KECE471 Computer Vision

EM Algorithm and Mean Shift

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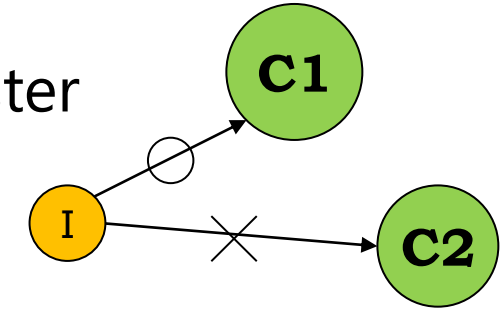
Chapter 14, Computer Vision by Forsyth and Ponce
Chapter 5, Computer Vision by Richard Szeliski
Note: Dr. Forsyth's notes are partly used.
The contents on meanshift part are copied from the ppt file of
Yaron Ukrainitz & Bernard Sarel

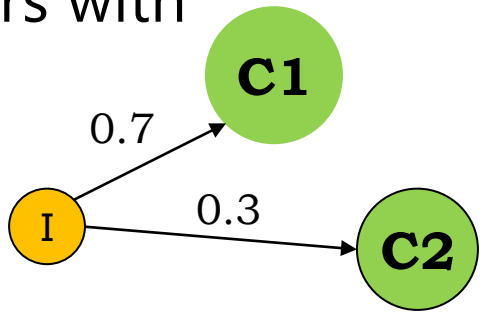
EM - EXPECTATION MAXIMIZATION

K-means and EM algorithm

- K-Means
  - Hard Clustering
    - An instance belongs to only one cluster

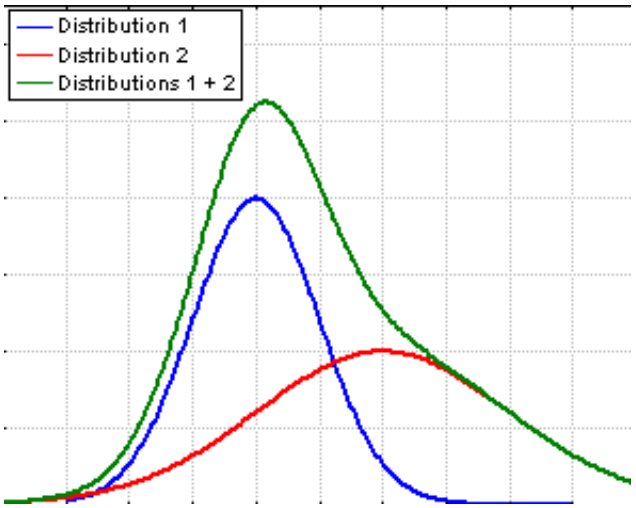


- EM
  - Soft Clustering.
    - An instance belongs to several clusters with membership probability



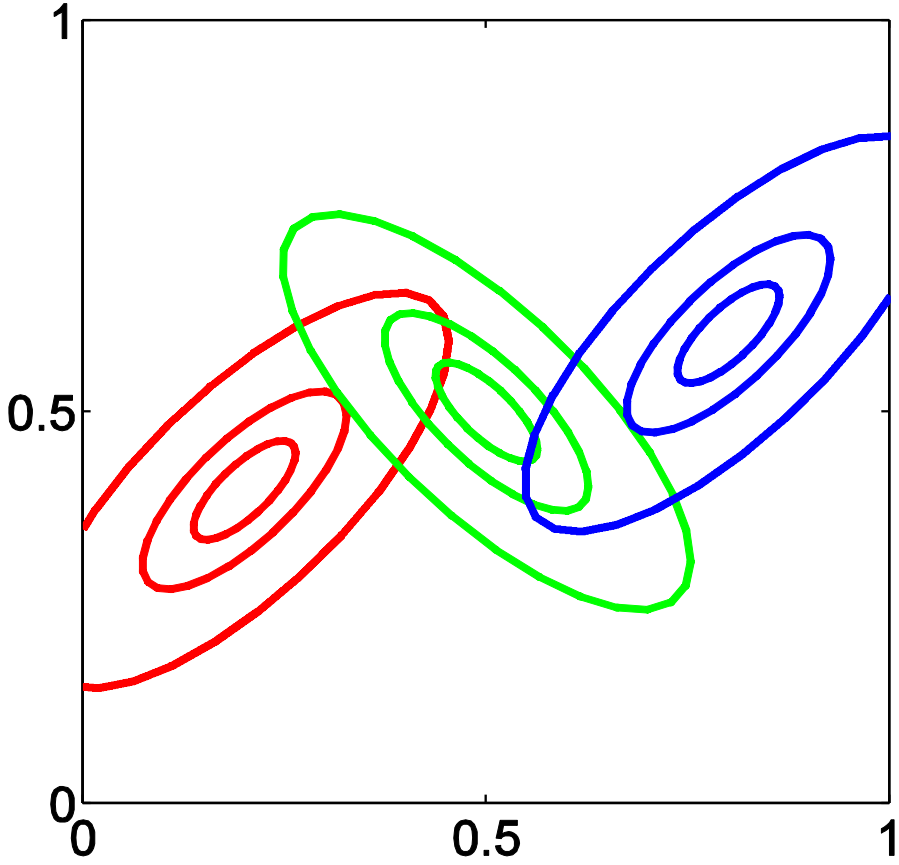
Clustering - EM algorithm

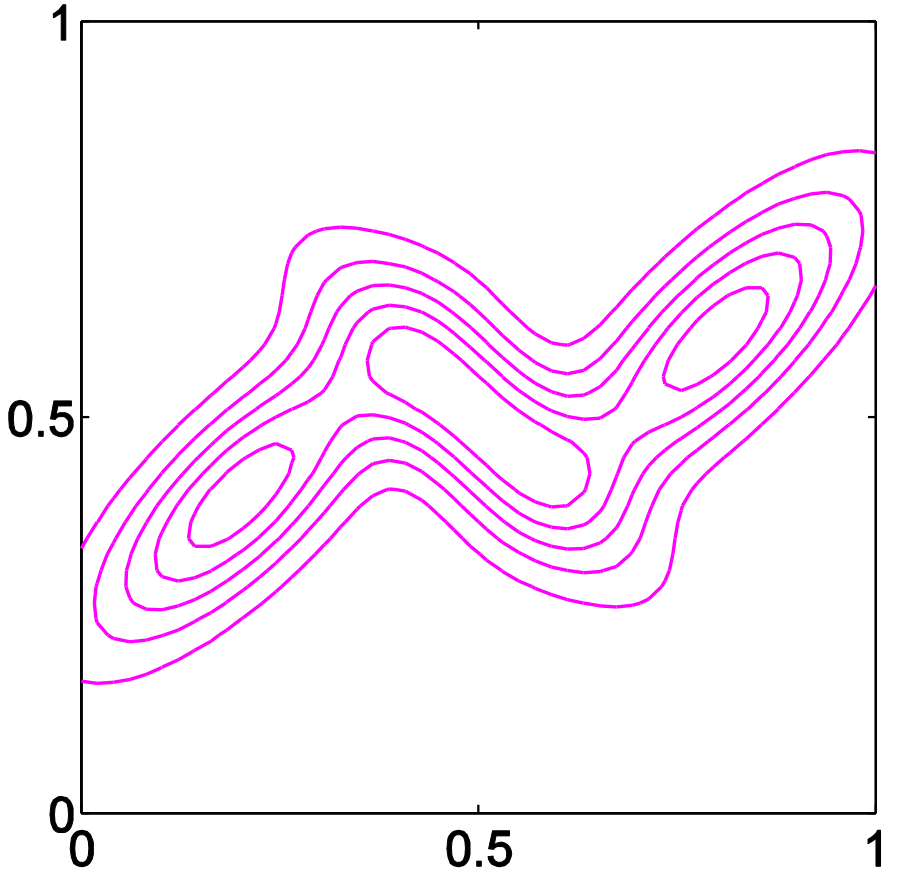
- Gaussian mixture model
  - A mixture is a set of *k* Gaussian distributions, representing *k* clusters.
  - Each Gaussian distribution is represented by mean and variance.
  - The mixture model combines several Gaussian distributions.



