

Digital Signal Processing

Chap 6. Structures for Discrete-Time Systems

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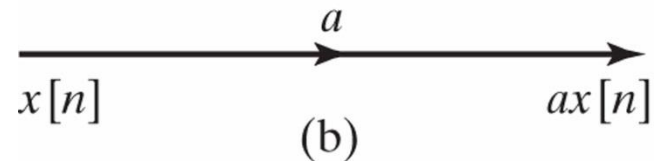
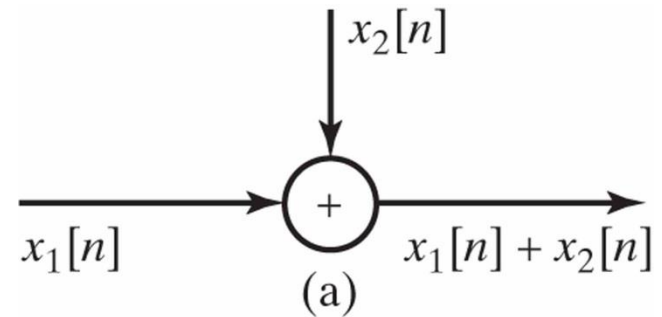
Summary

- We study various structures for implementing CCDE systems
 - Direct form I
 - Direct form II
 - Cascade form
 - Parallel form
 - Transposed form

Block Diagram Representation

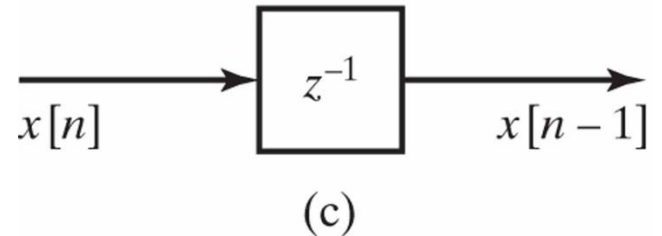
- Components

- Adder
- Multiplier
- Delay



- In implementation

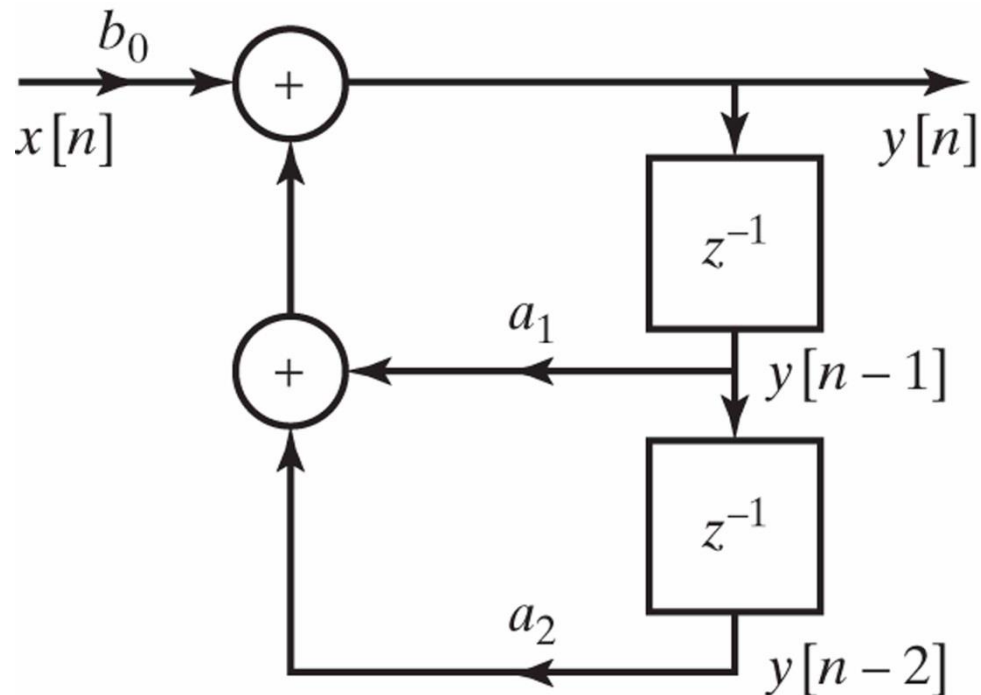
- Multipliers:
 computing resources
- Delays:
 memory resources



