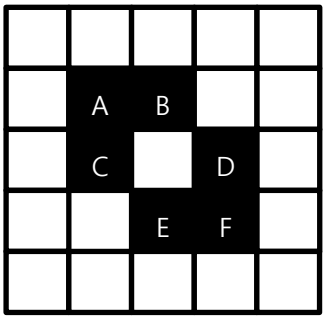
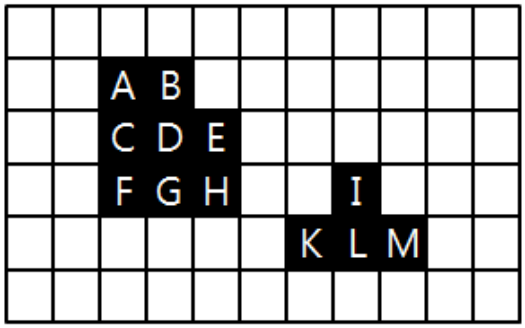
Computer Vision 2017: Assignment #2

Due 4 April 2017

1. Answer the questions in Yes or No.
   (a) In the 4-neighborhood definition, are A and F connected?
   (b) In the 4-neighborhood definition, is {A, B, C} a connected component?
   (c) In the 8-neighborhood definition, are A and F connected?
   (d) In the 8-neighborhood definition, is {A, B, C} a connected component?



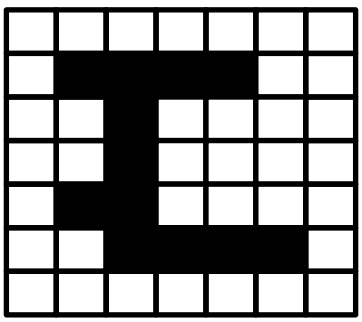
2. An image **B** and a structuring element **S** is given below. The origin of the structuring element is denoted by the letter 'O.'



Binary Image **B** Structuring Element **S**

   (a) In the 8-neighborhood sense, are 'A' and 'K' connected?
   (b) In the 8-neighborhood sense, is (A, B, D, H) a path?
   (c) In the 8-neighborhood sense, is (A, D, H) a path?
   (d) Let us define the length of a path $(A_0, A_1, \dots, A_n)$ as
   $$n = \text{the number of elements in the sequence} - 1.$$
   In the 4-neighborhood sense, what is the shortest length of a path between 'A' and 'H'?
   (e) Plot the dilation result $\mathbf{B} \oplus \mathbf{S}$.
   (f) In the dilation result $\mathbf{B} \oplus \mathbf{S}$, in the 8-neighborhood sense, are 'A' and 'K' connected?

3. Close the image B with the structuring element S. The origin of the structuring element is its center.



B

S

4. We learned the concepts of entropy and mutual information.
   (a) Let X be a random variable with $p(1) = \frac{1}{2}, p(2) = \frac{1}{4}, p(3) = \frac{1}{8}, p(4) = \frac{1}{8}$. What is the entropy H(X)?
   (b) Let Y be another random variable. Given the random variable X in (a), what is the maximum possible value of $I(X;Y)$? When is the maximum achieved?

5. Let X and Y be random variables with symbols {1, 2, 3} and joint distribution p(x,y), which is given by
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$
where the element in x-th row and y-th column is p(x,y). Determine H(X) and I(X;Y)

6. A method to design a decision tree is to use the concept of the mutual information. Specifically, we find the feature X, which maximizes the mutual information $I(X;C)$ with the class C, and use it in the root node. Suppose that you find the optimal feature X. A friend of yours told you that she made a better feature Y from X using the relation $Y = 1 - 2X$ and improved the classification performance. Did she tell you a lie or a truth?

7. The owner of a skip shop must order skis for the upcoming season. Orders must be placed in quantities of 25 pairs of skis. The cost per pair of skis is $50 if 25 are ordered, $45 if 50 are ordered, and $40 if 75 are ordered. The skis will be sold at $75 per pair. Any skis left over at the end of the year can be sold for sure at $25 a pair. If the owner runs out of skis during the season, he will suffer a loss of “goodwill” among unsatisfied customers. He rates this loss at $5 per unsatisfied customer. The owner feels that the demand for the skis will be 30, 40, 50 or 60 pairs with probabilities 0.2, 0.4, 0.2 and 0.2, respectively. What is the Bayesian (= as profitable as possible) action that the owner should take?