Clustering - EM algorithm

- Example : Mixture of 3 Gaussians



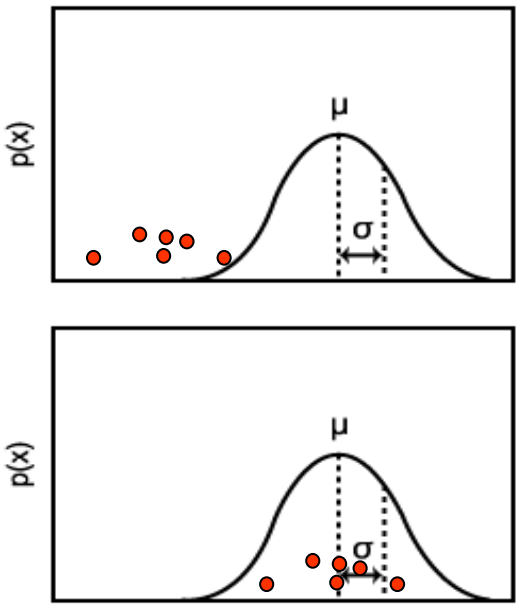


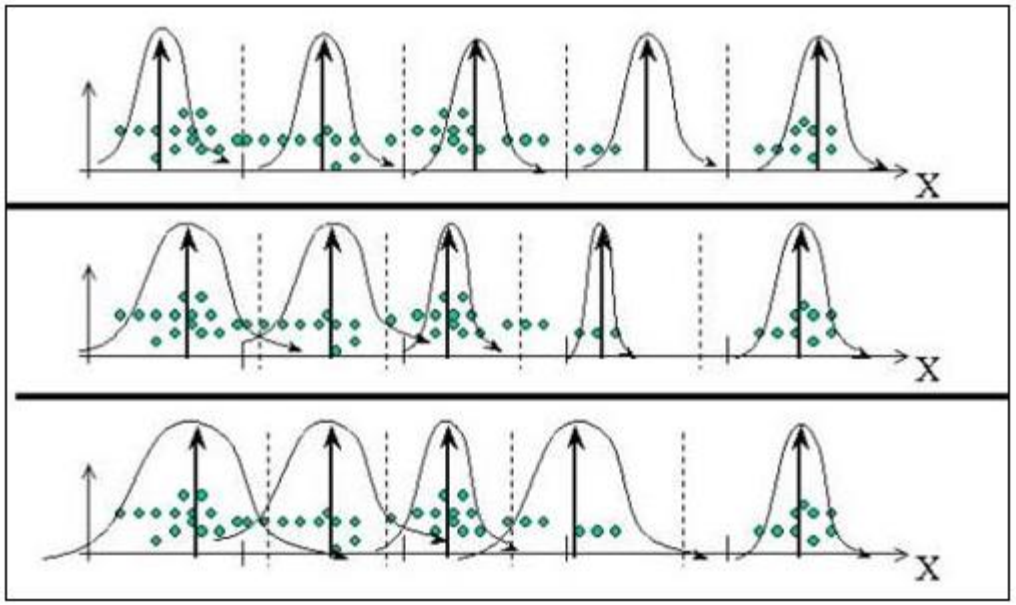
Fitting a Gaussian Mixture

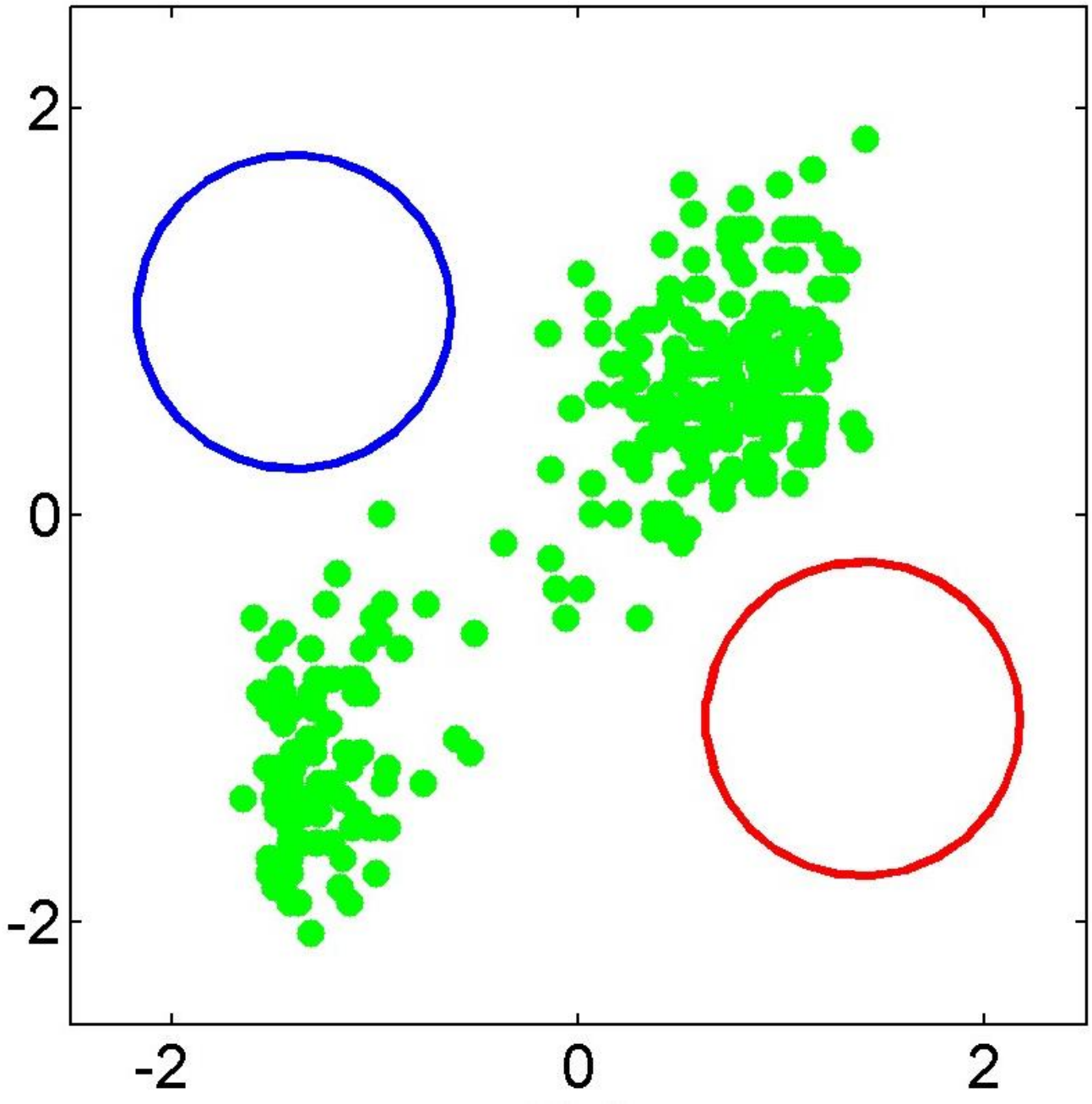
- Given the data set, estimate the parameters:
  - Mixing coefficients
  - Means
  - Variances
- If we know which cluster generates each data point, the estimation is very easy
- But the information is unknown

Informal derivation

- Iterative scheme
  - Make initial guesses for the parameters
  - Alternate between the following two stages:
    1. E-step: evaluate responsibilities
    2. M-step: update parameters using ML results