Block Diagram Representation

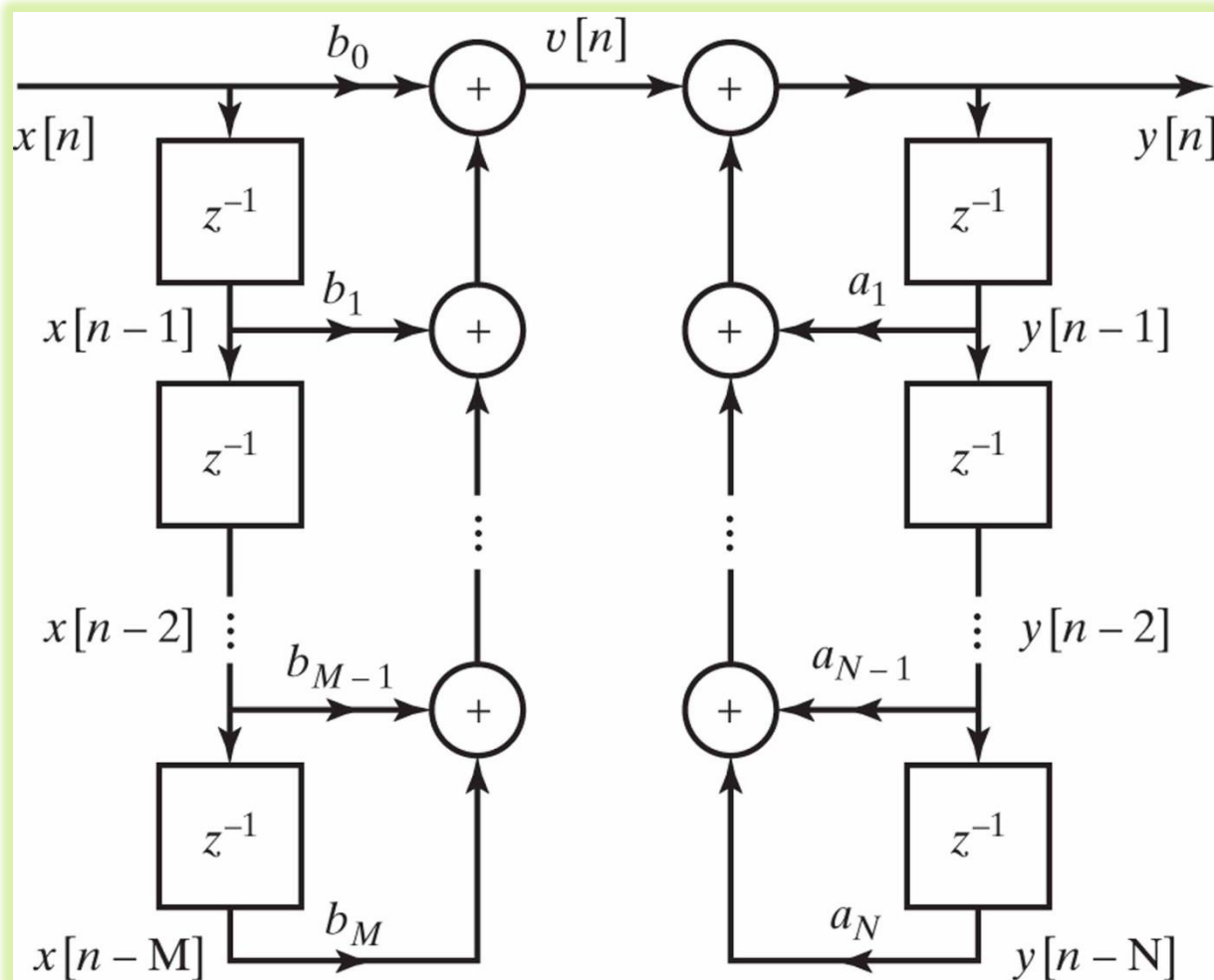
- Example

$$- H(z) = \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}}$$



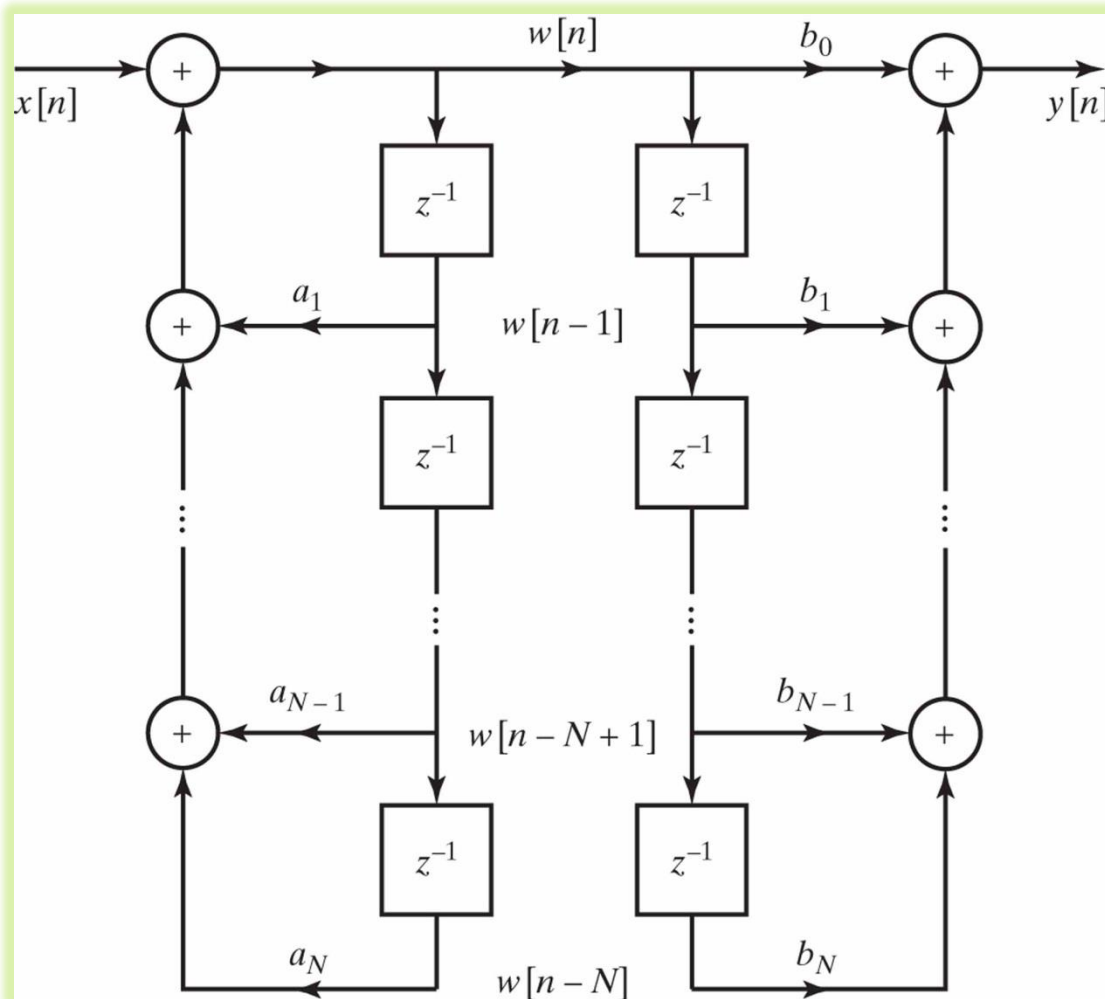
General CCDE's: Direct Form I

$$y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$



