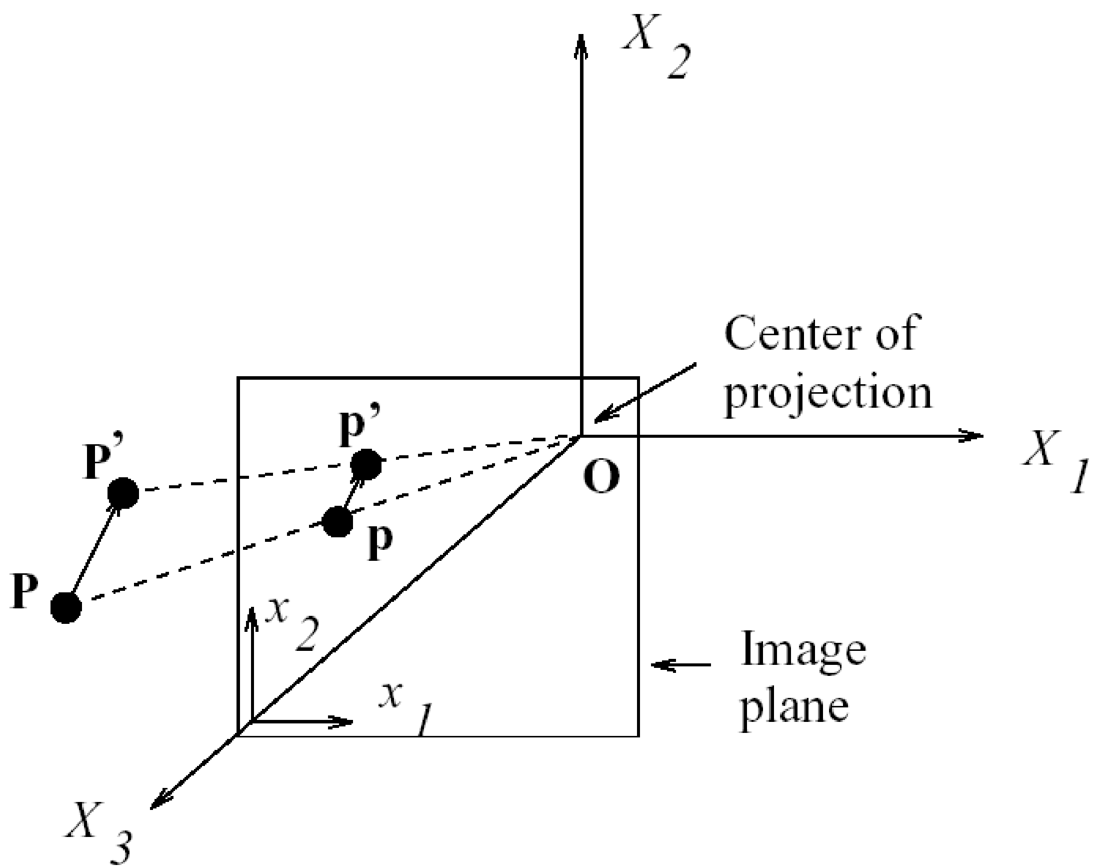
KECE471 Computer Vision

Motion

Chang-Su Kim

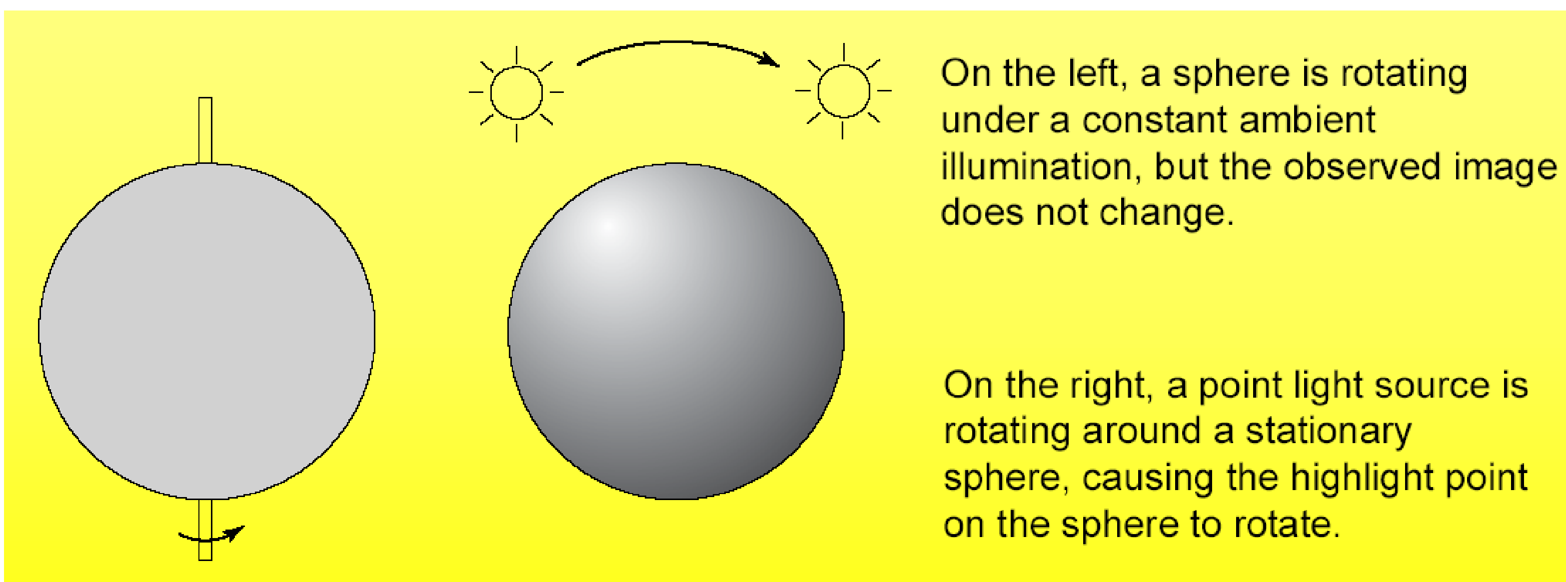
2D Motion vs. Optical Flow

- True 2D motion
  - There is 3D motion between object and camera
  - It is projected onto 2D imaging plane



2D Motion vs. Optical Flow

- Optical flow
  - observed, perceived, apparent 2D motion based on changes in pixel luminance
  - It also depends on illumination and object surface texture
  - It may not represent true 2D motion



Optical Flow Equation

- Given only video sequence without any other information (such as illumination condition), we cannot estimate true 2D motion.
- The best one can hope to estimate is optical flow
- Constant intensity assumption → optical flow equation

