Data Compression

2D Transforms

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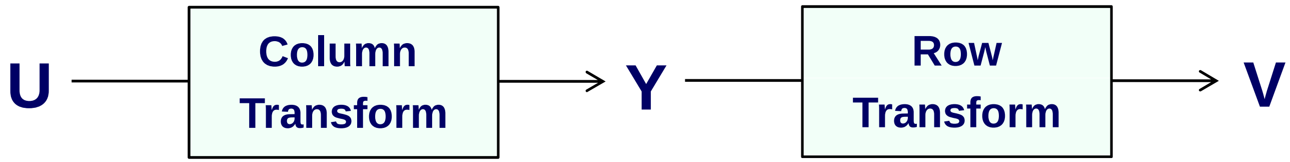
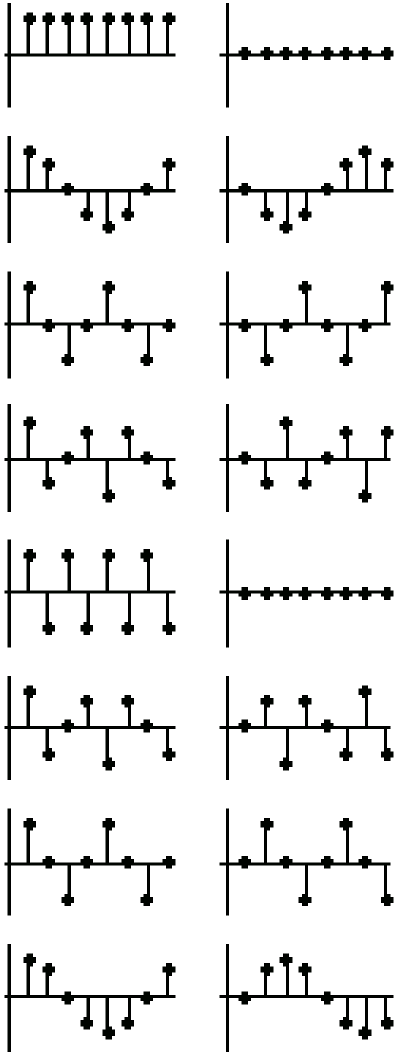
2D Separable Transforms



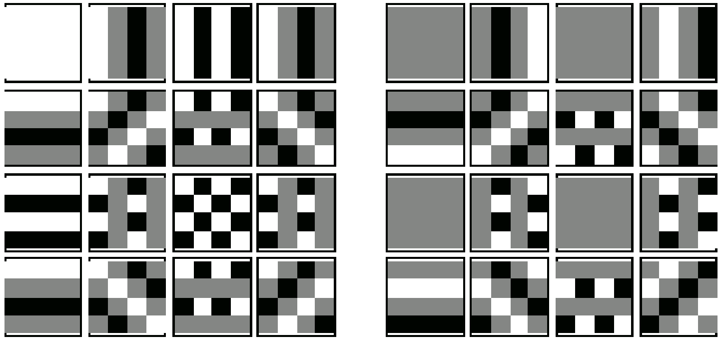
- $Y=AU$ and $V=YA^T$, therefore $V=AUA^T$
  - $U=A^{-1}V(A^T)^{-1}$
- If A is unitary, then $A^{-1}=A^H$
  - $U= A^H V(A^H)^T$
- If A is unitary and real (orthogonal), then
  - $U=A^T VA$
- $B^{i,j}$ = (i,j)th basis image = $b_i\, b_j^T$

DFT

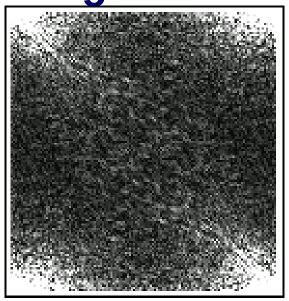
1D basis vectors (real, imaginary)



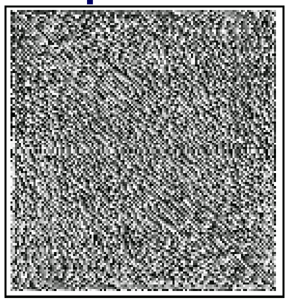
2D basis images (real, imaginary)