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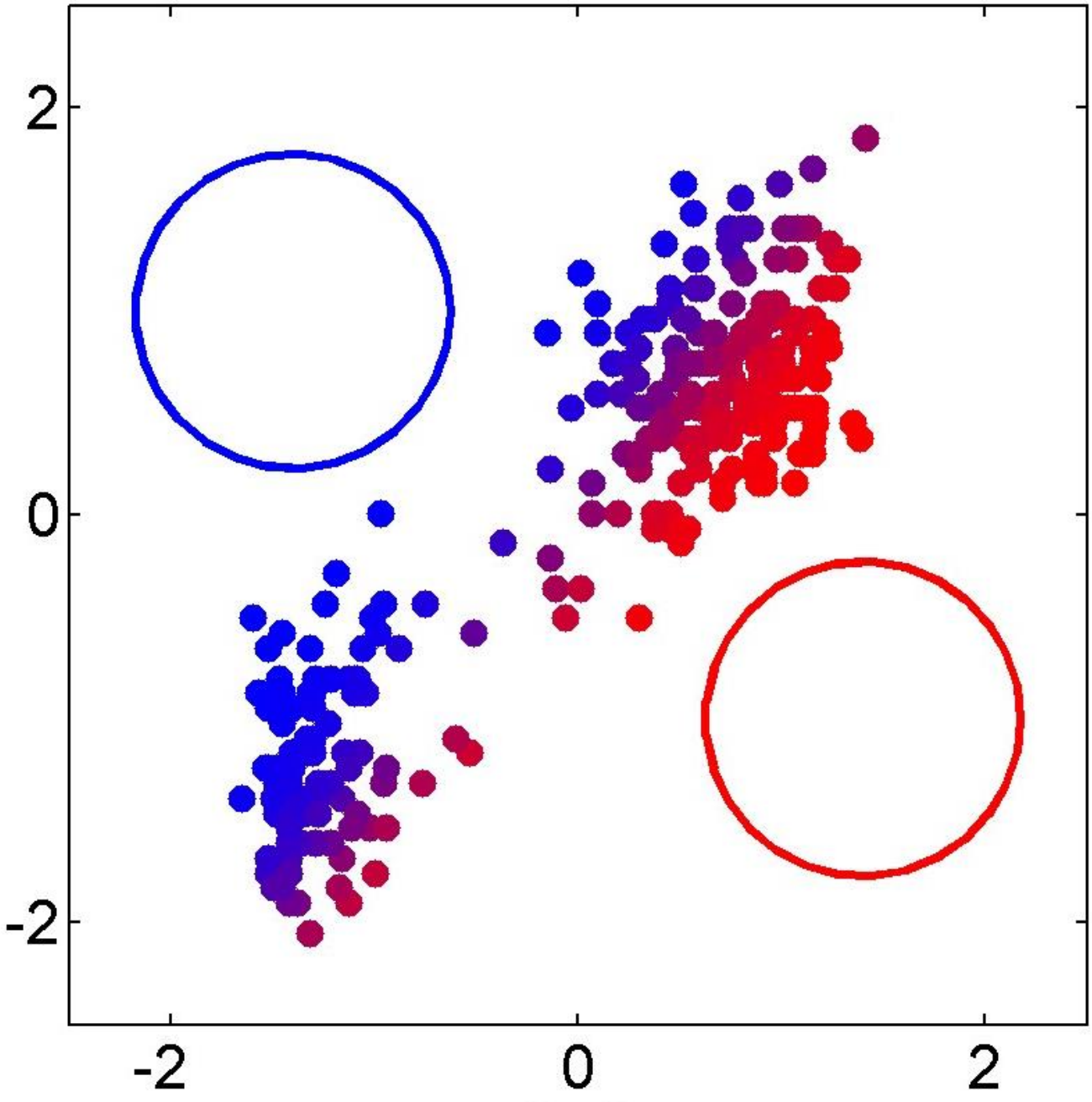
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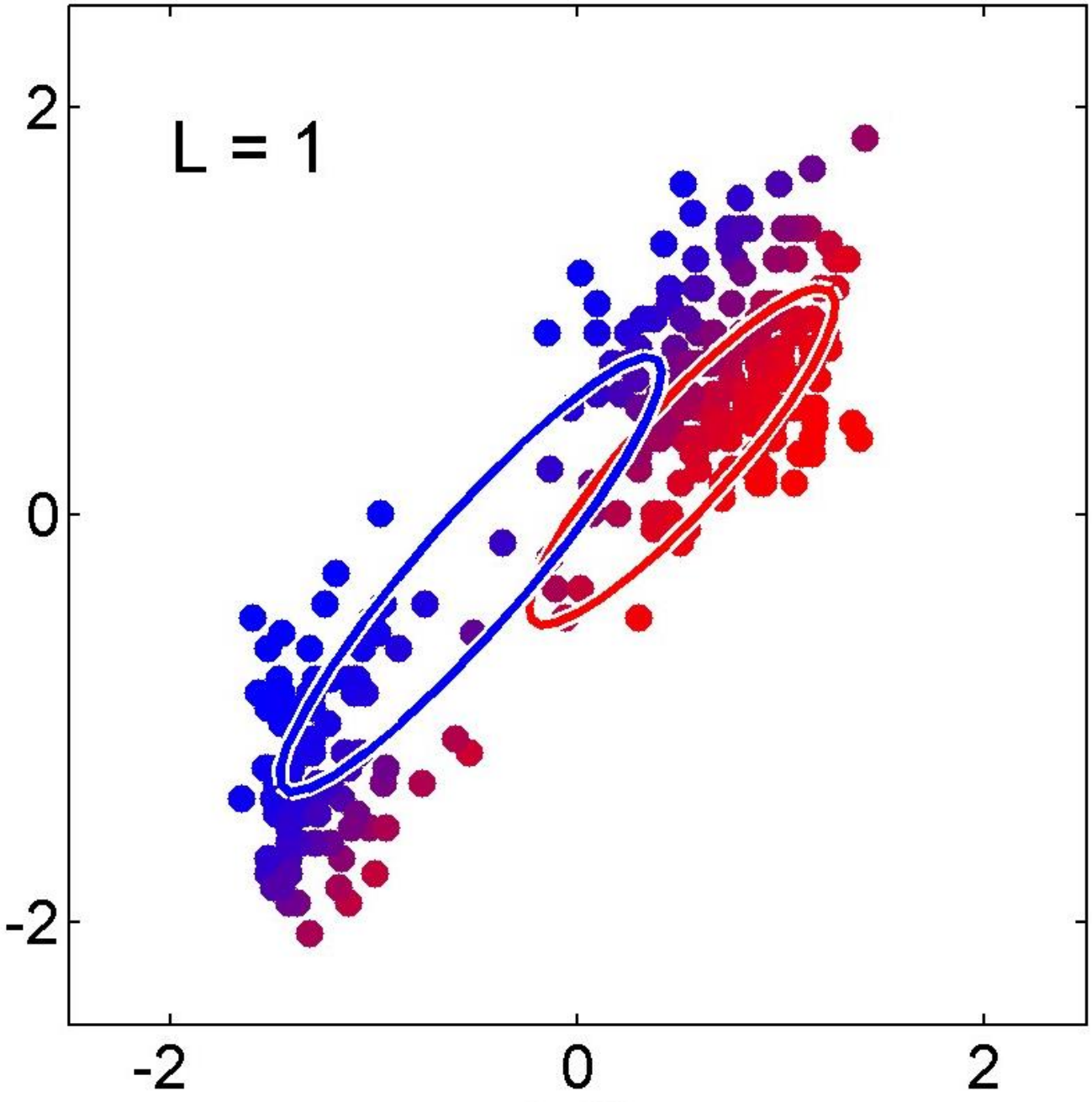
0
2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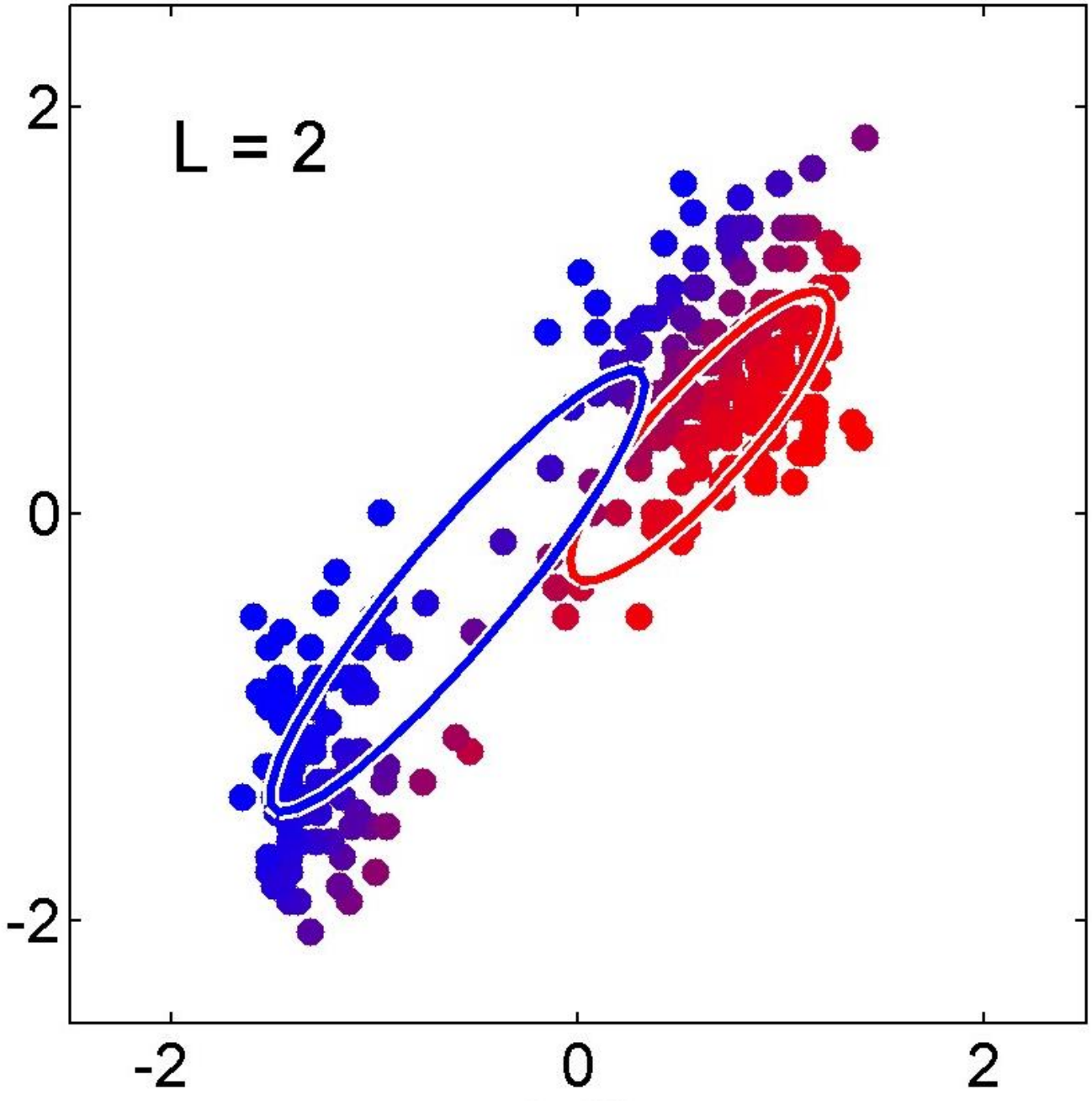
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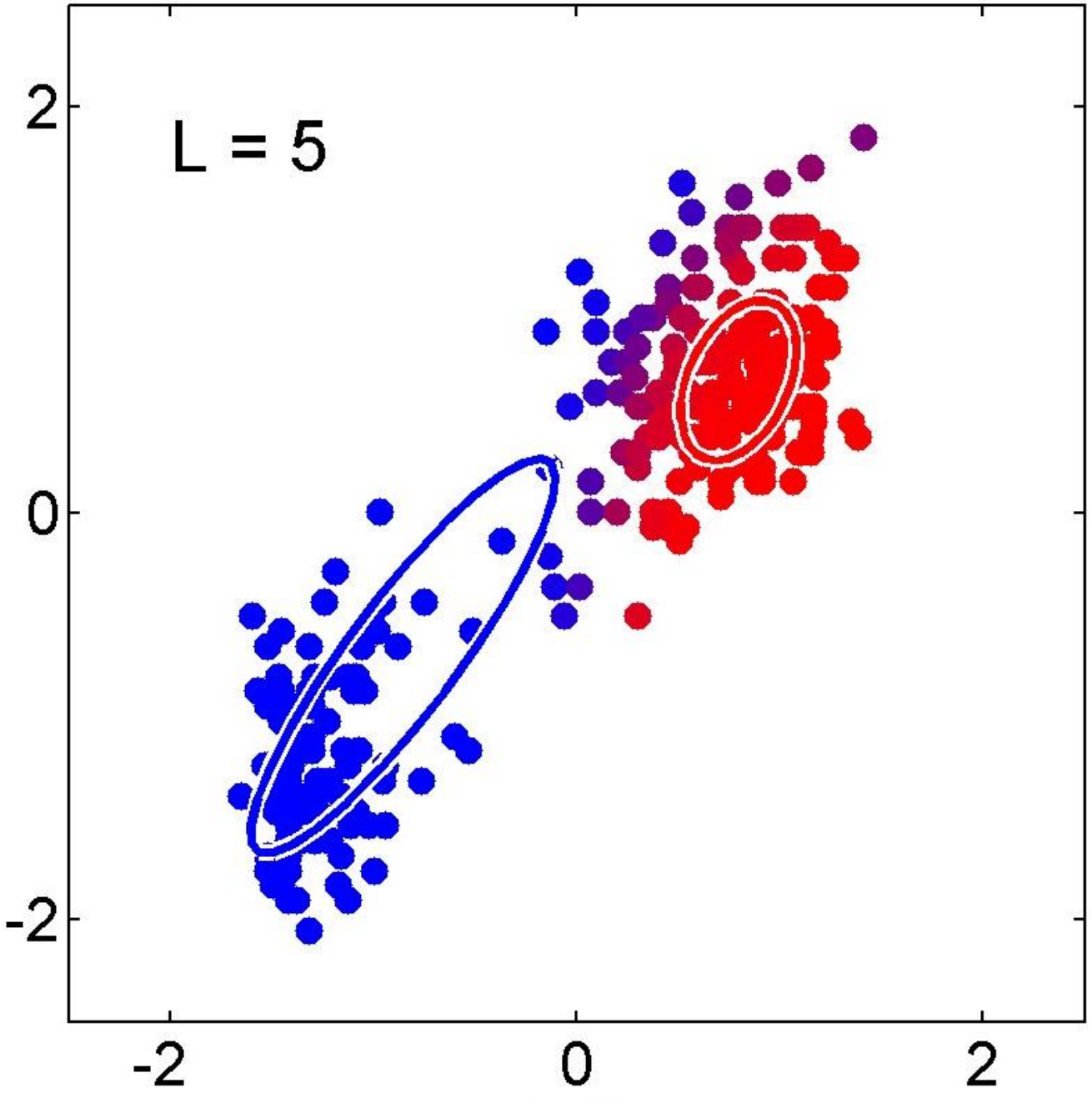
