학번 이름.

1. a) Verify that $A^2 = I$ when $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

b) Show that $M^2 = I$ when

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

c) Generalize the idea in b) by contructing a 6×6 matrix $M = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ C & 0 & D \end{bmatrix}$ such that $M^2 = I$.

Make C a nonzero 2×2 matrix.

2. Construct a 3×5 matrix A such that $\dim N(A) = 3$ and $\dim R(A) = 2$.

3. Suppose $A^n = 0$ for some $n > 1$. Find an inverse for $I - A$.

4. $V = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \; a - 3b + c = 0 \right\}$

(a) Find a basis of V.

(b) What is its dimension?

5. Prove that $Ax = 0$ if $A^TAx = 0$.

6. Describe all least-squares solutions of the system

$$\begin{cases} x+y=2 \\ x+y=4 \end{cases}$$

7. What can you say about the least-squares solution of $Ax=b$ when b is orthogonal to the columns of A?