Under "constant intensity assumption":

$$\psi(x+d_x, y+d_y, t+d_t) = \psi(x,y,t)$$

But, using Taylor's expansion :

$$\psi(x+d_x, y+d_y, t+d_t) = \psi(x,y,t) + \frac{\partial\psi}{\partial x} d_x + \frac{\partial\psi}{\partial y} d_y + \frac{\partial\psi}{\partial t} d_t$$

spatial gradient

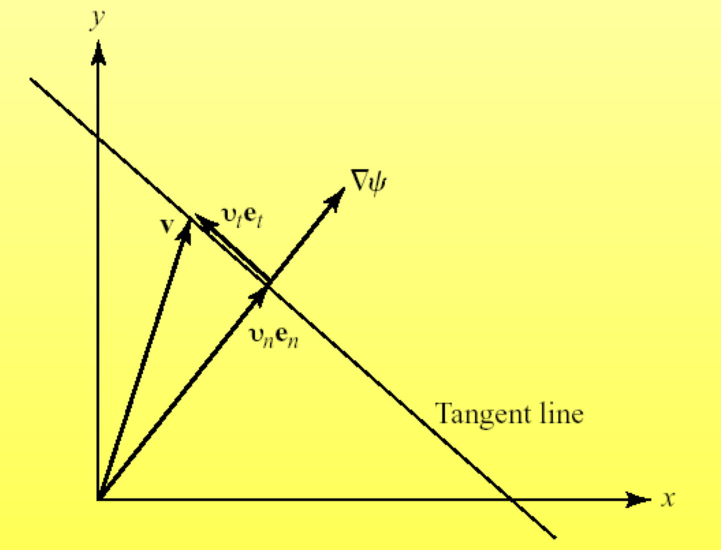
optical flow

Compare the above two, we have the ... at

$$\frac{\partial\psi}{\partial x} d_x + \frac{\partial\psi}{\partial y} d_y + \frac{\partial\psi}{\partial t} d_t = 0 \quad \text{or} \quad \frac{\partial\psi}{\partial x} v_x + \frac{\partial\psi}{\partial y} v_y + \frac{\partial\psi}{\partial t} = 0 \quad \text{or} \quad \boxed{\nabla\psi \cdot \mathbf{v} + \frac{\partial\psi}{\partial t} = 0}$$

