

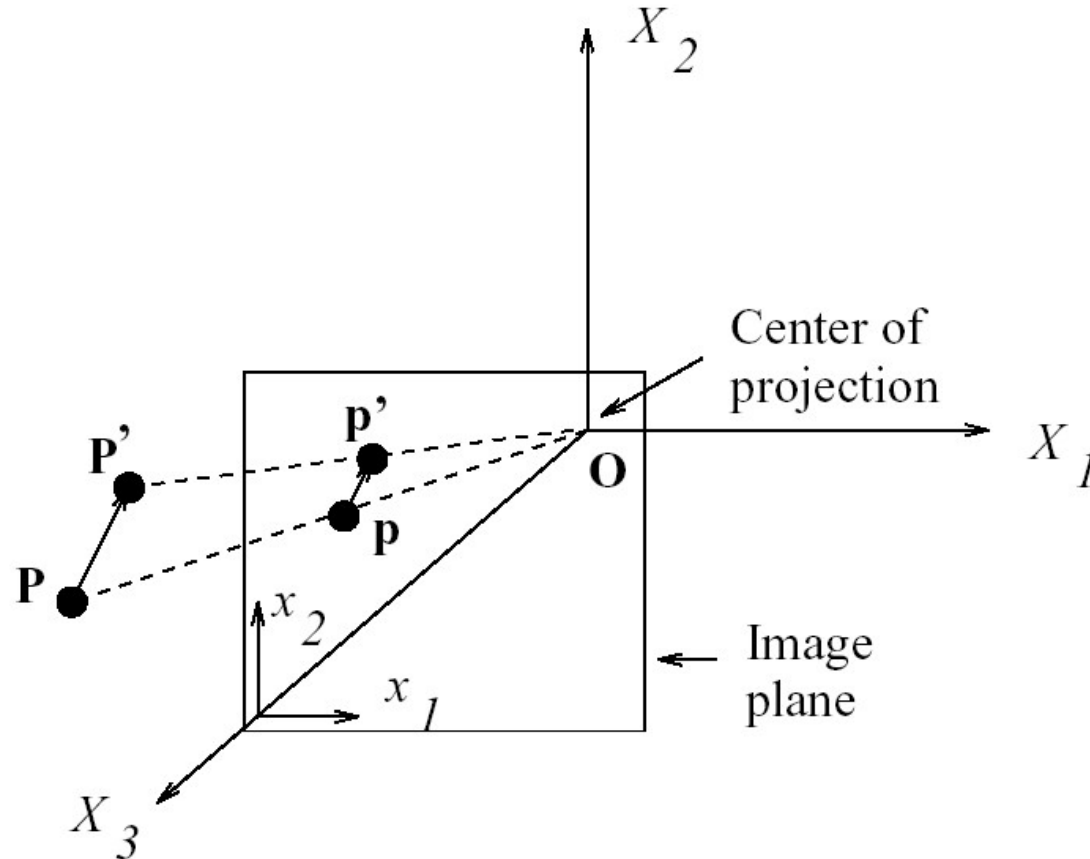
KECE471 Computer Vision

# Motion

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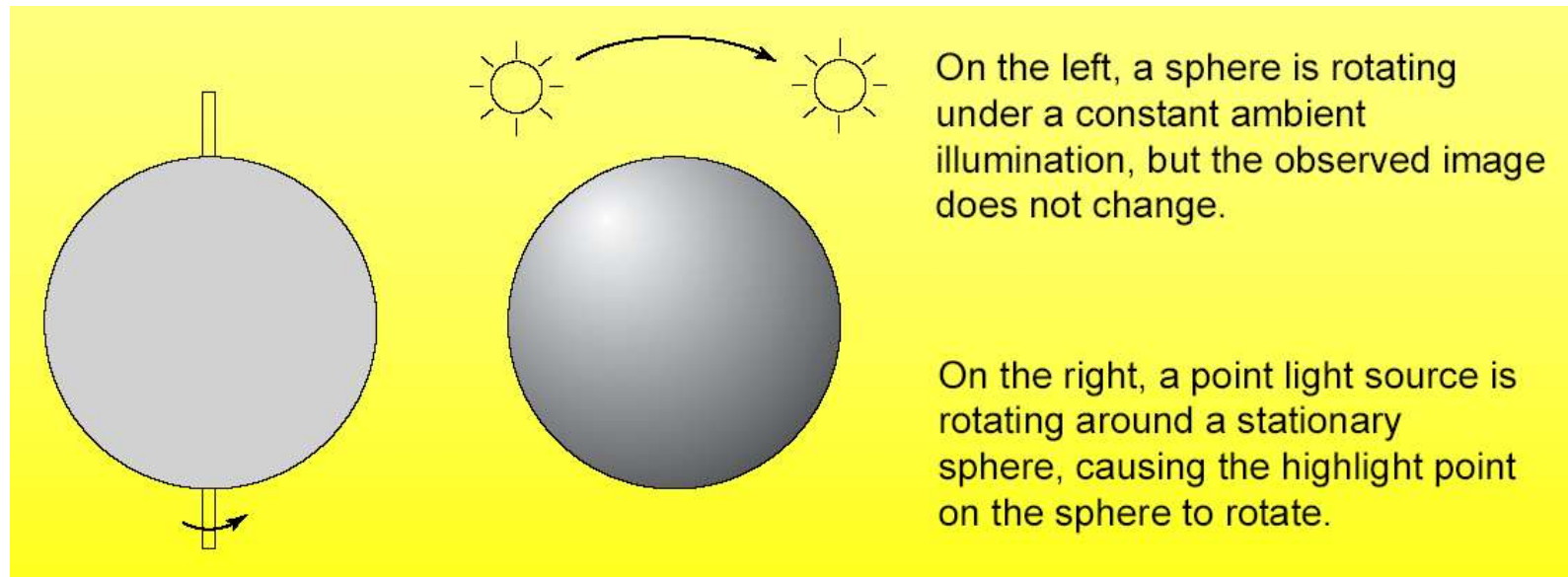
# 2D Motion vs. Optical Flow

- True 2D motion
  - There is 3D motion between object and camera
  - It is projected onto 2D imaging plane



# 2D Motion vs. Optical Flow

- Optical flow
  - **observed, perceived, apparent** 2D motion based on changes in pixel luminance
  - It also depends on illumination and object surface texture
  - It may **not** represent true 2D motion