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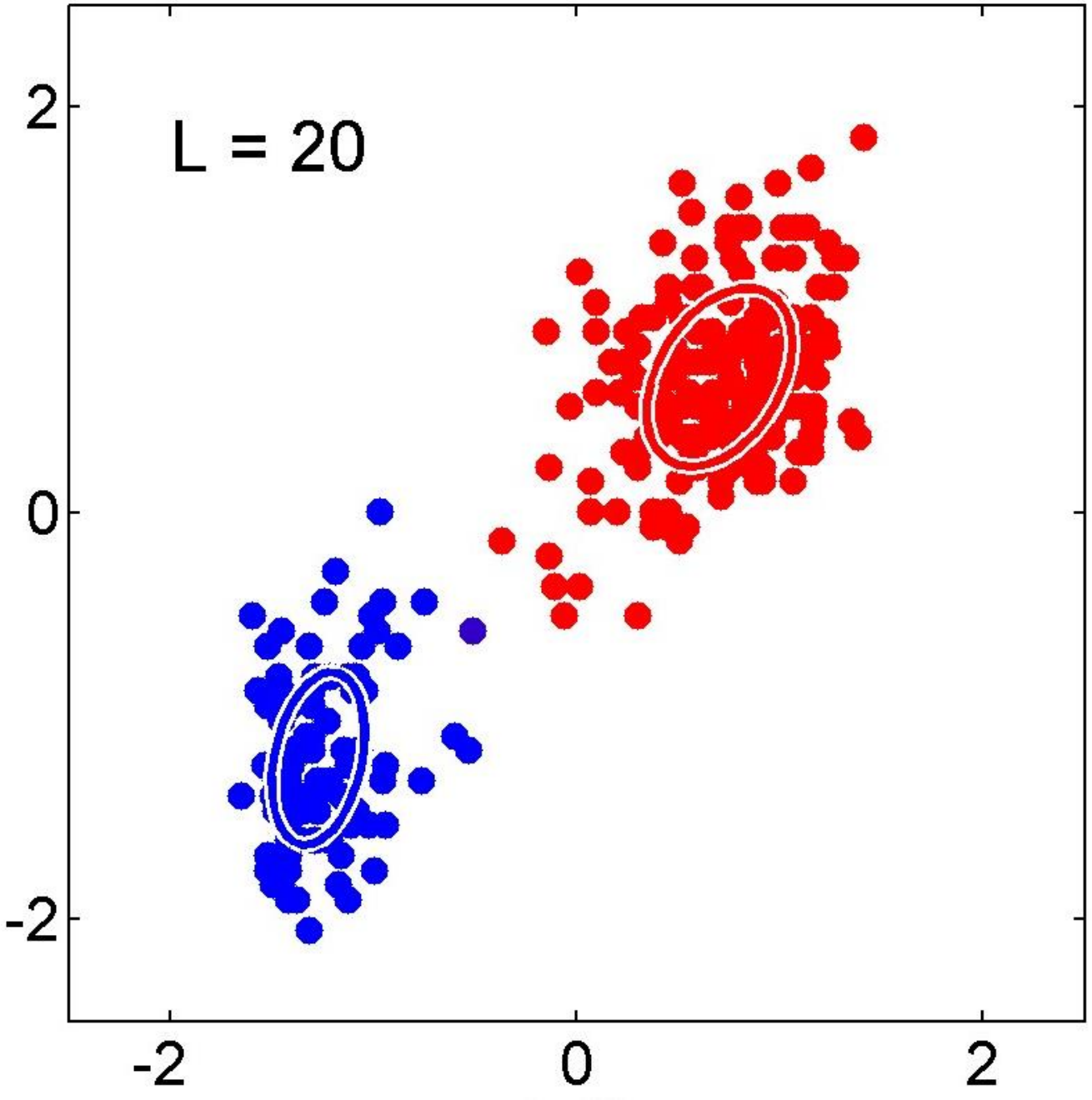
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-2
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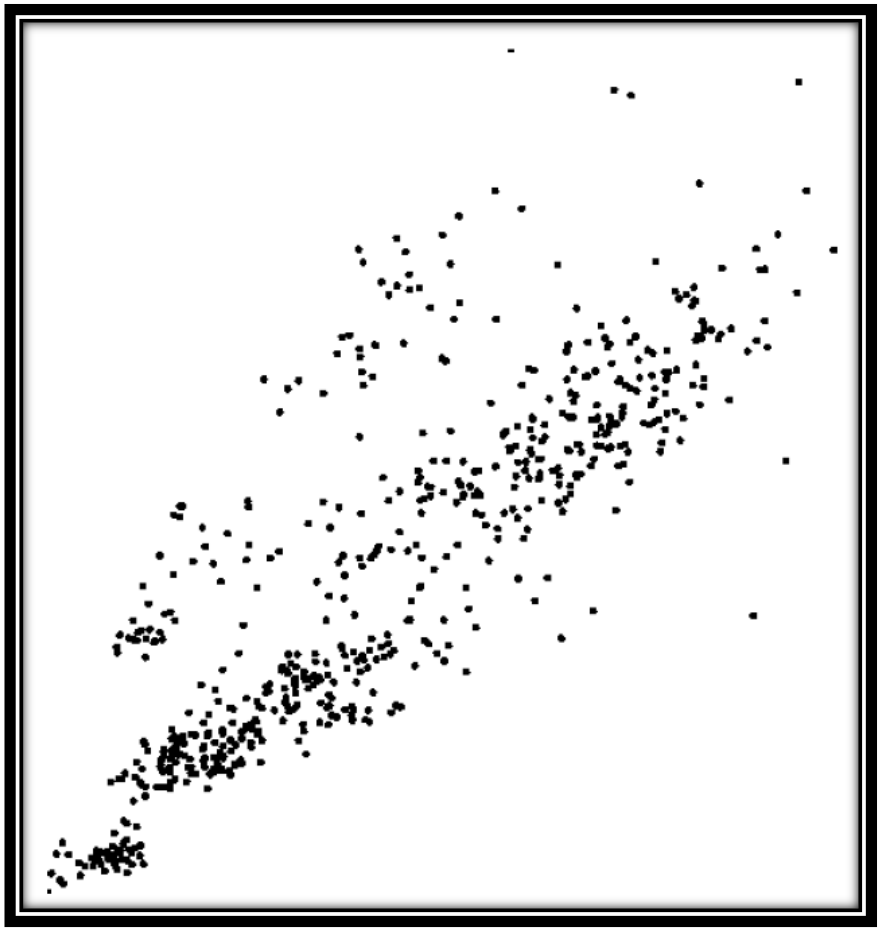
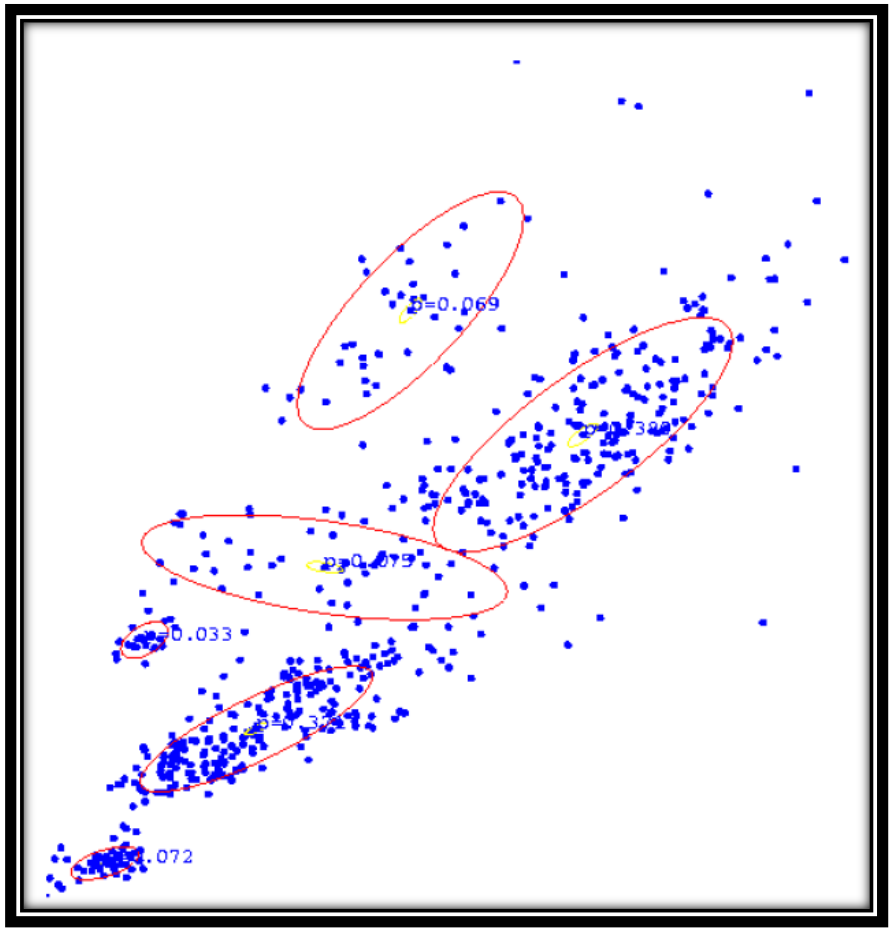
Another Example