General CCDE's

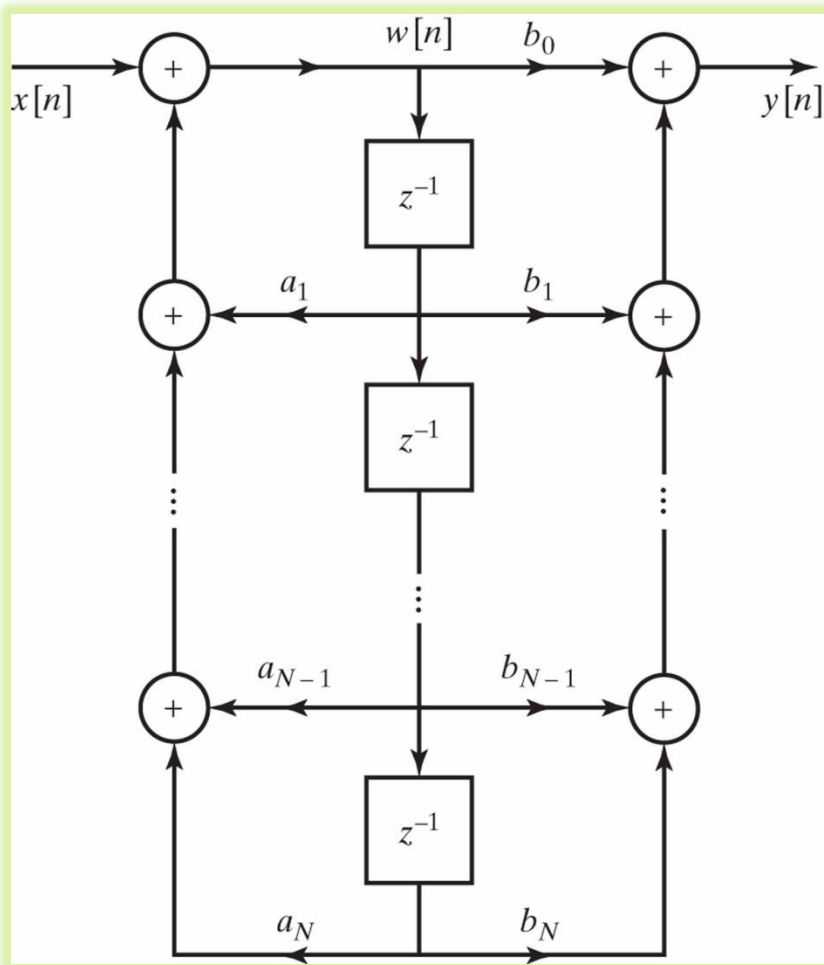
$$y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$



Assuming $M = N$

General CCDE's: Direct Form II

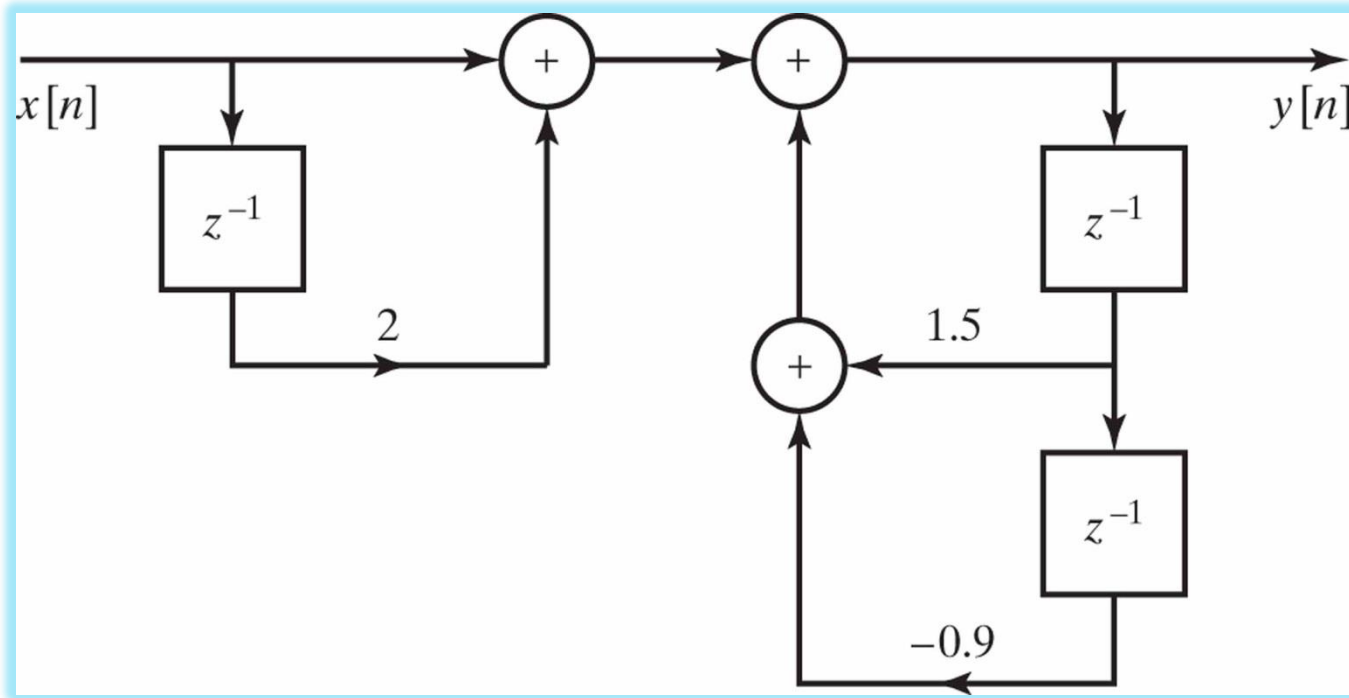
$$y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$