# Optical Flow Equation

- Given only video sequence without any other information (such as illumination condition), we cannot estimate true 2D motion.
- The best one can hope to estimate is optical flow
- Constant intensity assumption → optical flow equation

Under "constant intensity assumption":

$$\psi(x + d_x, y + d_y, t + d_t) = \psi(x, y, t)$$

But, using Taylor's expansion :

$$\psi(x + d_x, y + d_y, t + d_t) = \psi(x, y, t) + \frac{\partial \psi}{\partial x} d_x + \frac{\partial \psi}{\partial y} d_y + \frac{\partial \psi}{\partial t} d_t$$

Compare the above two, we have the

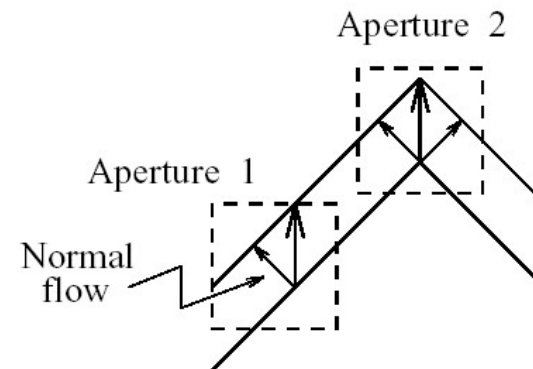
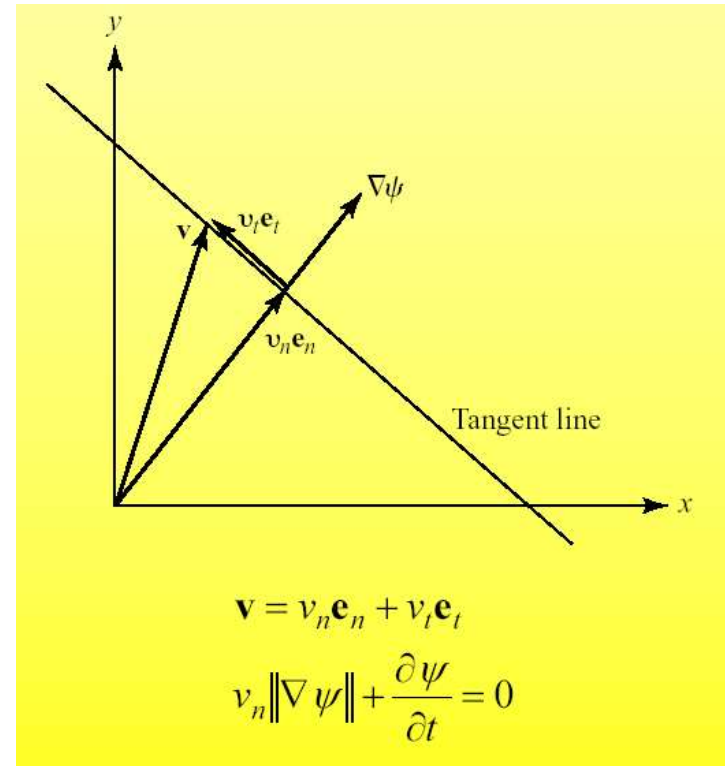
spatial  
gradient