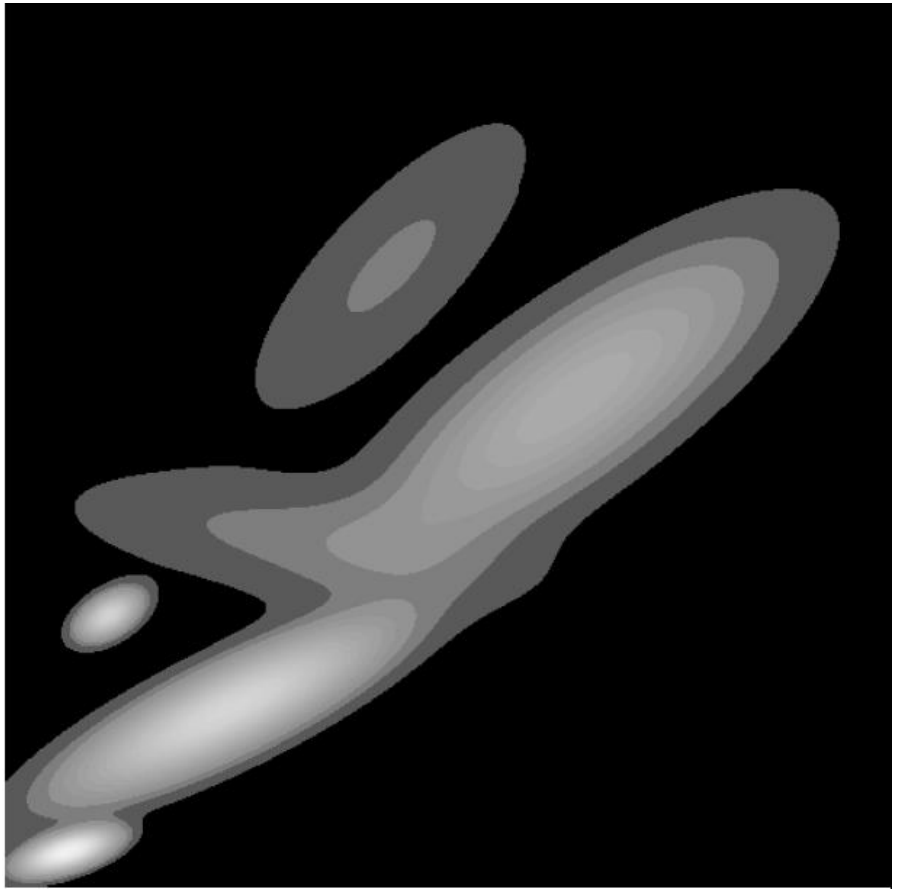
- Data set – 6 clusters





Fitting the Gaussian Mixture

- Resulting density estimation



Another Example

Original image

2 clusters

5 clusters

10 clusters

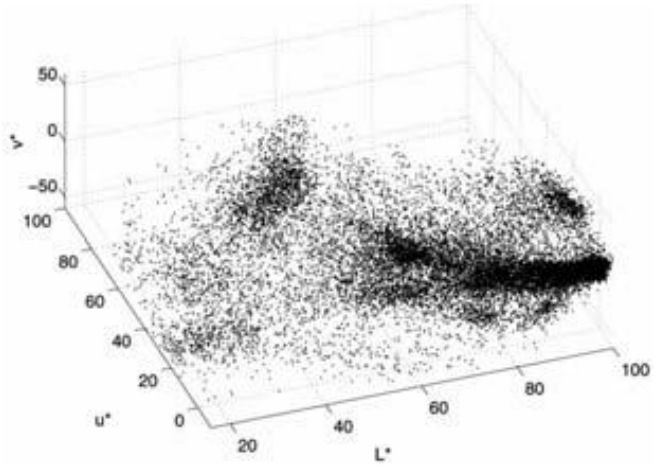
20 clusters

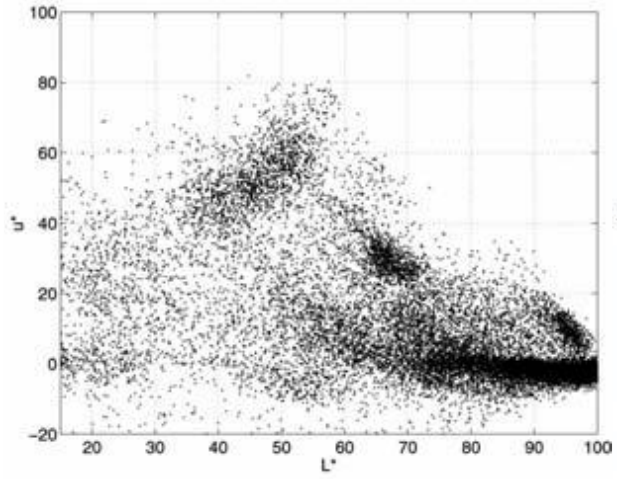
50 clusters

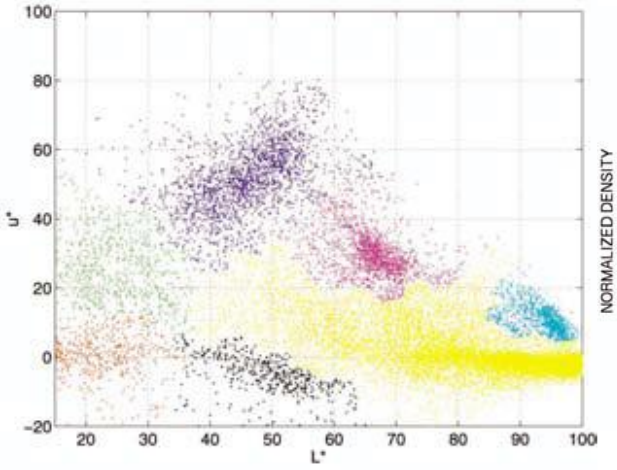
MEAN SHIFT AND MODE FINDING

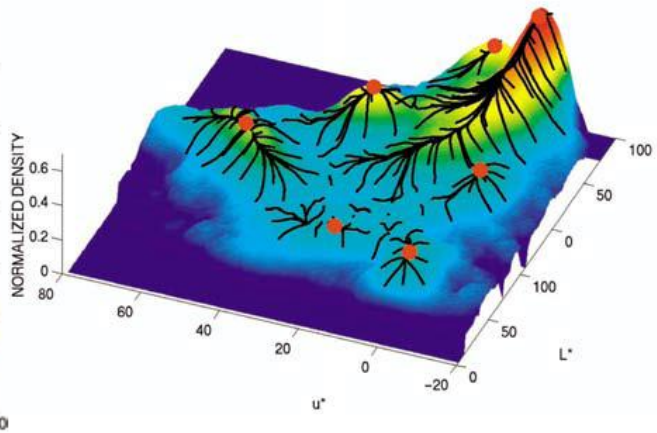
- D. Comaniciu and P. Meer, “Mean shift: A robust approach toward feature space analysis,” IEEE Trans. PAMI, vol. 24, no. 5, pp. 603-619, 2002.
- Slides are excerpted from those of Yaron Ukrainitz & Bernard Sarel

