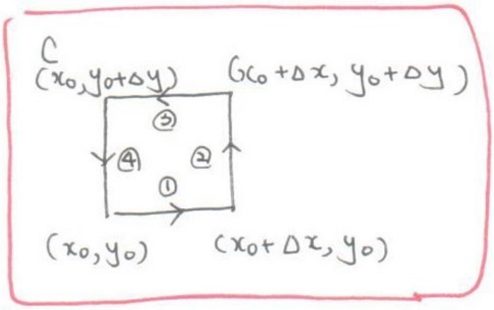
Heuristic proof of Cauchy's integral theorem

① Around a tiny rectangle C

$$\oint_C f(z)\,dz = \oint_C [u(z) + i v(z)](dx + i\,dy)$$

$$= \oint_C u\,dx - v\,dy + i \oint_C v\,dx + u\,dy$$



Real part

① $u(x_0, y_0)\,\Delta x$

② $-v(x_0 + \Delta x, y_0)\,\Delta y$

③ $u(x_0, y_0 + \Delta y)(-\Delta x)$

④ $-v(x_0, y_0)(-\Delta y)$

① + ② + ③ + ④

$$= -\Delta x\,[u(x_0, y_0 + \Delta y) - u(x_0, y_0)]$$
$$\quad - \Delta y\,[v(x_0 + \Delta x, y_0) - v(x_0, y_0)]$$

$$= -\Delta x\; u_y(x_0, y_0)\,\Delta y$$
$$\quad - \Delta y\; v_x(x_0, y_0)\,\Delta x$$

$$= [u_y(x_0, y_0) + v_x(x_0, y_0)] \times (-\Delta x\,\Delta y)$$

$$= 0 \qquad \because\; u_y = -v_x$$

Similarly the imaginary part is also 0. Q.E.D.

② It is true for every simple closed path C.

For example,