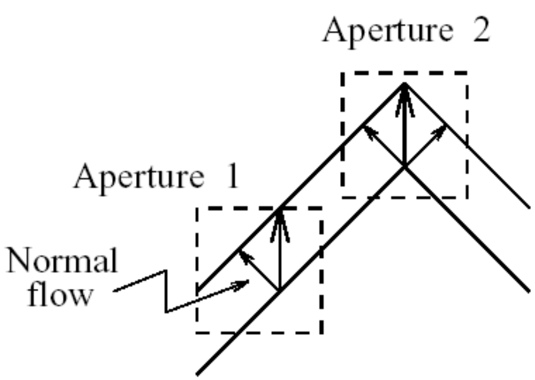
Ambiguities in Optical Flow Estimation

- Optical flow equation constrains the motion vector in the gradient direction v_n only
- The flow vector in the tangent direction v_t is under-determined
  - We can only determine the displacement that is orthogonal to the edges
- In regions with constant brightness $\nabla\psi = 0$, the flow is indeterminate
  - Optical flow estimation is unreliable in regions with flat texture and more reliable near edges

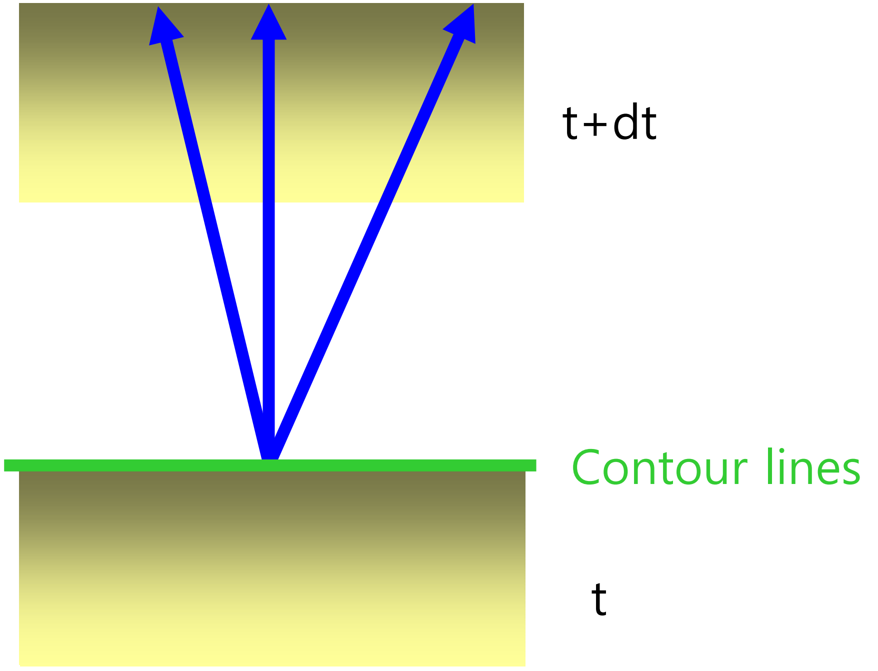


$$\mathbf{v} = v_n\mathbf{e}_n + v_t\mathbf{e}_t$$

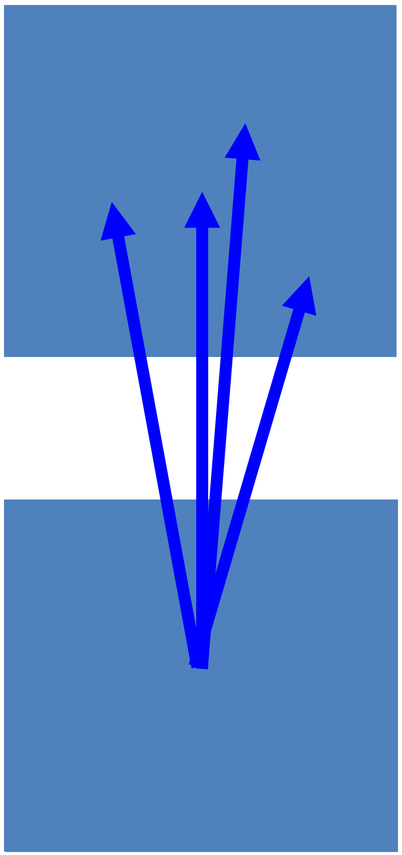
$$v_n\|\nabla\psi\| + \frac{\partial\psi}{\partial t} = 0$$