Mean shift and Mode Finding

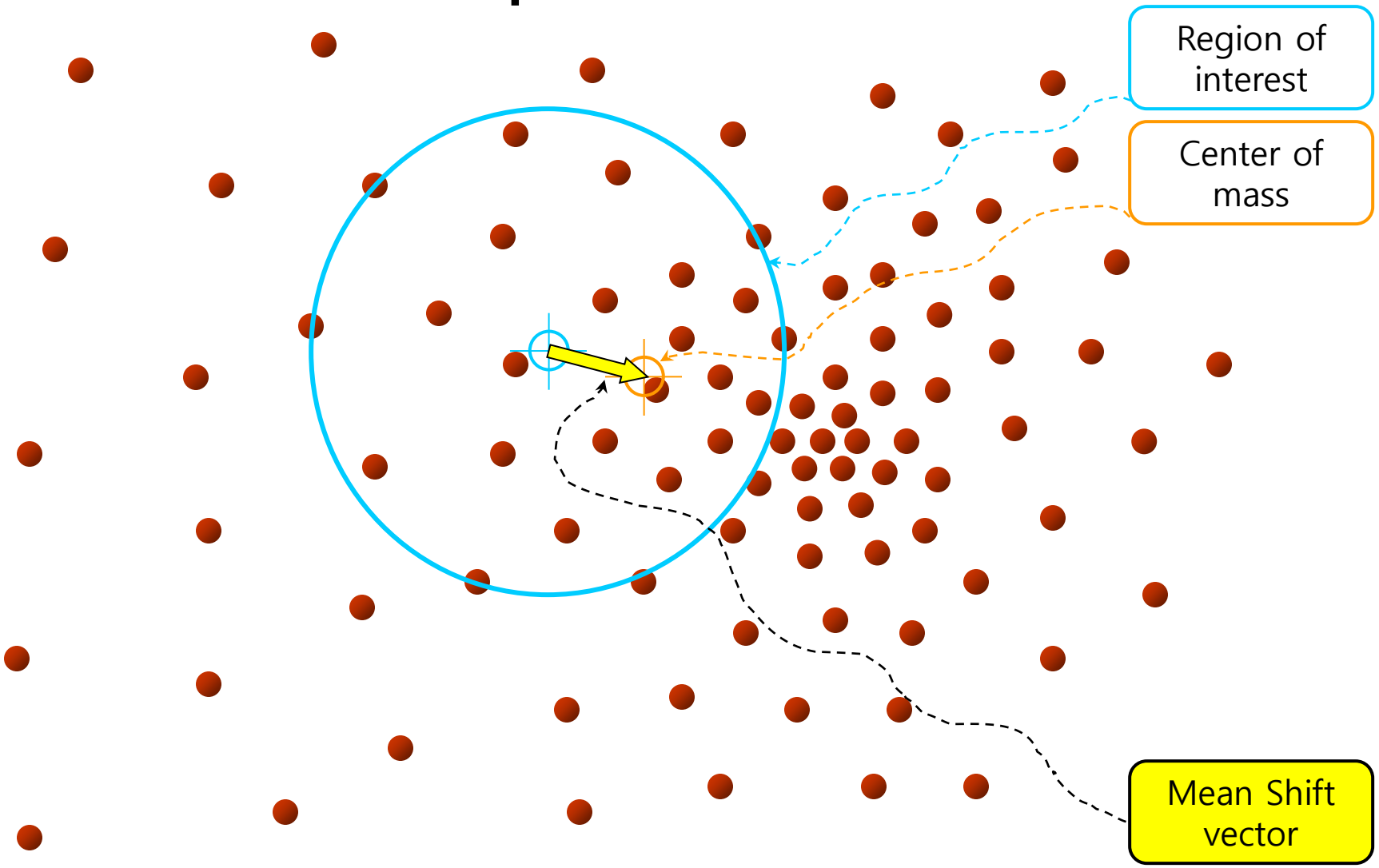






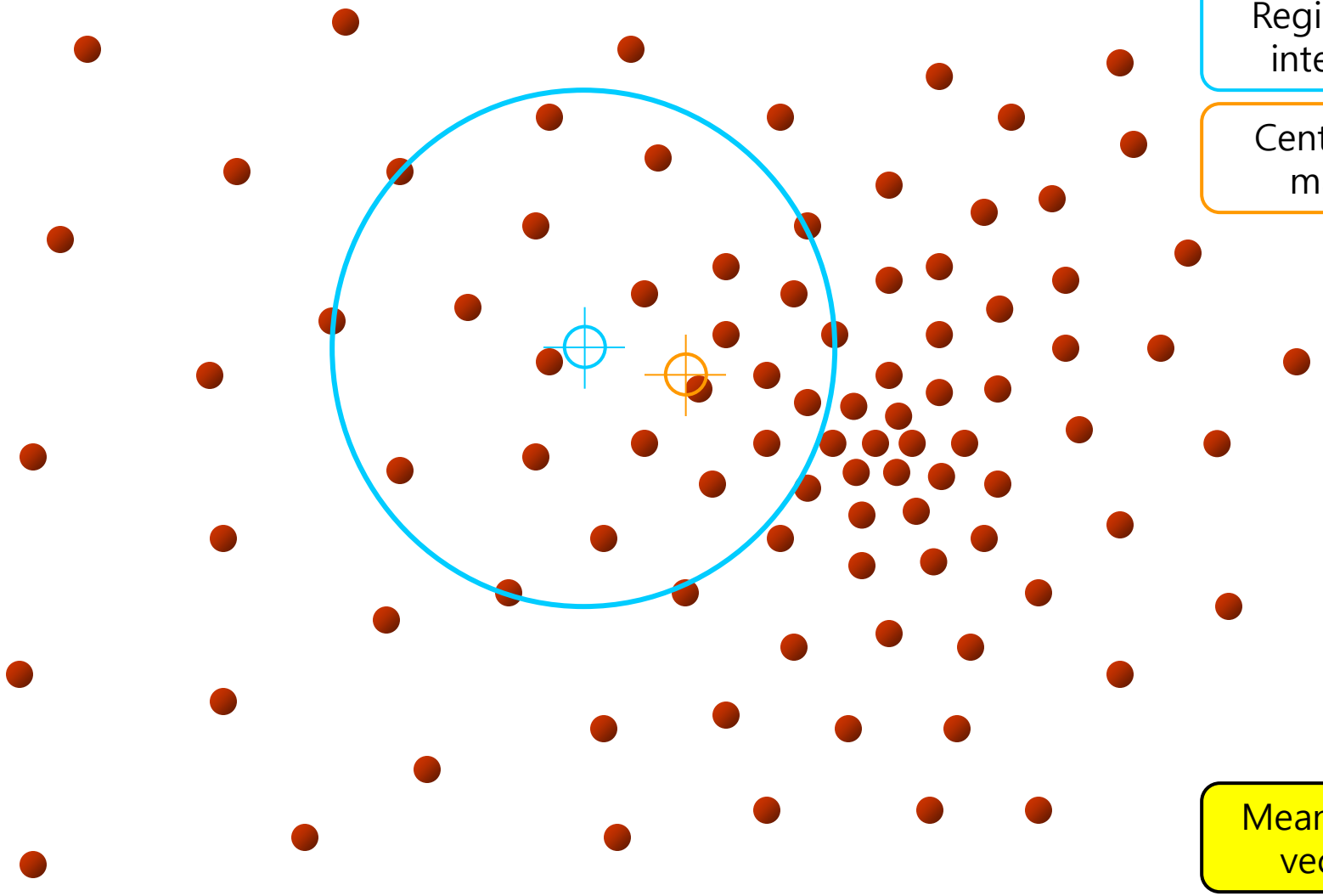


Intuitive Description



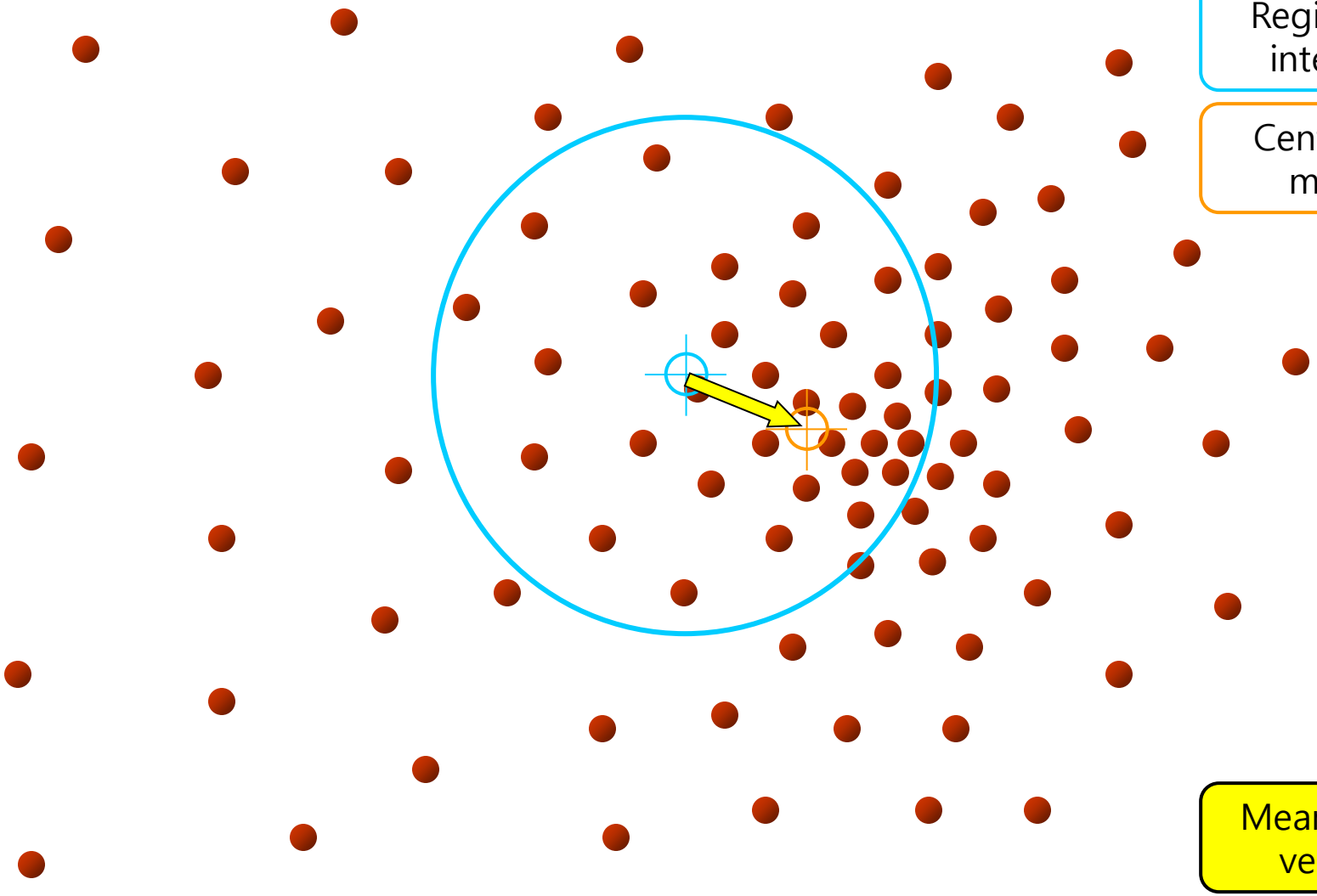
Objective : Find the densest region

Intuitive Description



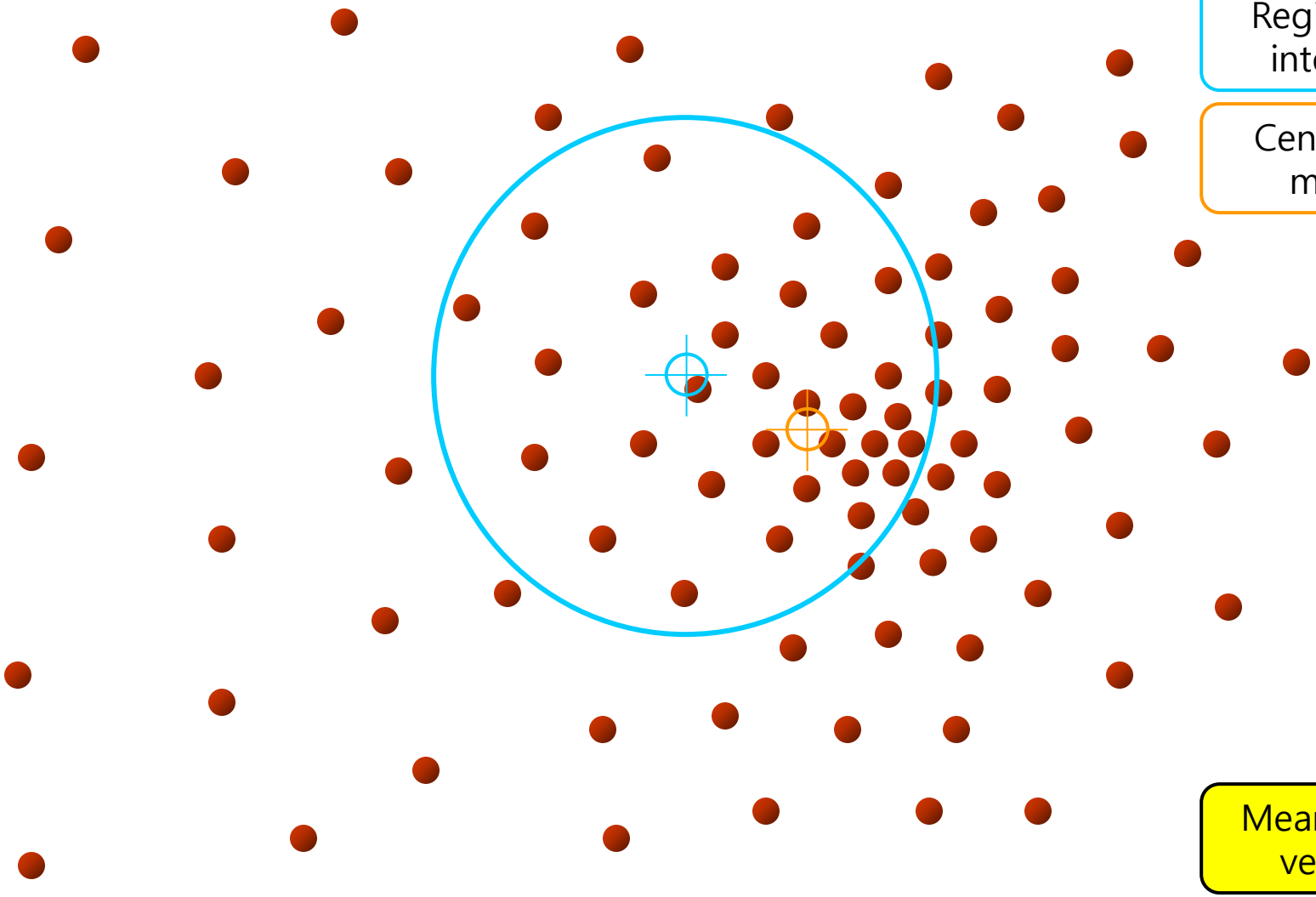
Objective : Find the densest region

Intuitive Description



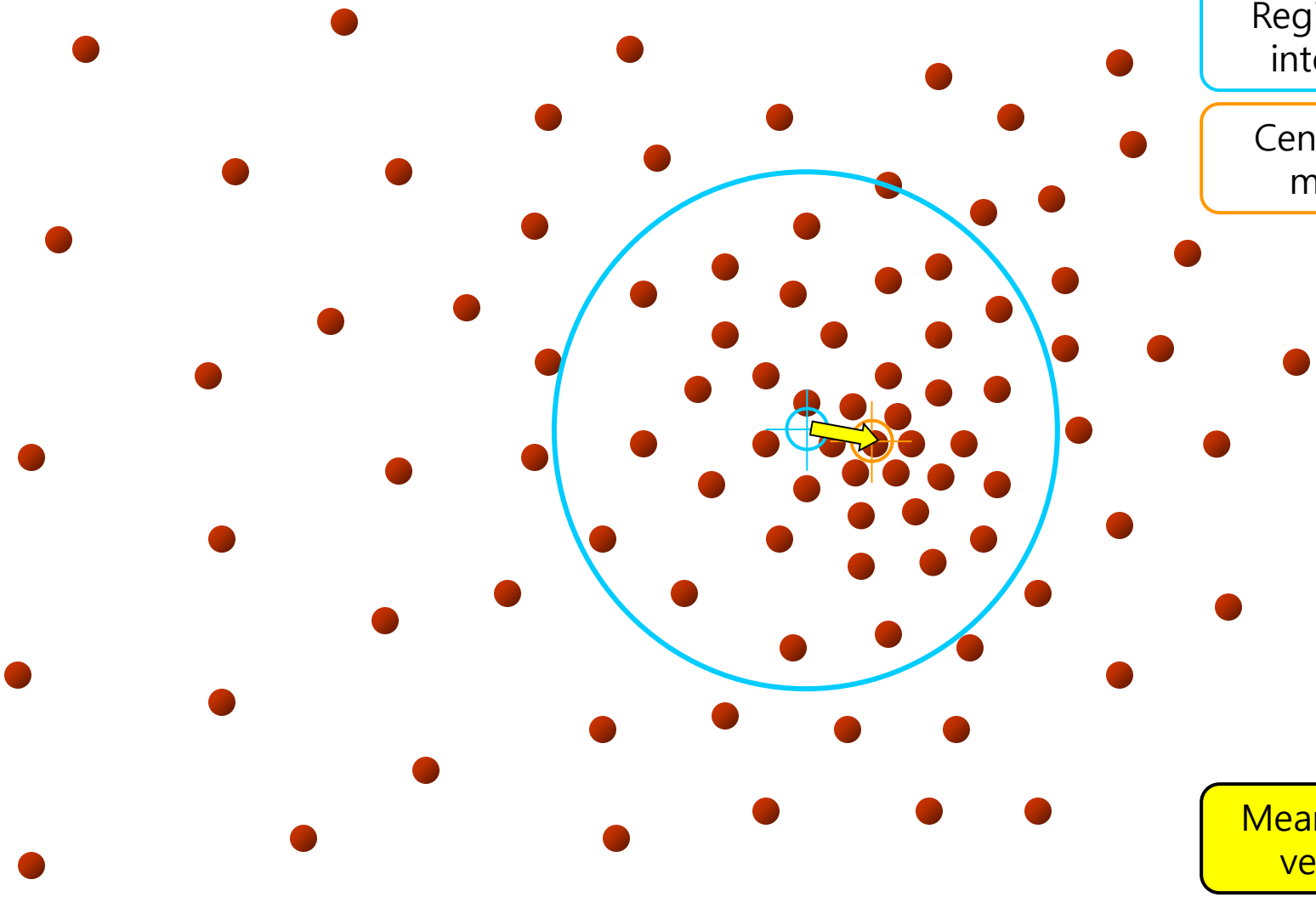
Objective : Find the densest region

Intuitive Description



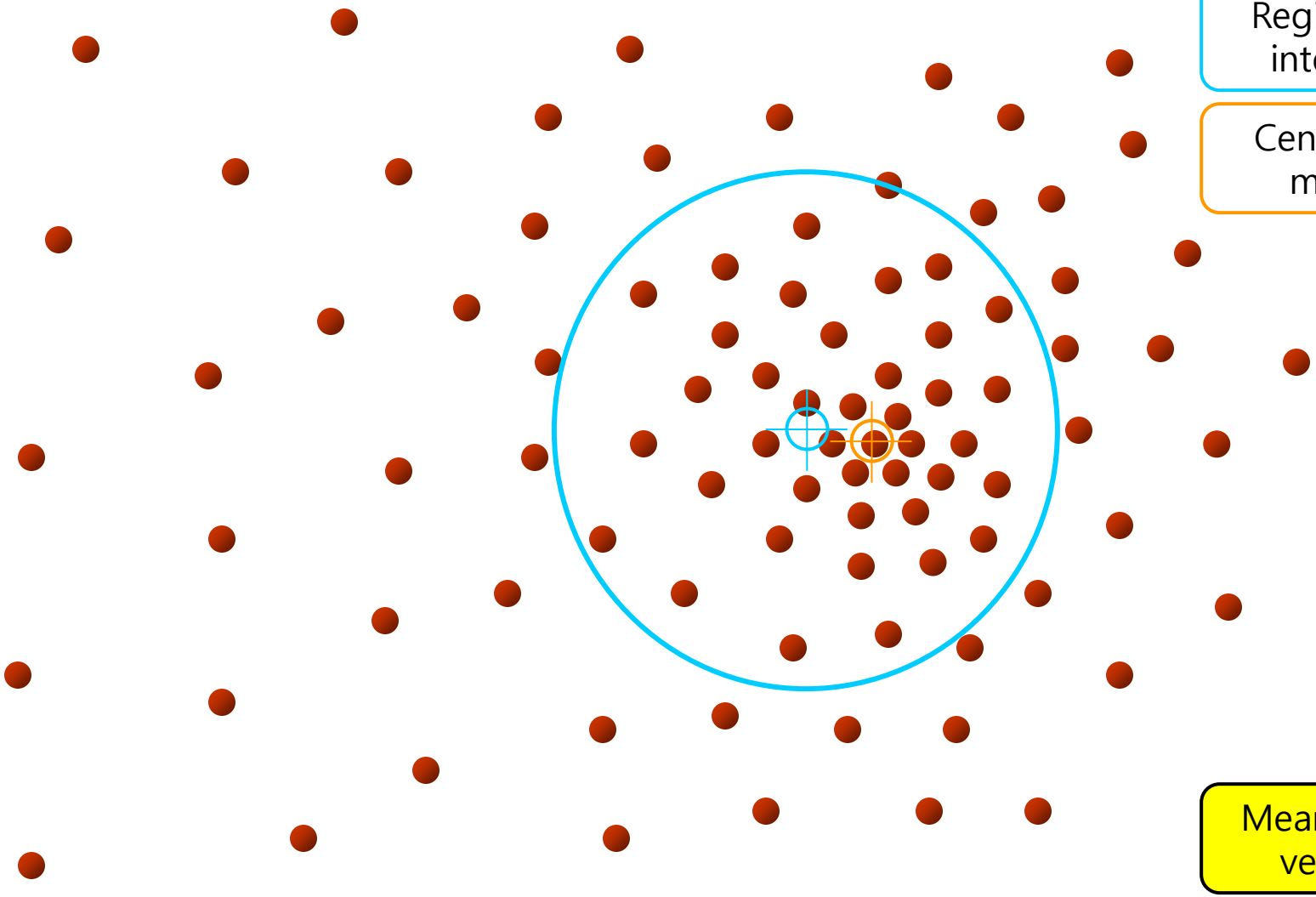
Objective : Find the densest region

Intuitive Description



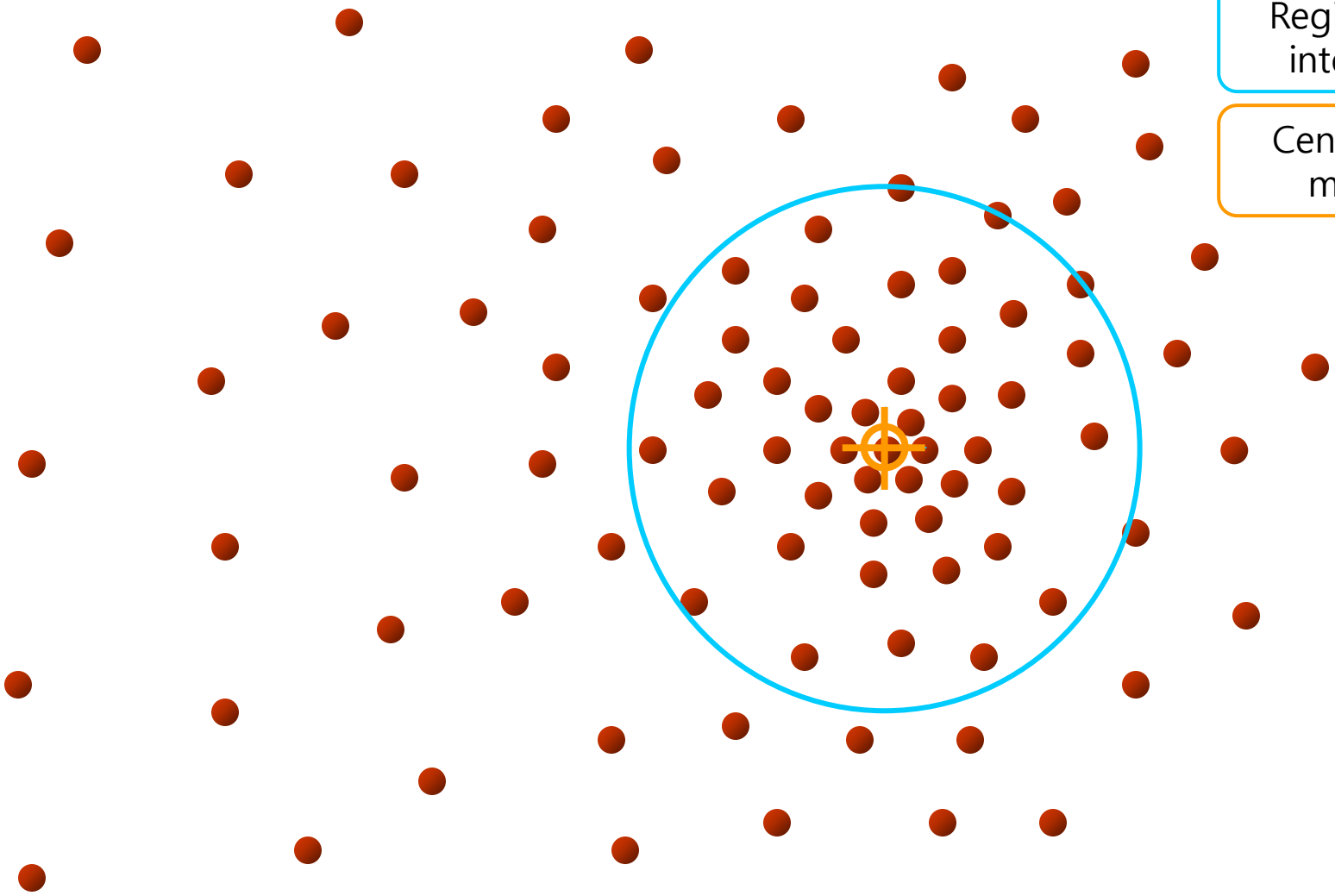
Objective : Find the densest region

Intuitive Description



Objective : Find the densest region

