

Computer Vision: Mid Exam

3:30 – 4:45 AM, 18 APR 2017

1. (20 pts) Answer whether each of the following statements is always true (A), sometimes true (S), or never true (N). You need not provide explanation.

- (a) $B \subset B \oplus S$
- (b) $B \cdot S = (B \cdot S) \cdot S$
- (c) A point operator for image enhancement can be replaced with a masking operator (*i.e.* linear filter).
- (d) The entropy $H(X)$ of a random variable X is negative.
- (e) Bayesian decision making minimizes the classification error probability.

2. (15 points) For a natural number n , D_n denotes the number of different ways to write n as the sum of 1, 3, 4. For example, $D_1 = 1, D_2 = 1, D_3 = 2, D_4 = 4$. Also, $D_5 = 6$ because

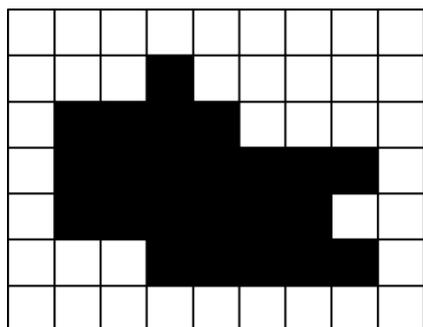
$$\begin{aligned} 5 &= 1 + 1 + 1 + 1 + 1 \\ &= 1 + 1 + 3 \\ &= 1 + 3 + 1 \\ &= 3 + 1 + 1 \\ &= 1 + 4 \\ &= 4 + 1 \end{aligned}$$

- (a) What is D_4 ?
- (b) What is D_6 ?
- (c) In general, what is the recurrence equation for solving this problem?

3. Find the mode, median, and mean of the following numbers.

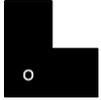
3, 4, 8, 7, 5, 3, 6

4. (5 points) Draw the distance transform result of the following image, where a black pixel has value '1' and a white pixel has value '0.'



5. (10 points) Open the binary image in Problem 2 with the following structuring element. The origin of the structuring element is denoted by 'o.'

YOU HAVE 8 PROBLEMS IN TOTAL IN FRONT AND BACK PAGES!!!!



6. (10 pts) $f(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{x}^T \mathbf{b} + c$, where $\mathbf{x} = \begin{bmatrix} u \\ v \end{bmatrix}$.

(a) Suppose that $A = \begin{bmatrix} 6 & -3 \\ 7 & 5 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and $c = -2$. Compute the gradient $\frac{\partial f}{\partial \mathbf{x}}$.

(b) Compute the gradient $\frac{\partial f}{\partial \mathbf{x}}$ for general A , \mathbf{b} , and c . Note that A need not be symmetric.

7. (20 pts) Let us consider a 1D classification problem with training samples

$$\text{Class } \omega_1: \quad x_1 = -5, x_2 = -3, x_3 = 3, x_4 = 5$$

$$\text{Class } \omega_2: \quad x_5 = 0, x_6 = 1$$

- (a) What is the nearest-neighbor (NN) classification rule?
(b) What is the 3-NN classification rule?

8. (20 points) In the single-link clustering, we learned the following notion of ‘distance’ between two sets of points.

$$d(A, B) = \min_{a \in A, b \in B} d(a, b) = \min_{a \in A, b \in B} \|a - b\|$$

Let us check whether it is indeed a distance function. Are the following statements true or false? Prove or disprove them.

- (a) $d(A, B) \geq 0$.
(b) $d(A, B) = 0$ if and only if $A = B$.
(c) $d(A, B) = d(B, A)$.
(d) $d(A, B) + d(B, C) \geq d(A, C)$.
9. (10 points) Suppose that we have 100 male and only 10 female students in our class. 90 of the male students have short hair, and 10 of them have long hair. 8 of the female students have long hair, and the other 2 have short hair. Suppose that a student has a long hair. Is the student male or female?
- (a) Provide the ML answer.
(b) Provide the MAP answer.