optical  
flow

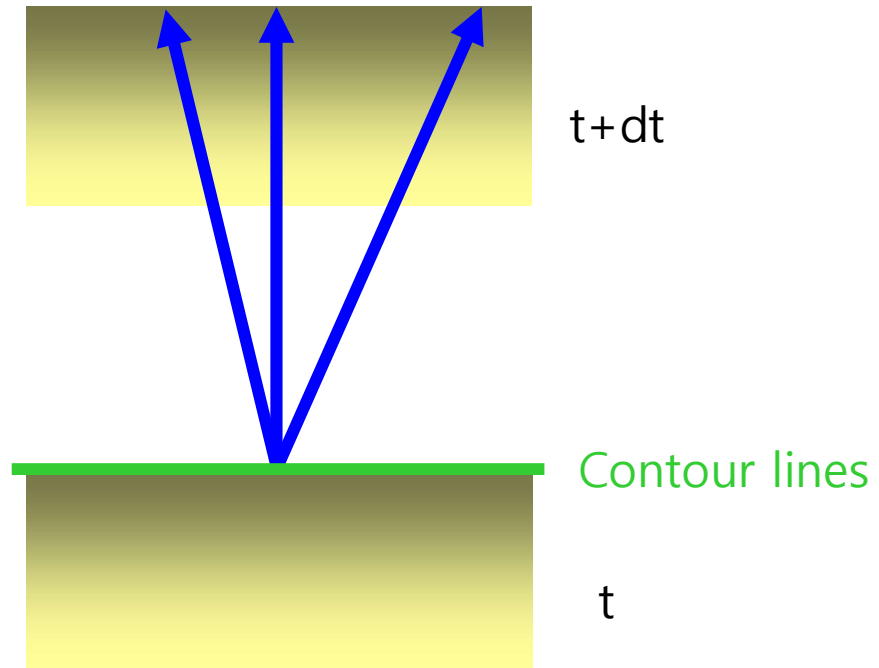
$$\frac{\partial \psi}{\partial x} d_x + \frac{\partial \psi}{\partial y} d_y + \frac{\partial \psi}{\partial t} d_t = 0 \quad \text{or} \quad \frac{\partial \psi}{\partial x} v_x + \frac{\partial \psi}{\partial y} v_y + \frac{\partial \psi}{\partial t} = 0 \quad \text{or} \quad \nabla \psi \cdot \mathbf{v} + \frac{\partial \psi}{\partial t} = 0$$

# Ambiguities in Optical Flow Estimation

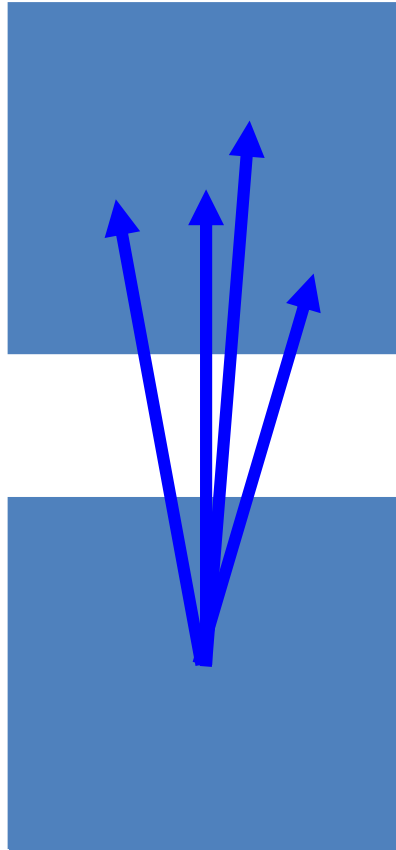
- Optical flow equation constrains the motion vector in the gradient direction  $v_n$  only
- The flow vector in the tangent direction  $v_t$  is under-determined
  - We can only determine the displacement that is orthogonal to the edges
- In regions with constant brightness  $\nabla\psi = 0$ , the flow is indeterminate
  - Optical flow estimation is unreliable in regions with flat texture and more reliable near edges