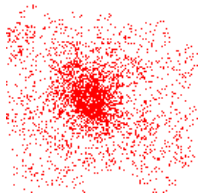
Intuitive Description



Objective : Find the densest region

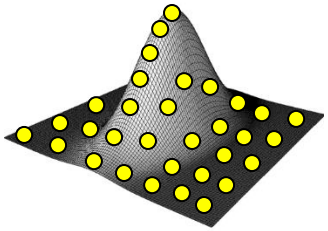
What is Mean Shift ?

A tool for:
Finding modes in a set of data samples, representing an underlying probability density function (PDF) in R^N



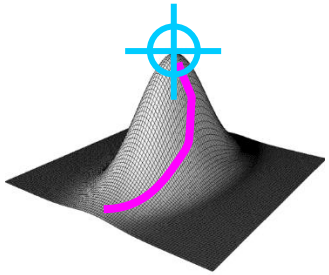
Data

Non-parametric
Density Estimation



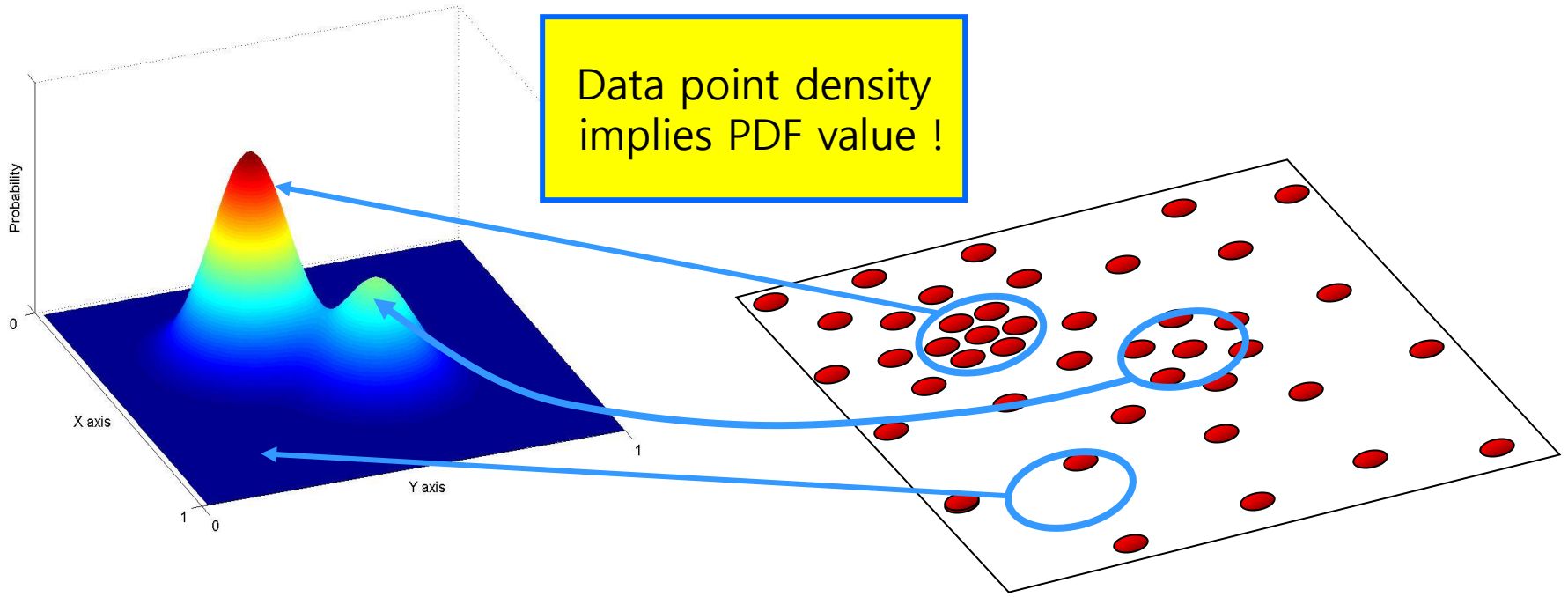
Discrete PDF Representation

Non-parametric
Density **GRADIENT** Estimation
(Mean Shift)



PDF Analysis

Density Estimation



Underlying PDF

Real Data Samples

Density Estimation

- Parametric density estimation

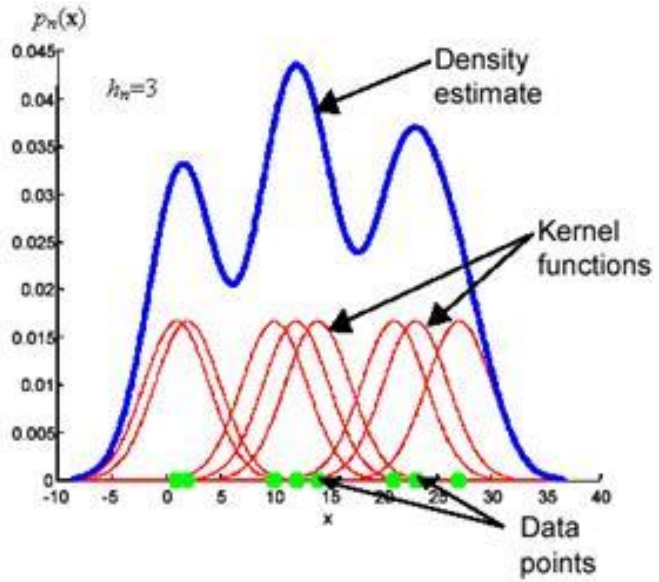
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