It requires the minimum number of delays, and is called a **canonic form**

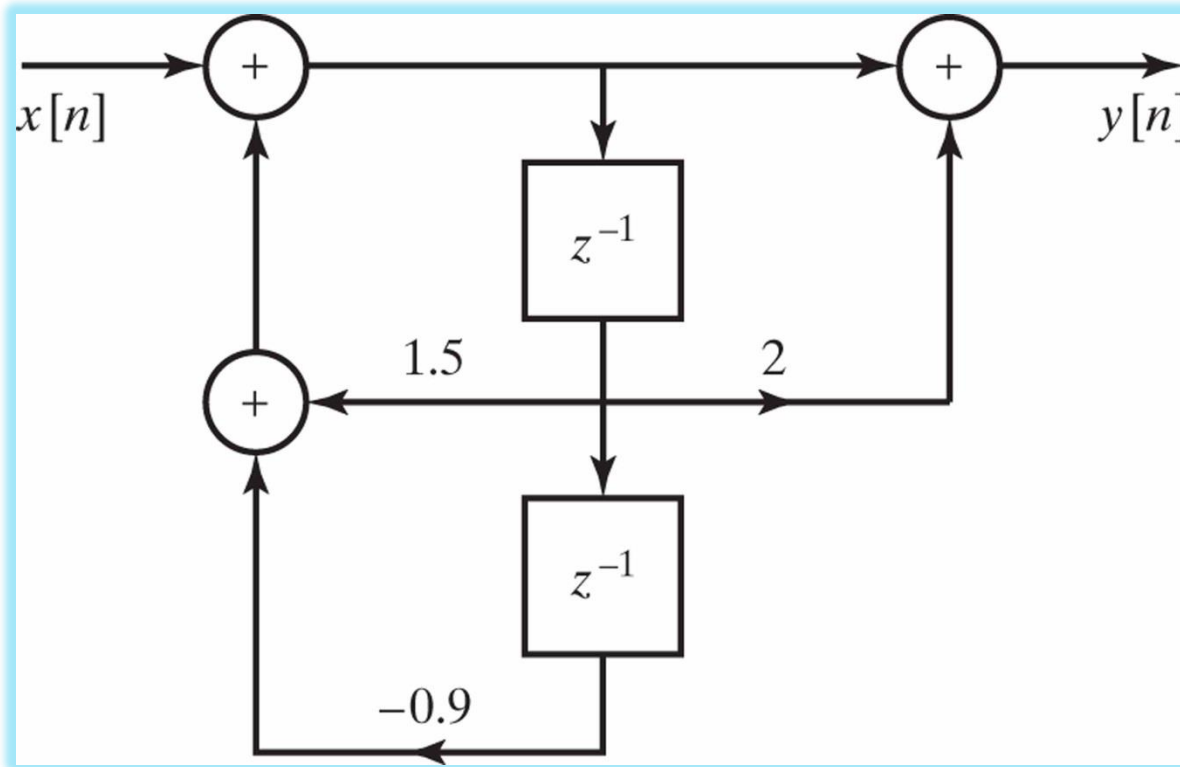
Direct Forms I and II

- $$H(z) = \frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$$

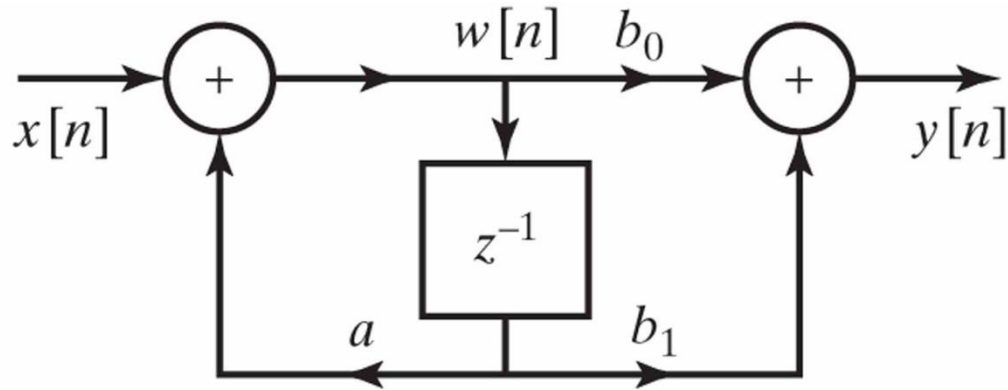


Direct Forms I and II

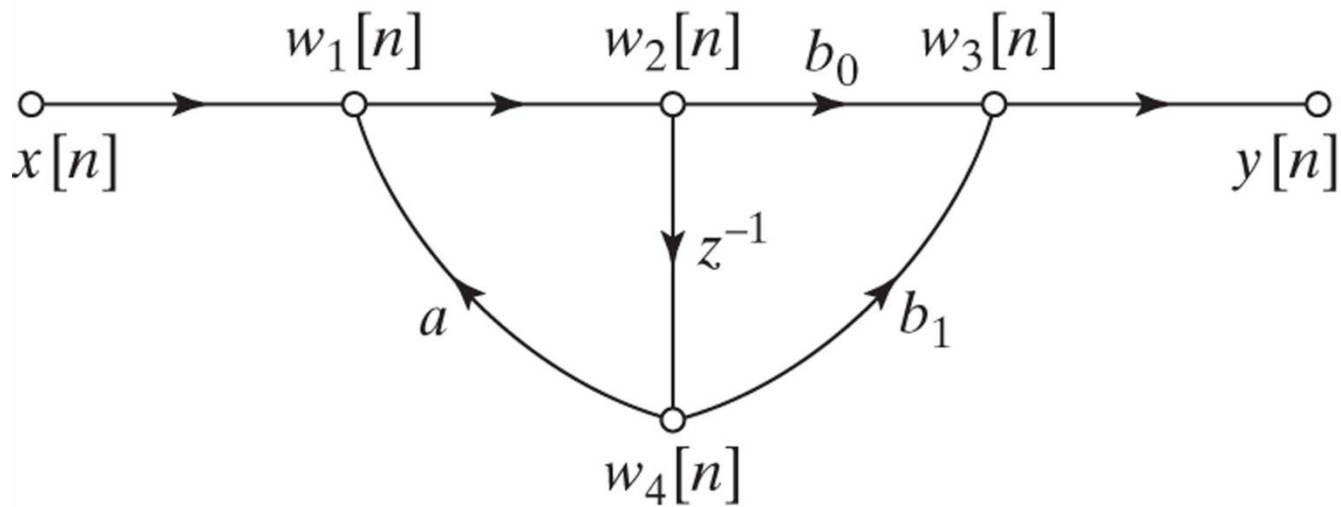
- $$H(z) = \frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$$