# Ambiguities in Optical Flow Estimation



# Ambiguities in Optical Flow Estimation



# Example

- Consider a video signal  $s(x, y, t)$  defined over the entire 3-D space  $(x, y, t)$ , where  $s(x, y, t)$  is generated from one object, which undergoes translational motion with a uniform constant velocity  $(v_x, v_y)$ . Suppose that  $s(x, y, 0) = x + y + xy$ .
  1. Determine  $s(x, y, t)$ .
  2. Show that in this case the following optical flow equation holds true.

$$v_x \frac{\partial s(x, y, t)}{\partial x} + v_y \frac{\partial s(x, y, t)}{\partial y} + \frac{\partial s(x, y, t)}{\partial t} = 0$$

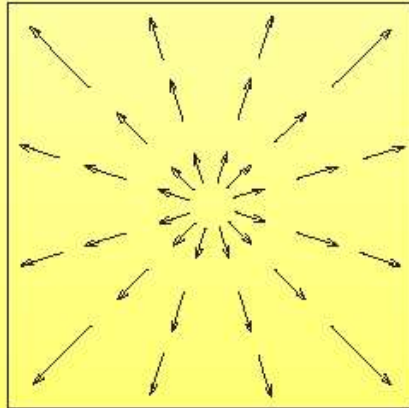


# General Considerations for Motion Estimation

- Two categories of approaches
  - Feature based
    - Correspondence between edges, points, etc
    - Object tracking, 3D reconstruction from 2D
  - Intensity based
    - Optical flow estimation based on constant intensity assumption
    - Focus in this class
- Three important questions
  - How to **represent** the motion field?
  - Which **cost function** (criterion) to use to estimate motion parameters?
  - Which **optimization** technique?

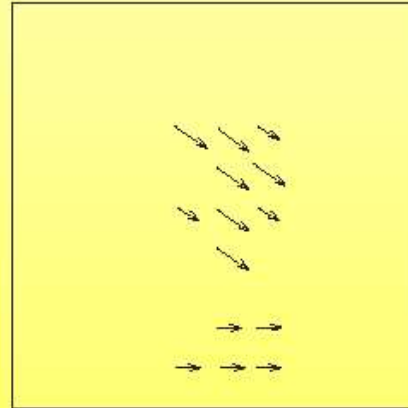
# Motion Representation

Global:  
Entire motion field is represented by a few global parameters



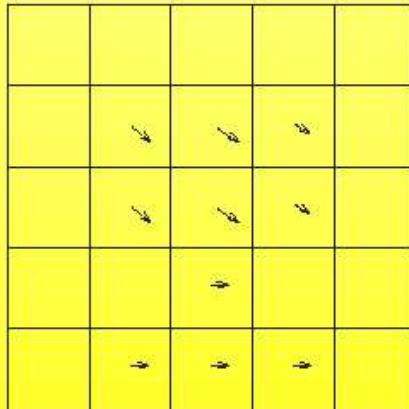
(a)

Pixel-based:  
One MV at each pixel, with some smoothness constraint between adjacent MVs.



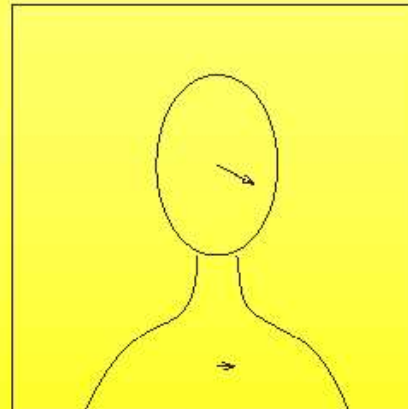
(b)

Block-based:  
Entire frame is divided into blocks, and motion in each block is characterized by a few parameters.