Ambiguities in Optical Flow Estimation



Ambiguities in Optical Flow Estimation



Example

- Consider a video signal $s(x, y, t)$ defined over the entire 3-D space (x, y, t), where $s(x, y, t)$ is generated from one object, which undergoes translational motion with a uniform constant velocity (v_x, v_y). Suppose that $s(x, y, 0) = x + y + xy$.
  1. Determine $s(x, y, t)$.
  2. Show that in this case the following optical flow equation holds true.

$$v_x \frac{\partial s(x, y, t)}{\partial x} + v_y \frac{\partial s(x, y, t)}{\partial y} + \frac{\partial s(x, y, t)}{\partial t} = 0$$

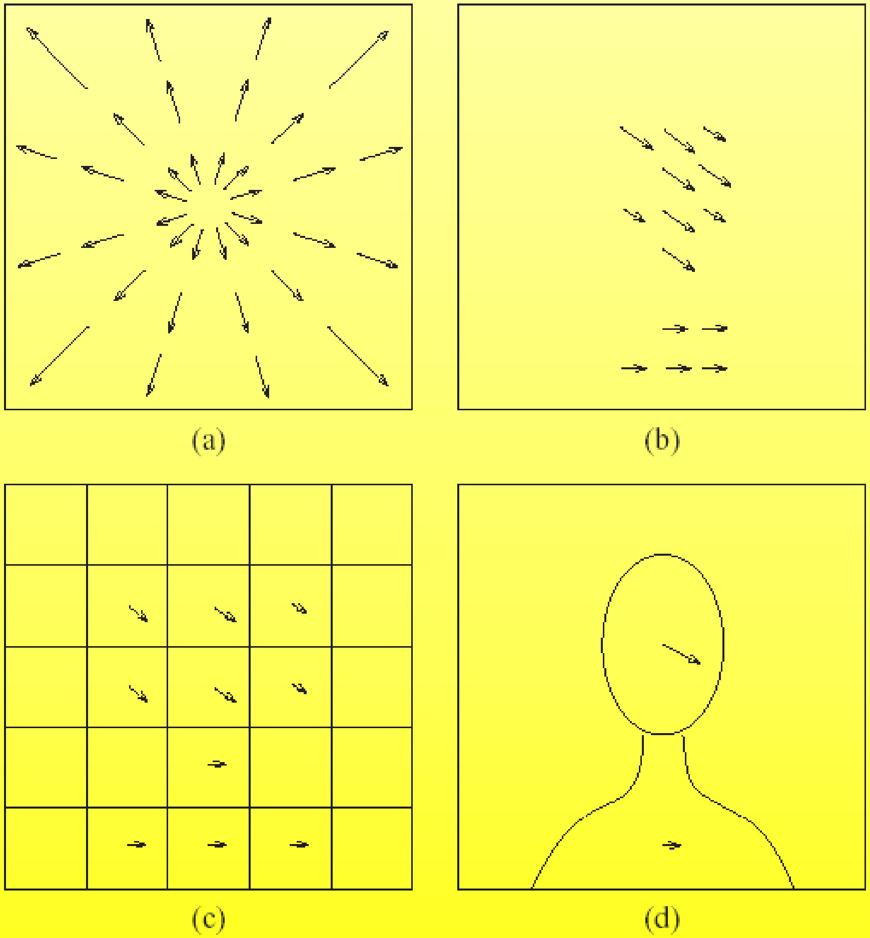
General Considerations for Motion Estimation

- Two categories of approaches
  - Feature based
    - Correspondence between edges, points, etc
    - Object tracking, 3D reconstruction from 2D
  - Intensity based
    - Optical flow estimation based on constant intensity assumption
    - Focus in this class

- Three important questions
  - How to represent the motion field?
  - Which cost function (criterion) to use to estimate motion parameters?
  - Which optimization technique?