Signal Flow Graph Representation

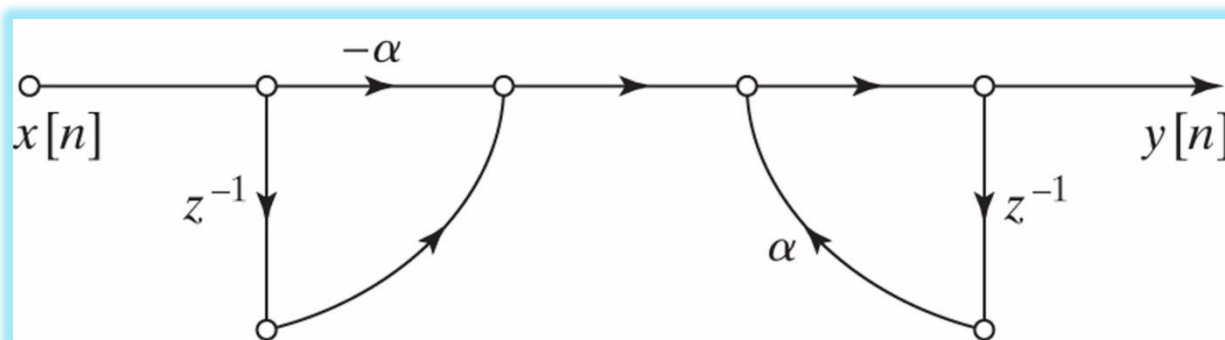
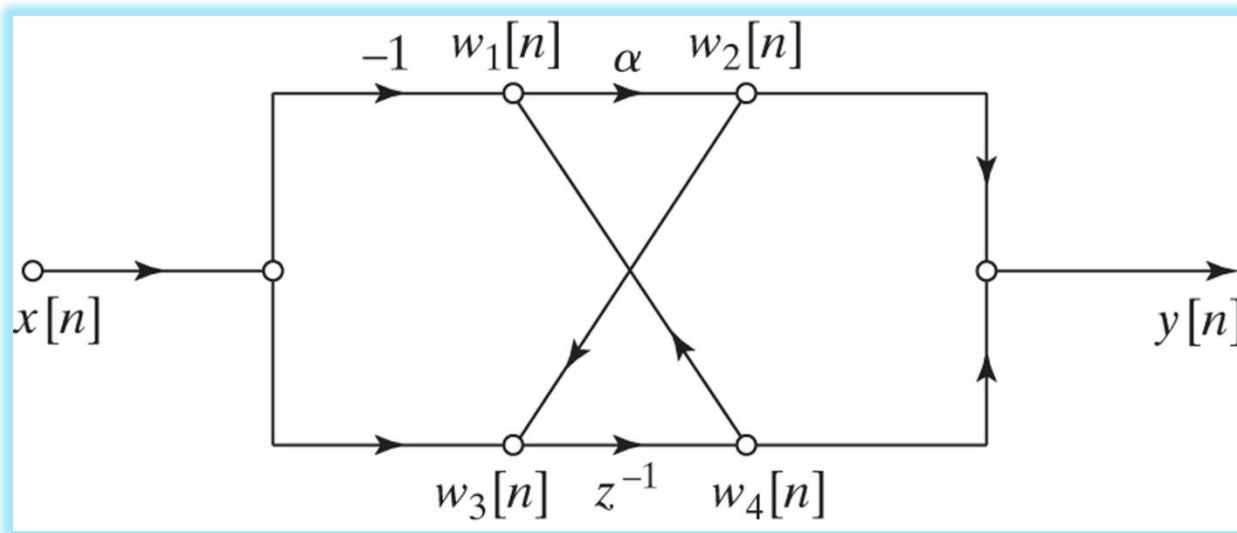


(a)



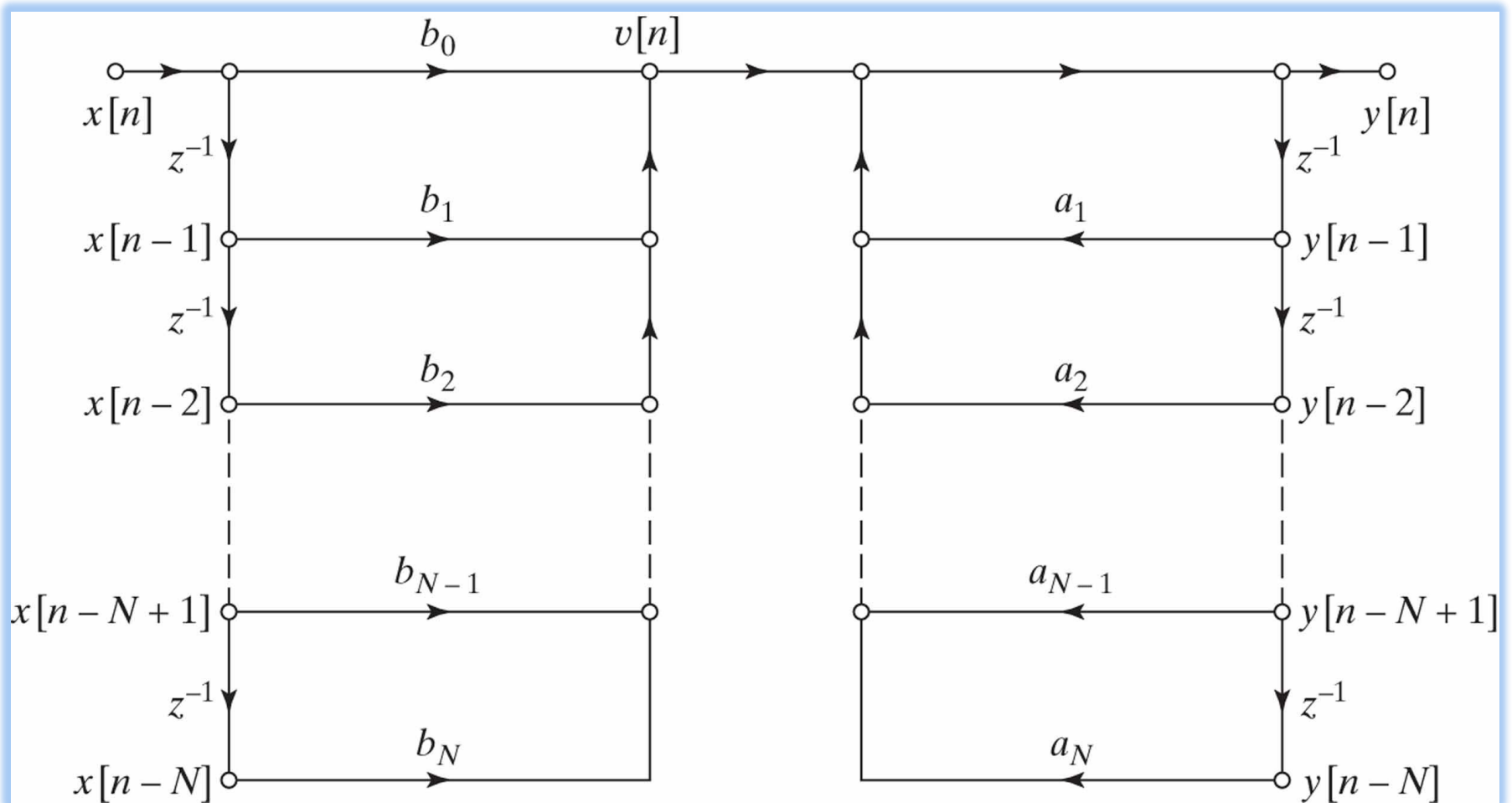
(b)