(c)

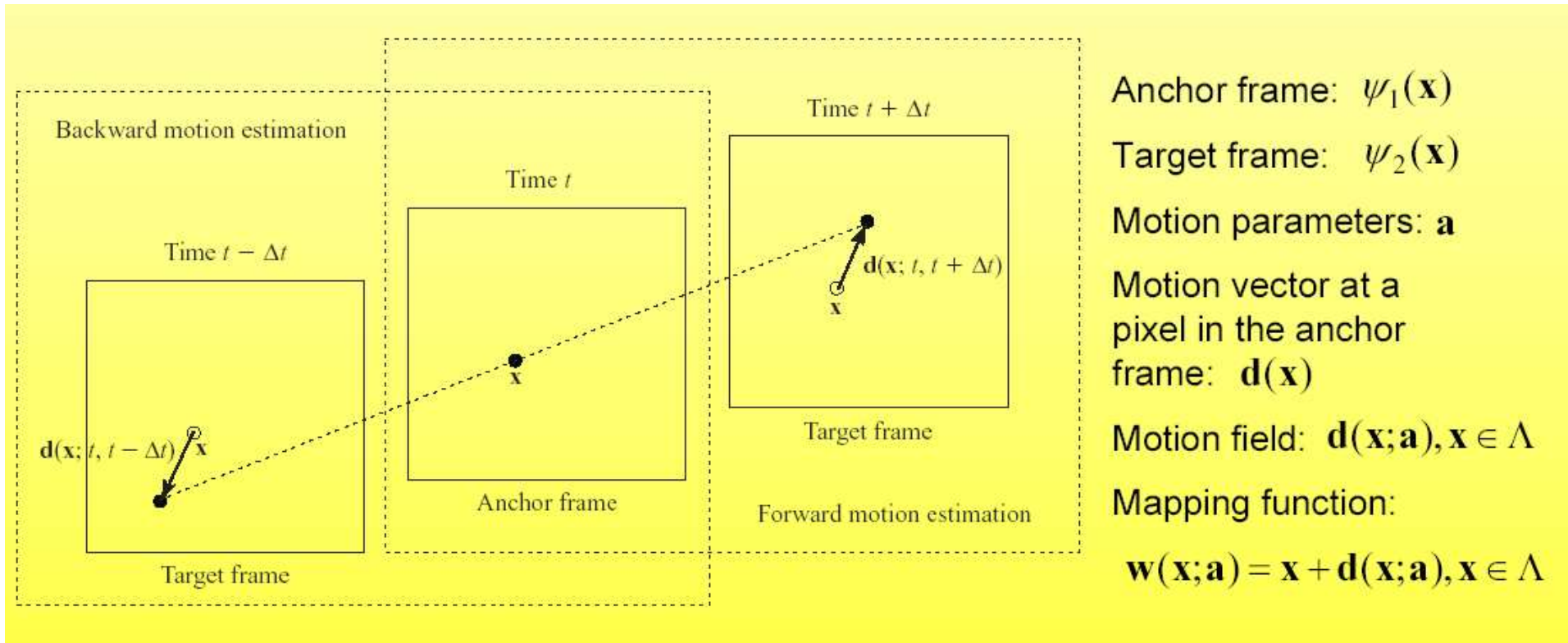
Region-based:  
Entire frame is divided into regions, each region corresponding to an object or sub-object with consistent motion, represented by a few parameters.



(d)

Other representation: mesh-based (control grid) (to be discussed later)

# Notations



# Motion Estimation Criteria

- Minimize displaced frame difference

$$E_{DFD}(\mathbf{a}) = \sum_{\mathbf{x} \in \Lambda} |\psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) - \psi_1(\mathbf{x})|^p$$

- For  $p = 2$  (MSE), the necessary condition for minimum is that the derivative is zero. Let  $\mathbf{z} = \mathbf{w}(\mathbf{x}; \mathbf{a}) = \mathbf{x} + \mathbf{d}(\mathbf{x}; \mathbf{a})$ . Then, the derivative is given by

$$\begin{aligned} \frac{\partial E_{DFD}}{\partial \mathbf{a}} &= 2 \sum_{\mathbf{x} \in \Lambda} (\psi_2(\mathbf{z}) - \psi_1(\mathbf{x})) \frac{\partial \psi_2}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{a}} \\ &= 2 \sum_{\mathbf{x} \in \Lambda} (\psi_2(\mathbf{z}) - \psi_1(\mathbf{x})) \frac{\partial \psi_2}{\partial \mathbf{z}} \frac{\partial \mathbf{d}}{\partial \mathbf{a}} \end{aligned}$$