Motion Representation

Global:
Entire motion field is represented by a few global parameters



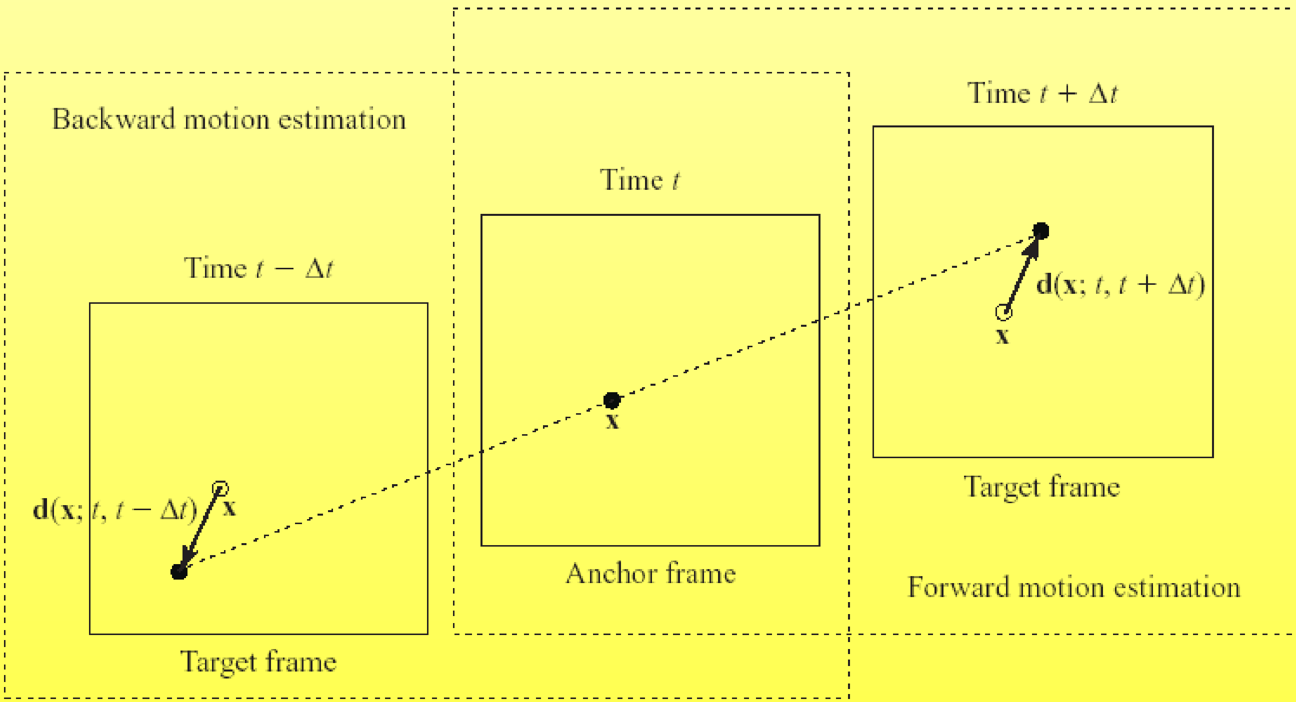
Pixel-based:
One MV at each pixel, with some smoothness constraint between adjacent MVs.

Block-based:
Entire frame is divided into blocks, and motion in each block is characterized by a few parameters.

Region-based:
Entire frame is divided into regions, each region corresponding to an object or sub-object with consistent motion, represented by a few parameters.

Other representation: mesh-based (control grid) (to be discussed later)

Notations



Anchor frame: $\psi_1(\mathbf{x})$

Target frame: $\psi_2(\mathbf{x})$

Motion parameters: $\mathbf{a}$

Motion vector at a pixel in the anchor frame: $\mathbf{d}(\mathbf{x})$

Motion field: $\mathbf{d}(\mathbf{x};\mathbf{a}), \mathbf{x} \in \Lambda$

Mapping function:

$$\mathbf{w}(\mathbf{x};\mathbf{a}) = \mathbf{x} + \mathbf{d}(\mathbf{x};\mathbf{a}), \mathbf{x} \in \Lambda$$

Motion Estimation Criteria

- Minimize displaced frame difference

$$E_{DFD}(\mathbf{a}) = \sum_{\mathbf{x}\in\Lambda} |\psi_2(\mathbf{w}(\mathbf{x};\mathbf{a})) - \psi_1(\mathbf{x})|^p$$