System Function from Flow Graph



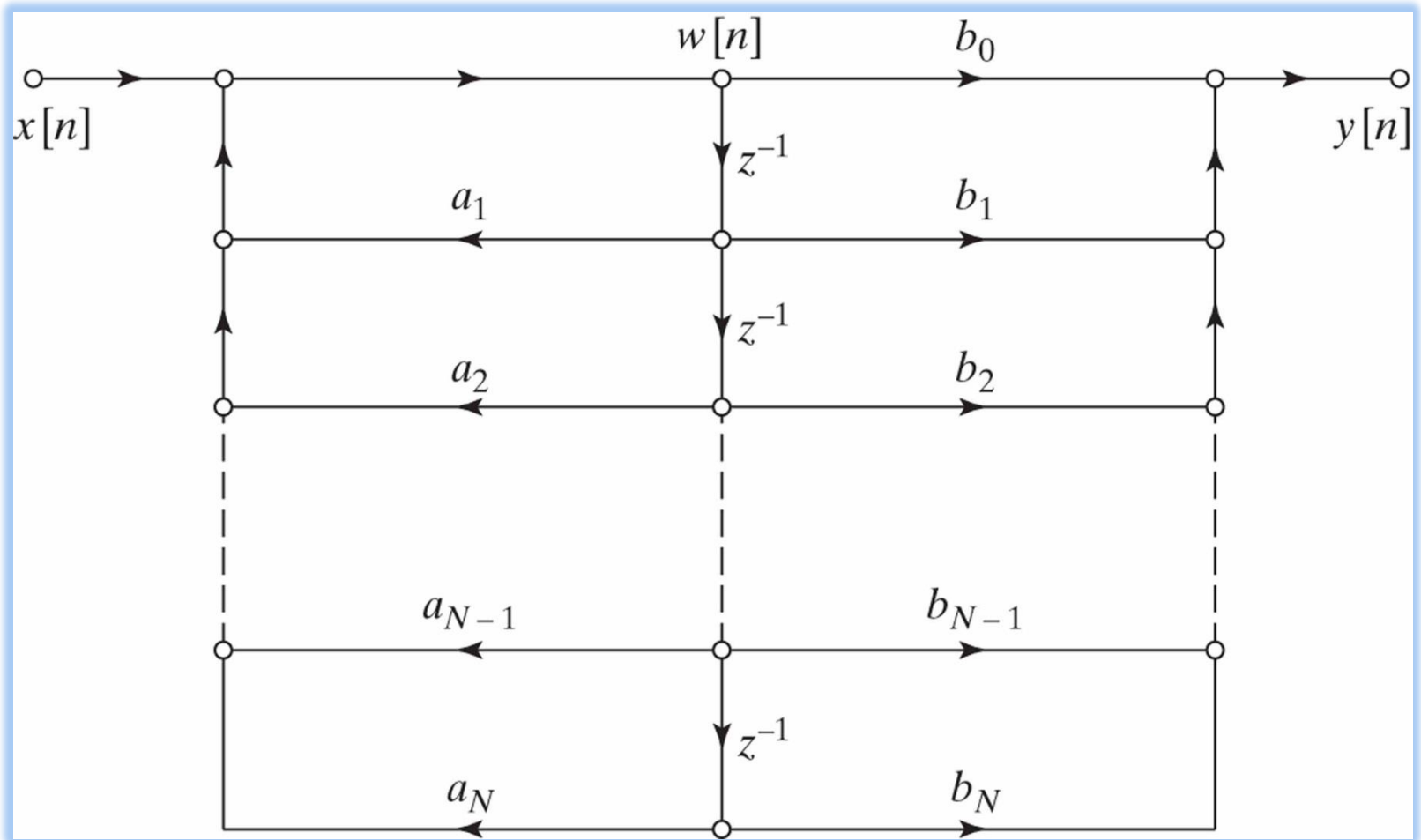
Direct Form I

$$y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

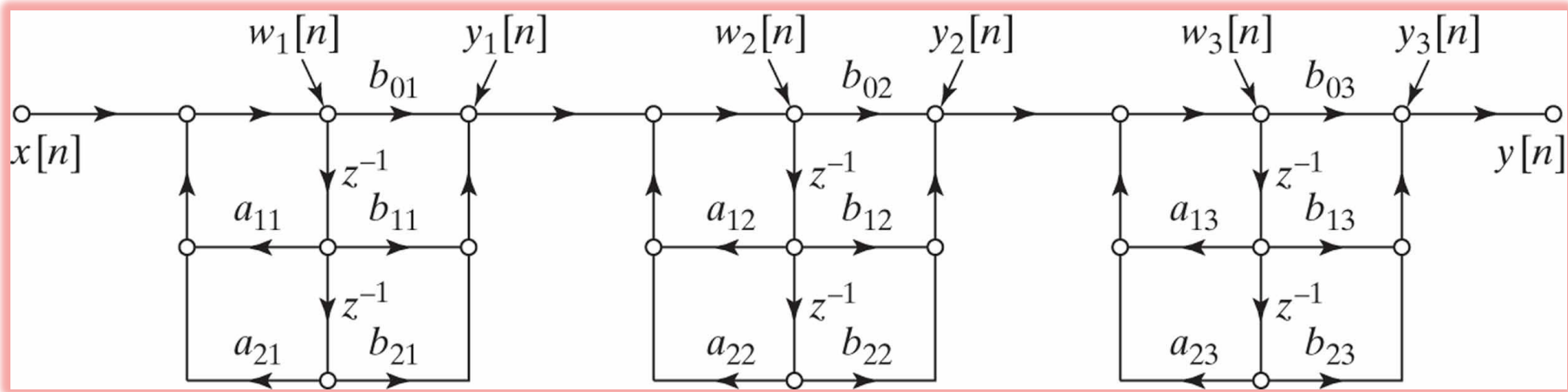


Direct Form II

$$y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$



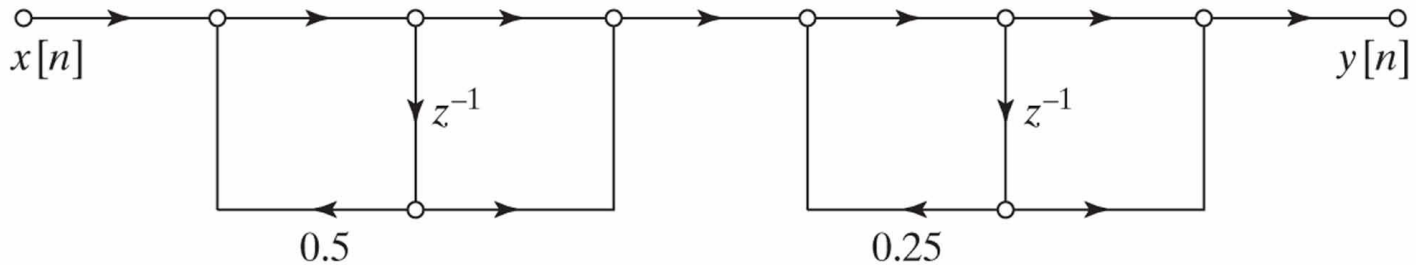
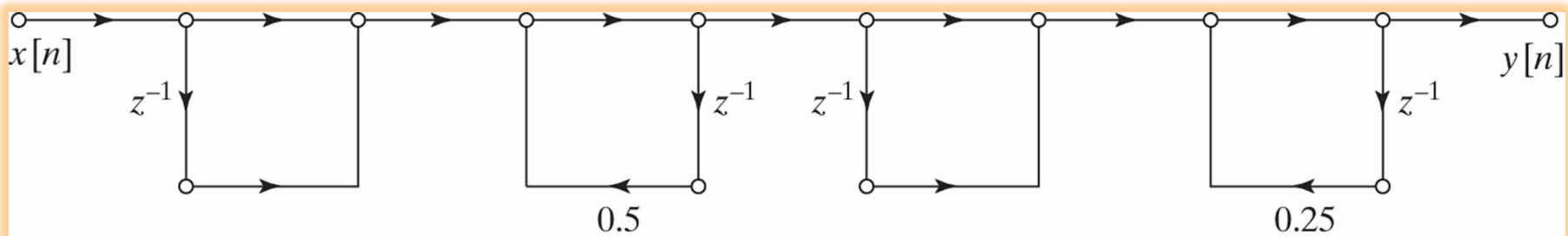