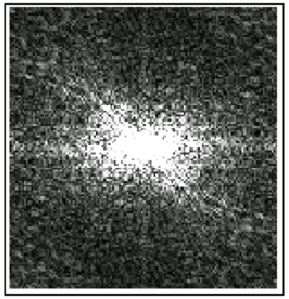


magnitude



phase

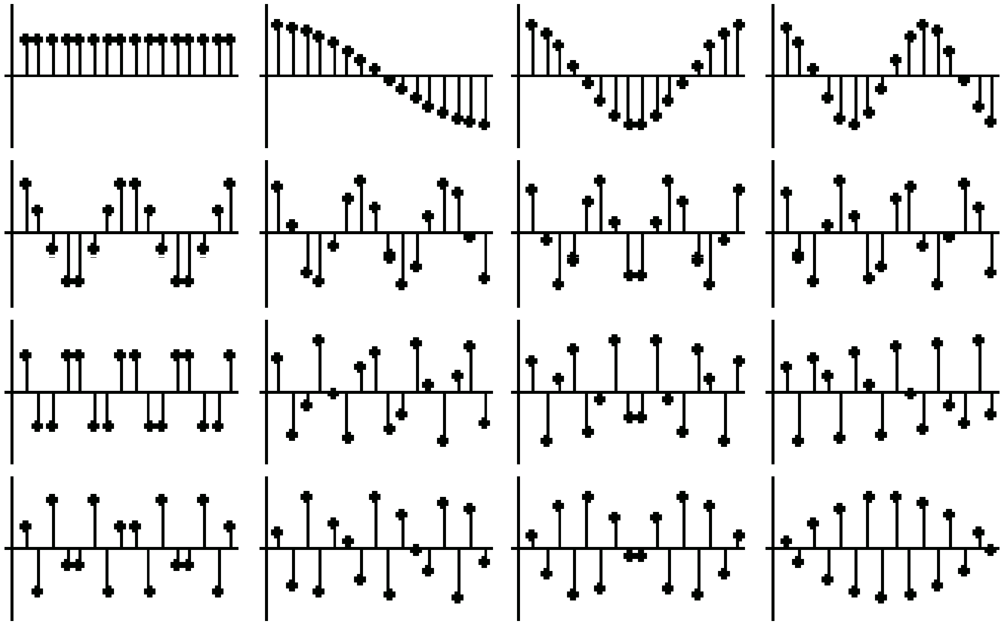




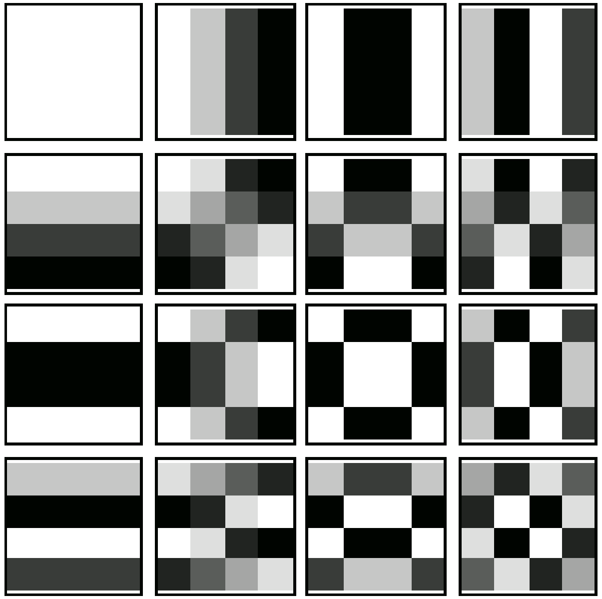
shifted magnitude

DCT

1D basis vectors



2D basis images



Hadamard Transform

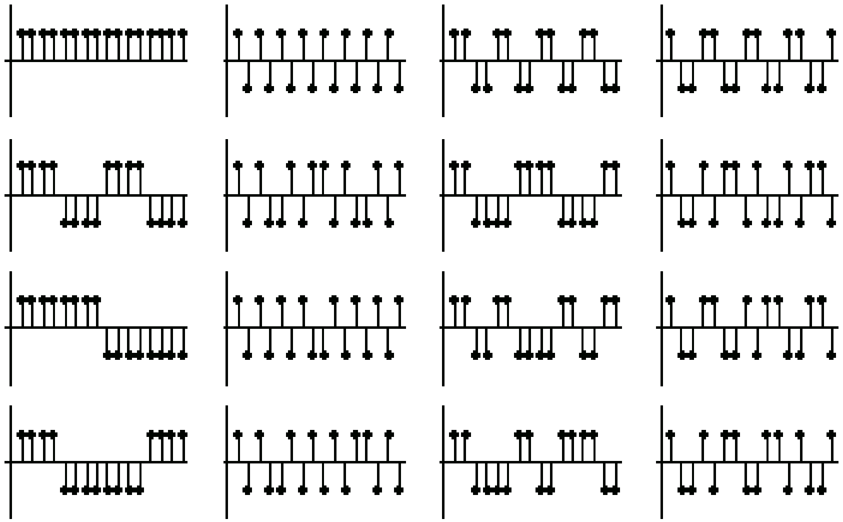
1D transform matrix

$$H_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

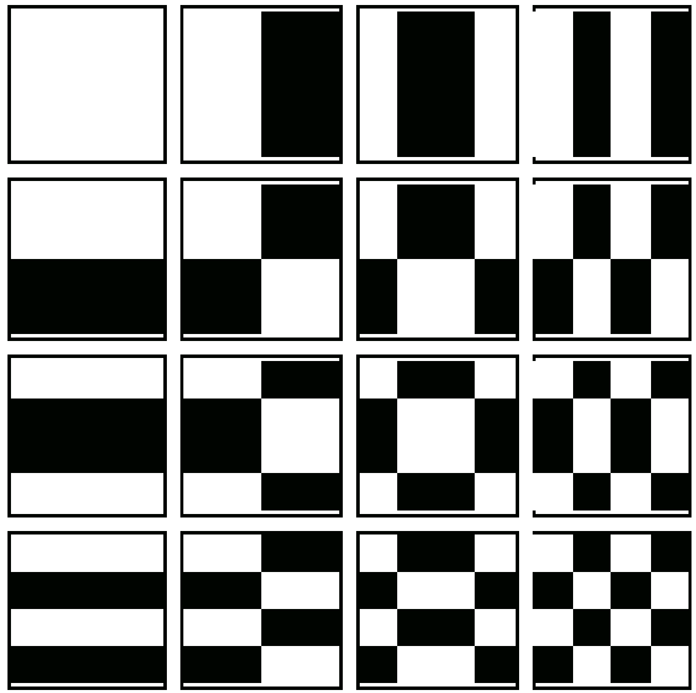
$$\tilde{H}_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

1D basis vectors

for 16-point Hadamard transform



2D basis images



Haar Transform

1D transform matrix

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

2D basis images