# Motion Estimation Criteria

- Optical Flow Equation

$$\begin{aligned}\frac{\partial \psi}{\partial x} d_x + \frac{\partial \psi}{\partial y} d_y + \frac{\partial \psi}{\partial t} d_t &= 0 \\ \Rightarrow \frac{\partial \psi}{\partial x} v_x + \frac{\partial \psi}{\partial y} v_y + \frac{\partial \psi}{\partial t} &= 0\end{aligned}$$

where  $\mathbf{d}(\mathbf{x}; \mathbf{a}) = [d_x, d_y]^T$  and

$\mathbf{v}(\mathbf{x}; \mathbf{a}) = [v_x, v_y]^T = [d_x/d_t, d_y/d_t]^T$ .

- Thus, obtain the motion vector parameters  $\mathbf{a}$  which minimizes the cost function

$$E_{OF}(\mathbf{a}) = \sum_{\mathbf{x} \in \Lambda} \left| \frac{\partial \psi}{\partial \mathbf{x}} \mathbf{v}(\mathbf{x}; \mathbf{a}) + \frac{\partial \psi}{\partial t} \right|^p$$

# Lucas-Kanade Method

- Based on optical flow equation
- Assuming all pixels in a small block surrounding a pixel have the same motion vector

# Lucas-Kanade Method

■ Optical Flow Equation  $\frac{\partial \psi}{\partial x} v_x + \frac{\partial \psi}{\partial y} v_y + \frac{\partial \psi}{\partial t} = 0$

■ For each pixel one equation two unknowns  $(v_x, v_y)$

■ Under-determined system

■ Idea

■ Assume that a block  $\mathcal{B}$  centered around the current pixel has the same motion and then minimize

$$E(v_x, v_y) = \sum_{\mathbf{x} \in \mathcal{B}} \left( \frac{\partial \psi}{\partial x}(\mathbf{x}) v_x + \frac{\partial \psi}{\partial y}(\mathbf{x}) v_y + \frac{\partial \psi}{\partial t}(\mathbf{x}) \right)^2$$

■ Then, the optimal vector is set as the vector of the current pixel

# Lucas-Kanade Method

## ■ Solution

■ By setting the partial derivatives with respect to  $v_x$  and  $v_y$  to zeros

$$\sum_{\mathbf{x} \in \mathcal{B}} \left( \frac{\partial \psi}{\partial x}(\mathbf{x}) v_x + \frac{\partial \psi}{\partial y}(\mathbf{x}) v_y + \frac{\partial \psi}{\partial t}(\mathbf{x}) \right) \frac{\partial \psi}{\partial x}(\mathbf{x}) = 0,$$

$$\sum_{\mathbf{x} \in \mathcal{B}} \left( \frac{\partial \psi}{\partial x}(\mathbf{x}) v_x + \frac{\partial \psi}{\partial y}(\mathbf{x}) v_y + \frac{\partial \psi}{\partial t}(\mathbf{x}) \right) \frac{\partial \psi}{\partial y}(\mathbf{x}) = 0,$$

■ we have

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = - \begin{bmatrix} \sum_{\mathbf{x} \in \mathcal{B}} \frac{\partial \psi}{\partial x}(\mathbf{x}) \frac{\partial \psi}{\partial x}(\mathbf{x}) & \sum_{\mathbf{x} \in \mathcal{B}} \frac{\partial \psi}{\partial x}(\mathbf{x}) \frac{\partial \psi}{\partial y}(\mathbf{x}) \\ \sum_{\mathbf{x} \in \mathcal{B}} \frac{\partial \psi}{\partial x}(\mathbf{x}) \frac{\partial \psi}{\partial y}(\mathbf{x}) & \sum_{\mathbf{x} \in \mathcal{B}} \frac{\partial \psi}{\partial y}(\mathbf{x}) \frac{\partial \psi}{\partial y}(\mathbf{x}) \end{bmatrix}^{-1} \begin{bmatrix} \sum_{\mathbf{x} \in \mathcal{B}} \frac{\partial \psi}{\partial x}(\mathbf{x}) \frac{\partial \psi}{\partial t}(\mathbf{x}) \\ \sum_{\mathbf{x} \in \mathcal{B}} \frac{\partial \psi}{\partial y}(\mathbf{x}) \frac{\partial \psi}{\partial t}(\mathbf{x}) \end{bmatrix}$$