Cascade Form



- Ex)
$$H(z) = \frac{2(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})(1-j\frac{1}{4}z^{-1})(1+j\frac{1}{4}z^{-1})}{(1-z^{-1})(1-2z^{-1})(1-3e^{j\frac{1}{4}\pi}z^{-1})(1-3e^{-j\frac{1}{4}\pi}z^{-1})}$$

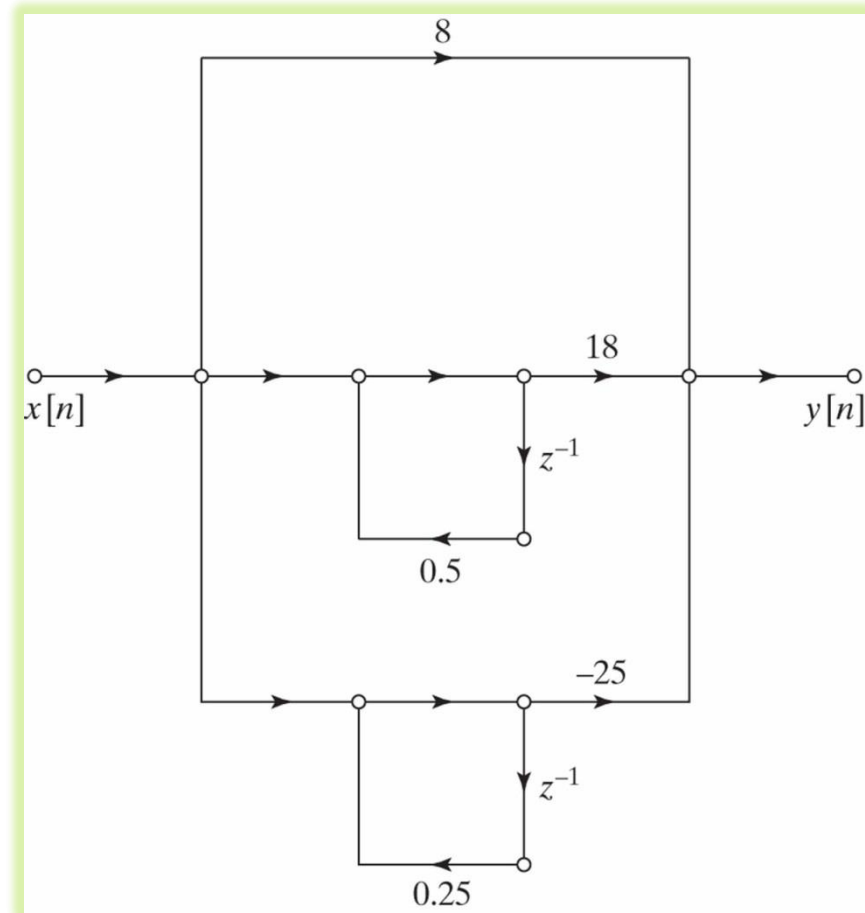
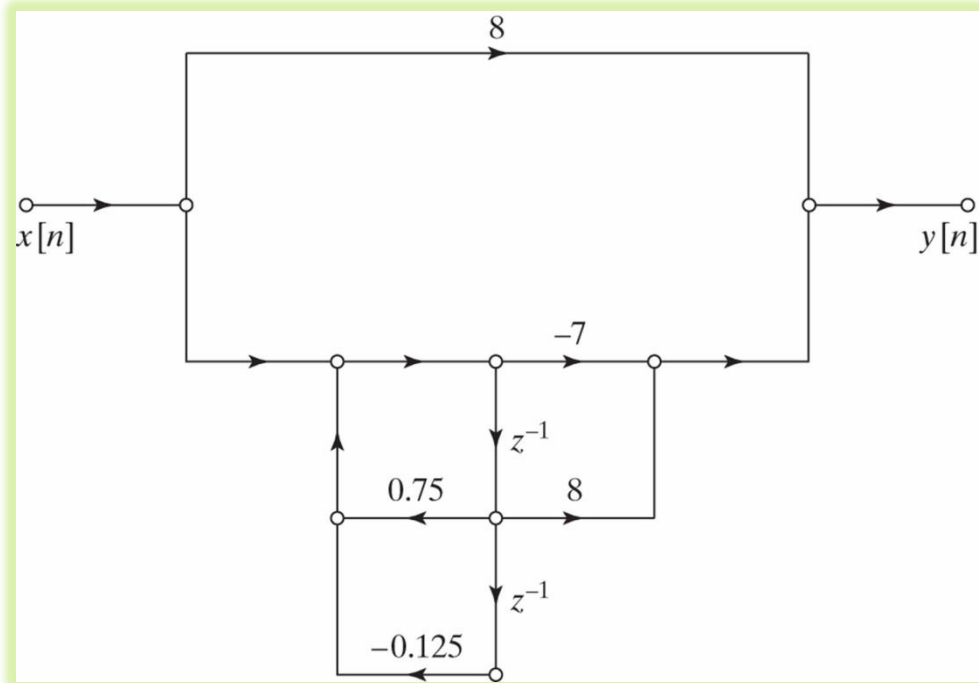
Cascade Form