- For $p = 2$ (MSE), the necessary condition for minimum is that the derivative is zero. Let $\mathbf{z} = \mathbf{w}(\mathbf{x};\mathbf{a}) = \mathbf{x} + \mathbf{d}(\mathbf{x};\mathbf{a})$. Then, the derivative is given by

$$\begin{aligned}\frac{\partial E_{DFD}}{\partial \mathbf{a}} &= 2\sum_{\mathbf{x}\in\Lambda}(\psi_2(\mathbf{z}) - \psi_1(\mathbf{x}))\frac{\partial \psi_2}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{a}} \\ &= 2\sum_{\mathbf{x}\in\Lambda}(\psi_2(\mathbf{z}) - \psi_1(\mathbf{x}))\frac{\partial \psi_2}{\partial \mathbf{z}}\frac{\partial \mathbf{d}}{\partial \mathbf{a}}\end{aligned}$$

Motion Estimation Criteria

- Optical Flow Equation

$$\frac{\partial \psi}{\partial x} d_x + \frac{\partial \psi}{\partial y} d_y + \frac{\partial \psi}{\partial t} d_t = 0$$

$$\Rightarrow \quad \frac{\partial \psi}{\partial x} v_x + \frac{\partial \psi}{\partial y} v_y + \frac{\partial \psi}{\partial t} = 0$$

where $\mathbf{d}(\mathbf{x};\mathbf{a}) = [d_x, d_y]^T$ and
$\mathbf{v}(\mathbf{x};\mathbf{a}) = [v_x, v_y]^T = [d_x/d_t, d_y/d_t]^T$.

- Thus, obtain the motion vector parameters $\mathbf{a}$ which minimizes the cost function

$$E_{OF}(\mathbf{a}) = \sum_{\mathbf{x} \in \Lambda} \left| \frac{\partial \psi}{\partial \mathbf{x}} \mathbf{v}(\mathbf{x};\mathbf{a}) + \frac{\partial \psi}{\partial t} \right|^p$$

Lucas-Kanade Method

- Based on optical flow equation
- Assuming all pixels in a small block surrounding a pixel have the same motion vector

Lucas-Kanade Method

- Optical Flow Equation $\frac{\partial\psi}{\partial x}v_x + \frac{\partial\psi}{\partial y}v_y + \frac{\partial\psi}{\partial t} = 0$
  - For each pixel one equation two unknowns (v_x, v_y)
  - Under-determined system
- Idea
  - Assume that a block $\mathcal{B}$ centered around the current pixel has the same motion and then minimize

$$E(v_x, v_y) = \sum_{\mathbf{x}\in\mathcal{B}} \left(\frac{\partial\psi}{\partial x}(\mathbf{x})v_x + \frac{\partial\psi}{\partial y}(\mathbf{x})v_y + \frac{\partial\psi}{\partial t}(\mathbf{x}) \right)^2$$