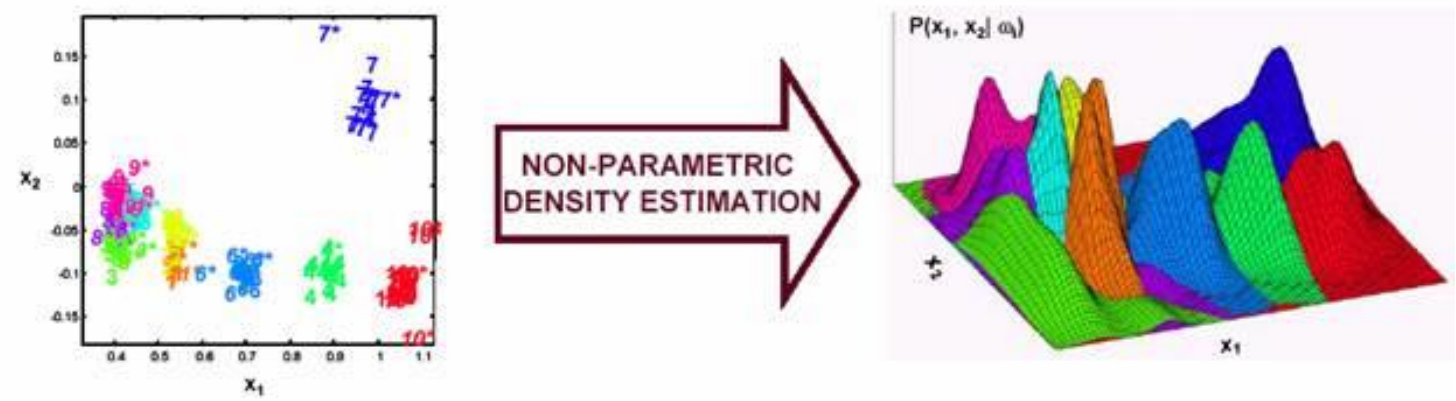
- Nonparametric density estimation

$$f(x) = \sum_i k(x - x_i)$$

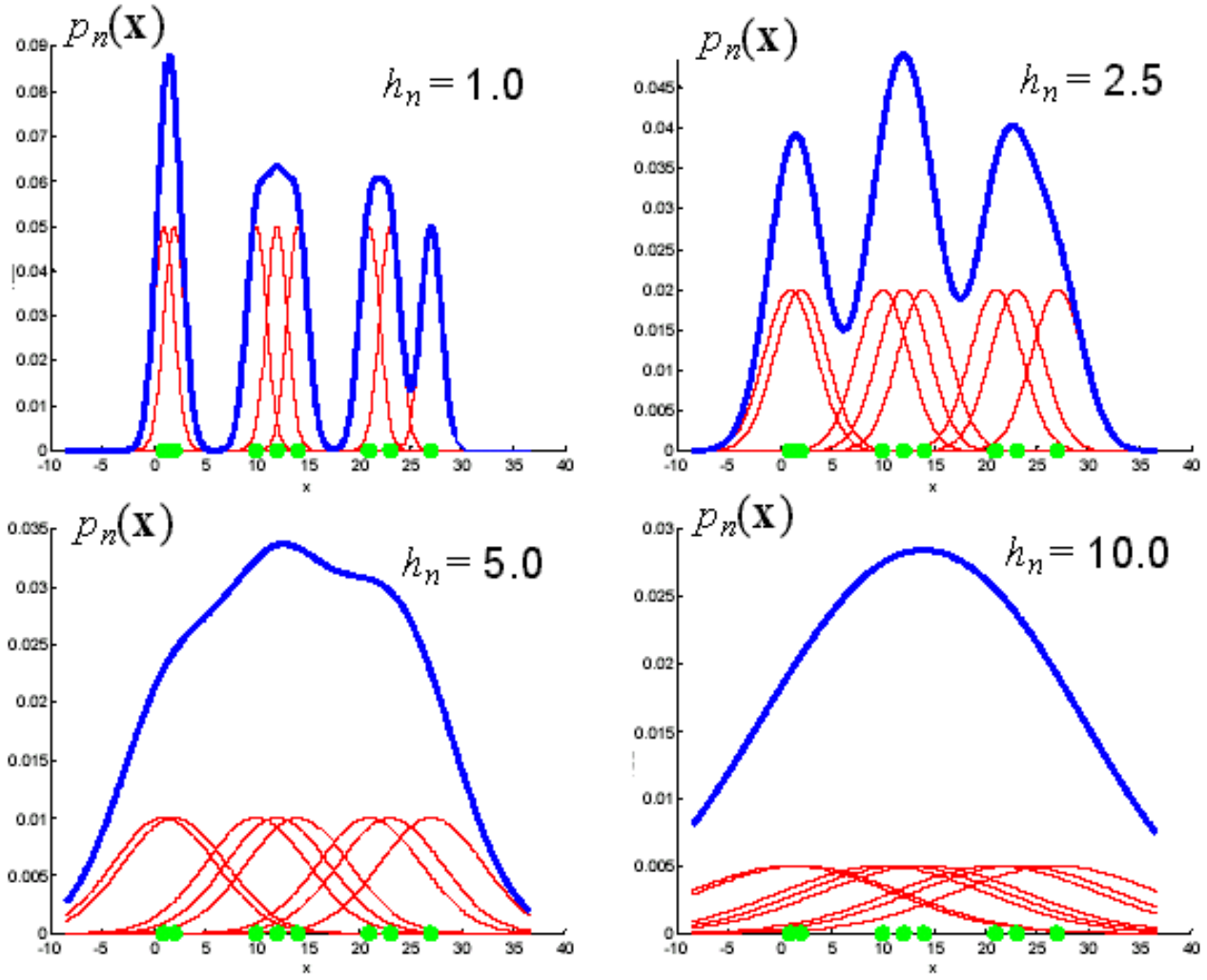
  - Kernel density estimation
  - Parzen window technique

Non-Parametric Density Estimation





Non-Parametric Density Estimation



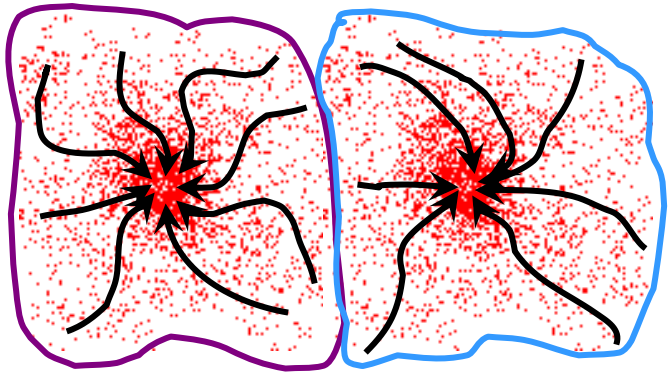
Mean Shift Operation

- In the mean shift procedure, we obtain not the density $f(x)$, but its gradient $\nabla f(x)$
- Clustering
  - Record the moving trajectory of each point
  - All points leading to a mode form a cluster
- A faster approach
  - Randomly subsample input points
  - Keep track of the temporal evolution of each point.
  - Classify the remaining points based on the nearest evolution path

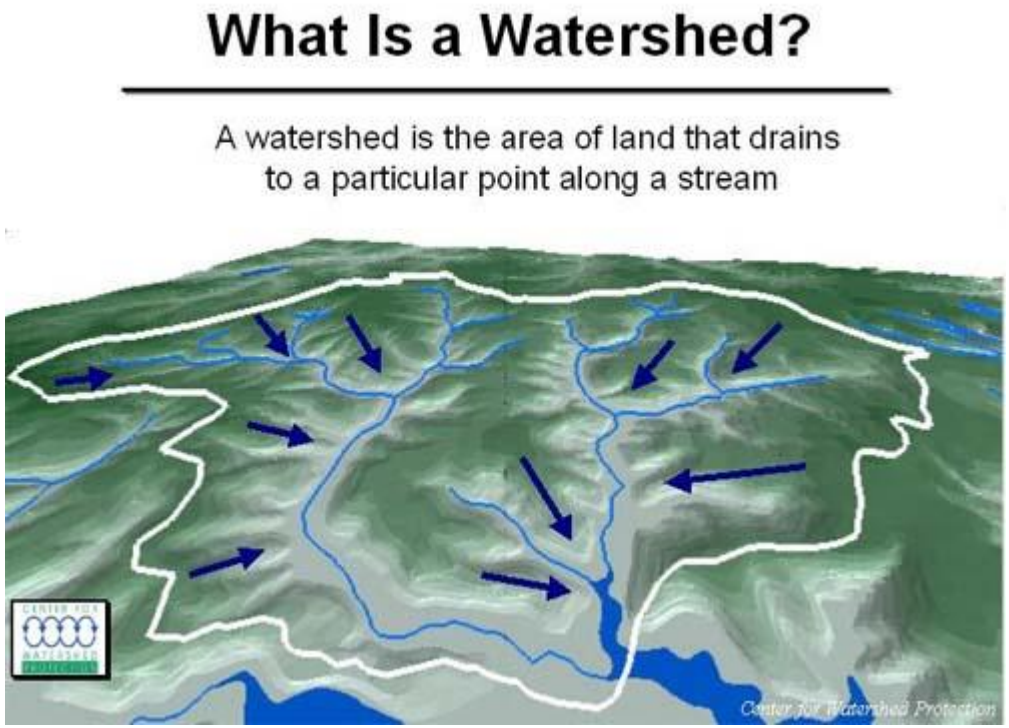
Clustering

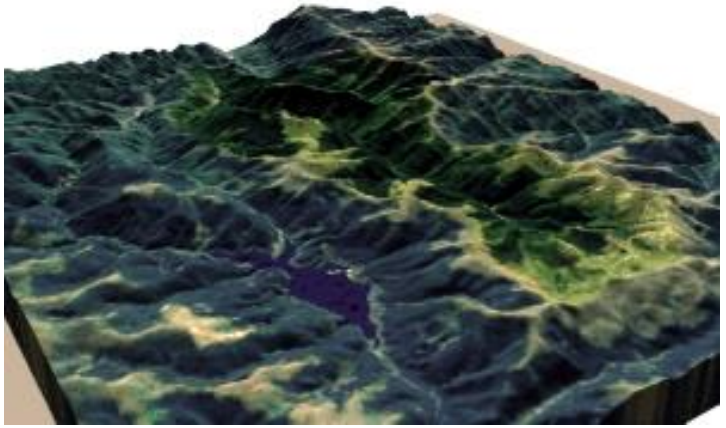
Cluster : All data points in the ***attraction basin*** of a mode

Attraction basin : the region for which all trajectories lead to the same mode



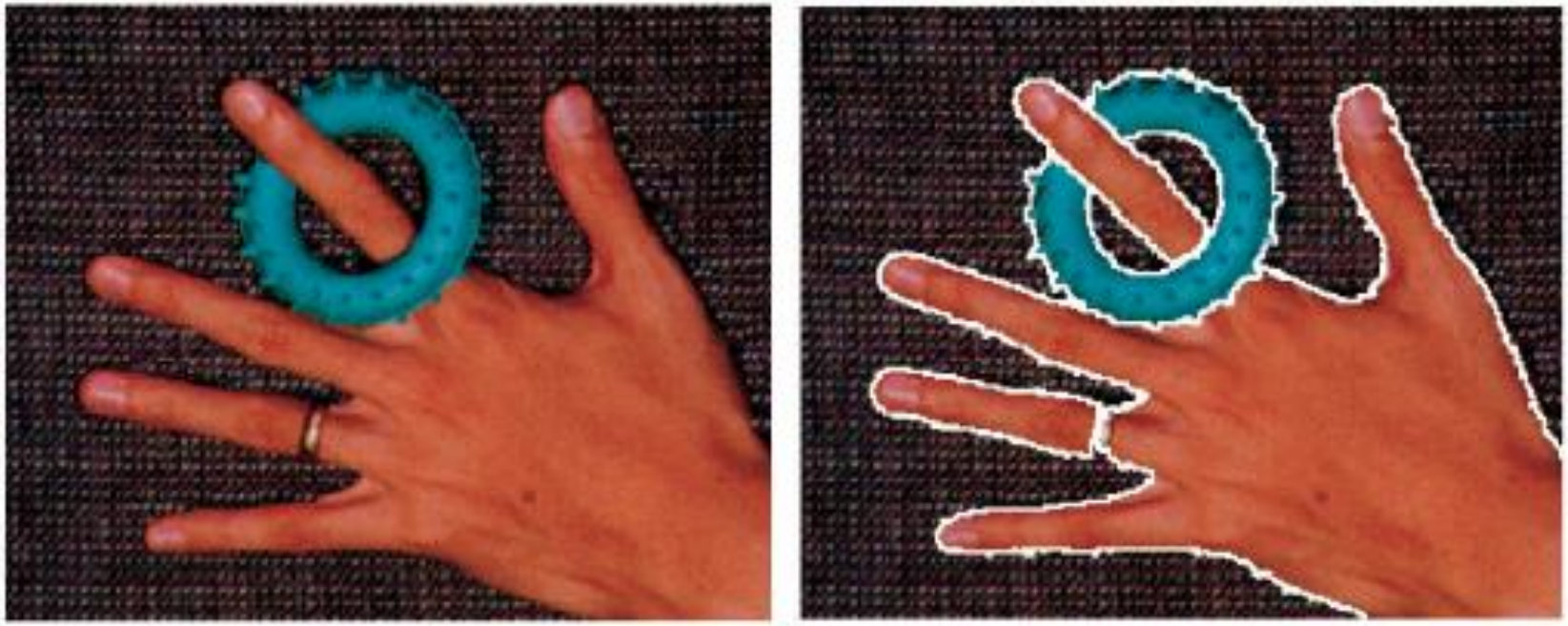
Drainage Basin (Watershed)





Segmentation

Example



Segmentation

Example