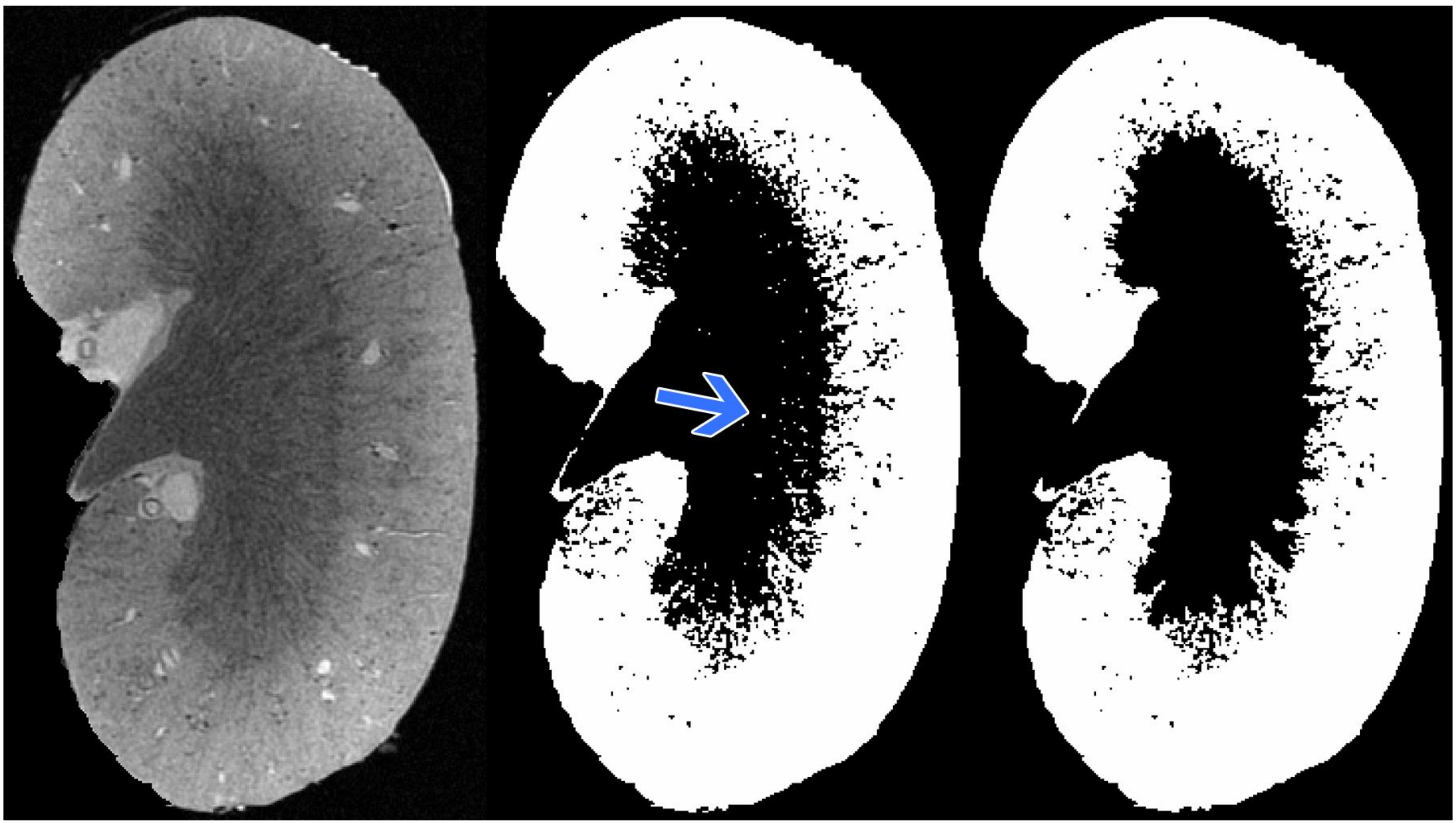


  - Recursive search of 8 neighbors



Hysteresis Thresholding

original *simple thresholding* *hystereisis*



Challenges or Opportunities?

Edges are really at the lower level?
Can we find better edges or silhouettes?