  - Then, the optimal vector is set as the vector of the current pixel

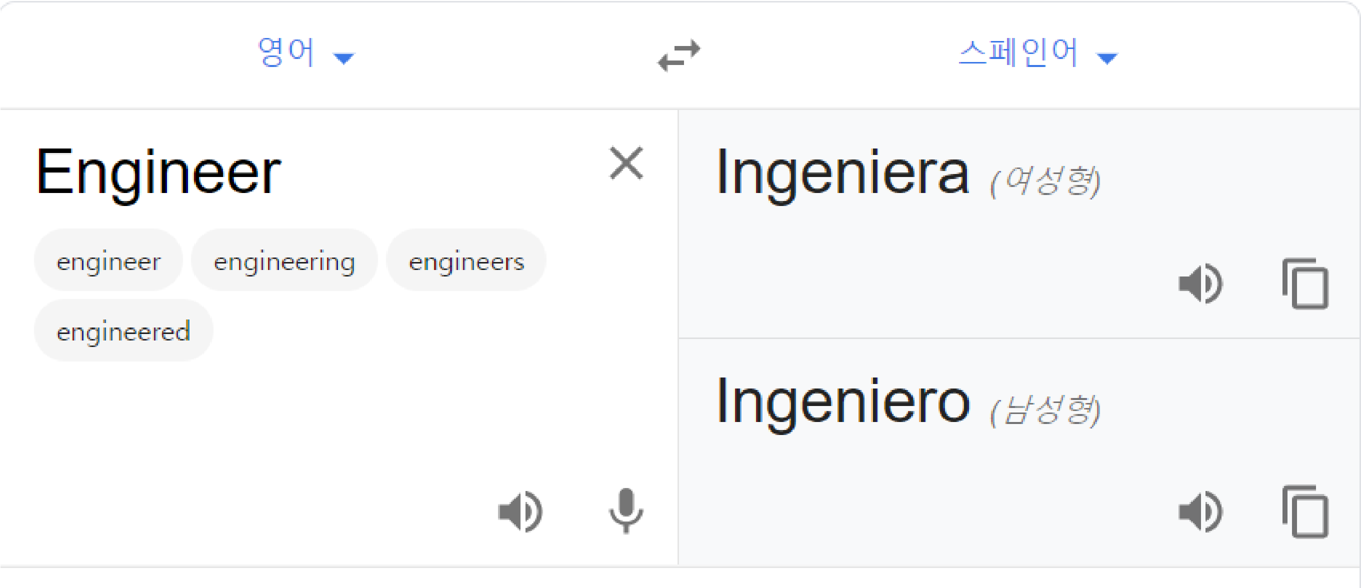
Lucas-Kanade Method

- Solution
  - By setting the partial derivatives with respect to v_x and v_y to zeros

$$
\begin{aligned}
\sum_{\mathbf{x}\in\mathcal{B}} \left(\frac{\partial\psi}{\partial x}(\mathbf{x}) v_x + \frac{\partial\psi}{\partial y}(\mathbf{x}) v_y + \frac{\partial\psi}{\partial t}(\mathbf{x}) \right) \frac{\partial\psi}{\partial x}(\mathbf{x}) &= 0, \\
\sum_{\mathbf{x}\in\mathcal{B}} \left(\frac{\partial\psi}{\partial x}(\mathbf{x}) v_x + \frac{\partial\psi}{\partial y}(\mathbf{x}) v_y + \frac{\partial\psi}{\partial t}(\mathbf{x}) \right) \frac{\partial\psi}{\partial y}(\mathbf{x}) &= 0,
\end{aligned}
$$

  - we have

$$
\begin{bmatrix} v_x \\ v_y \end{bmatrix} = - \begin{bmatrix} \sum_{\mathbf{x}\in\mathcal{B}} \frac{\partial\psi}{\partial x}(\mathbf{x}) \frac{\partial\psi}{\partial x}(\mathbf{x}) & \sum_{\mathbf{x}\in\mathcal{B}} \frac{\partial\psi}{\partial x}(\mathbf{x}) \frac{\partial\psi}{\partial y}(\mathbf{x}) \\ \sum_{\mathbf{x}\in\mathcal{B}} \frac{\partial\psi}{\partial x}(\mathbf{x}) \frac{\partial\psi}{\partial y}(\mathbf{x}) & \sum_{\mathbf{x}\in\mathcal{B}} \frac{\partial\psi}{\partial y}(\mathbf{x}) \frac{\partial\psi}{\partial y}(\mathbf{x}) \end{bmatrix}^{-1} \begin{bmatrix} \sum_{\mathbf{x}\in\mathcal{B}} \frac{\partial\psi}{\partial x}(\mathbf{x}) \frac{\partial\psi}{\partial t}(\mathbf{x}) \\ \sum_{\mathbf{x}\in\mathcal{B}} \frac{\partial\psi}{\partial y}(\mathbf{x}) \frac{\partial\psi}{\partial t}(\mathbf{x}) \end{bmatrix}
$$



영어
스페인어
Engineer
engineer
engineering
engineers
engineered
Ingeniera (여성형)
Ingeniero (남성형)

영어 ⇄ 스페인어

Engineer

engineer engineering engineers engineered

Ingeniera (여성형)

ingenious *adjective*

/ɪnˈdʒiːniəs/

/ɪnˈdʒiːniəs/

1 (of an object, a plan, an idea, etc.) very suitable for a particular purpose and resulting from clever new ideas

- *an ingenious device/invention/experiment*
- *ingenious ways of saving energy*
- *His plots are always very ingenious.*

\+ Oxford Collocations Dictionary

2 (of a person) having a lot of clever new ideas and good at inventing things

- *an ingenious cook*
- *She's very ingenious when it comes to finding excuses.*

TOPICS **Personal qualities** C1