- Ex) $H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$



Parallel Form

- Ex) $H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$

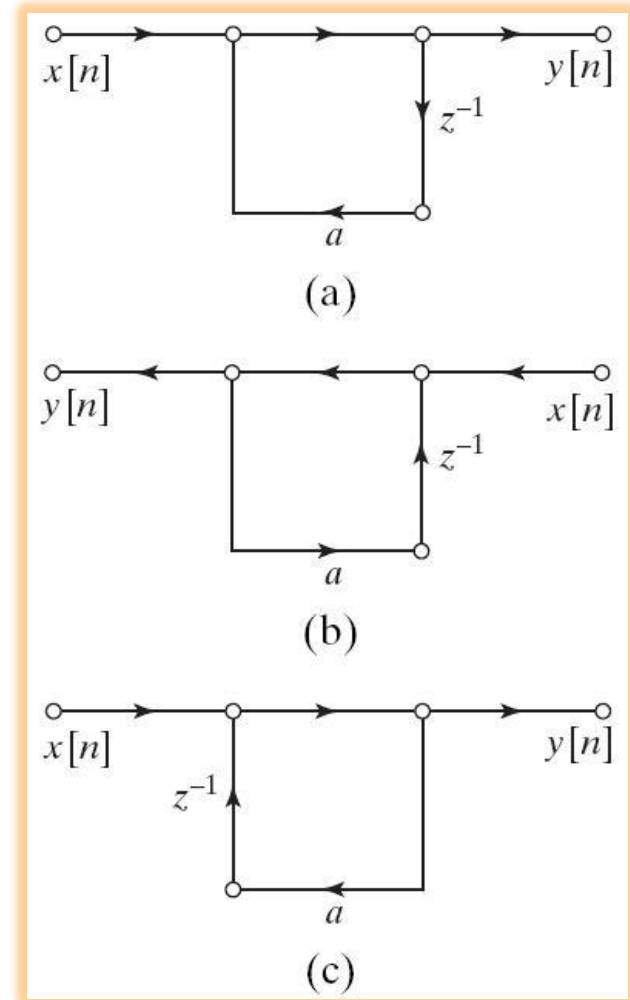


Transposed Form

- Flow graph reversal
 - Reverse the directions of all branches
 - Reverse the roles of input and output nodes

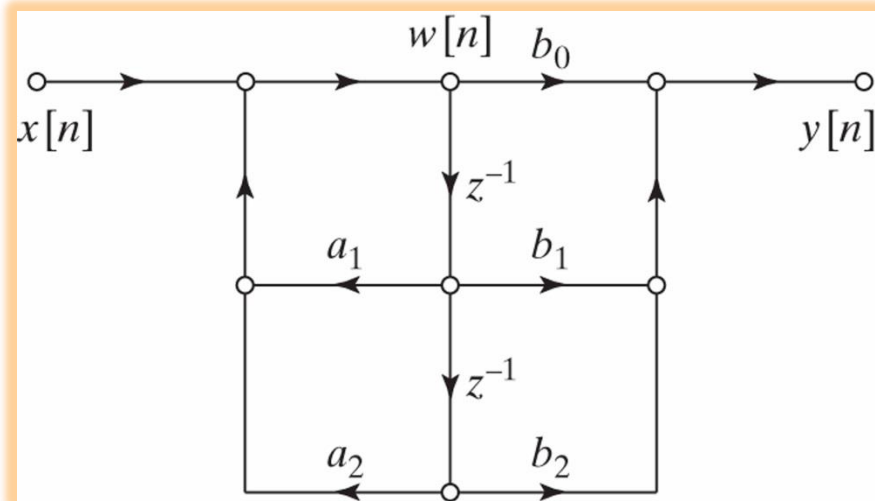
(we are skipping the proof)

- Ex) $H(z) = \frac{1}{1-az^{-1}}$

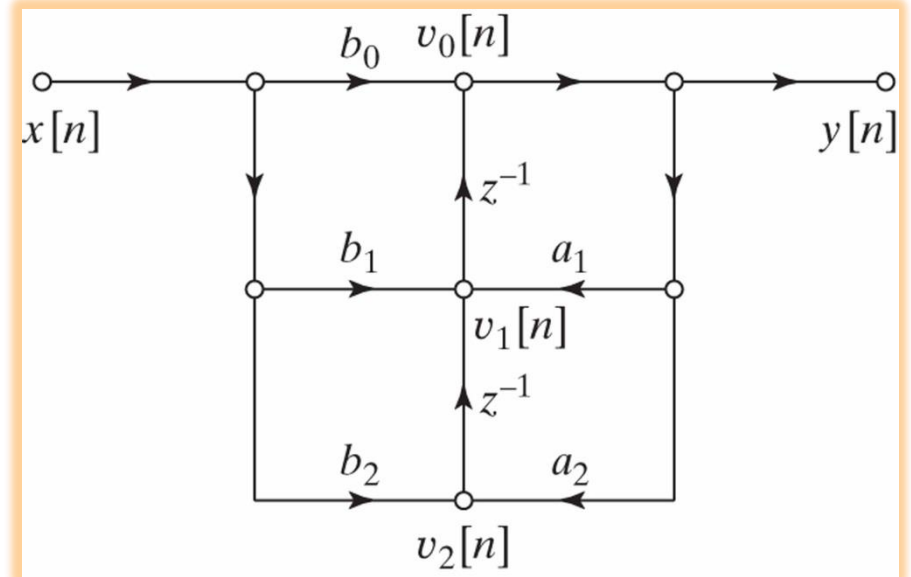


Transposed Form

- Direct form II

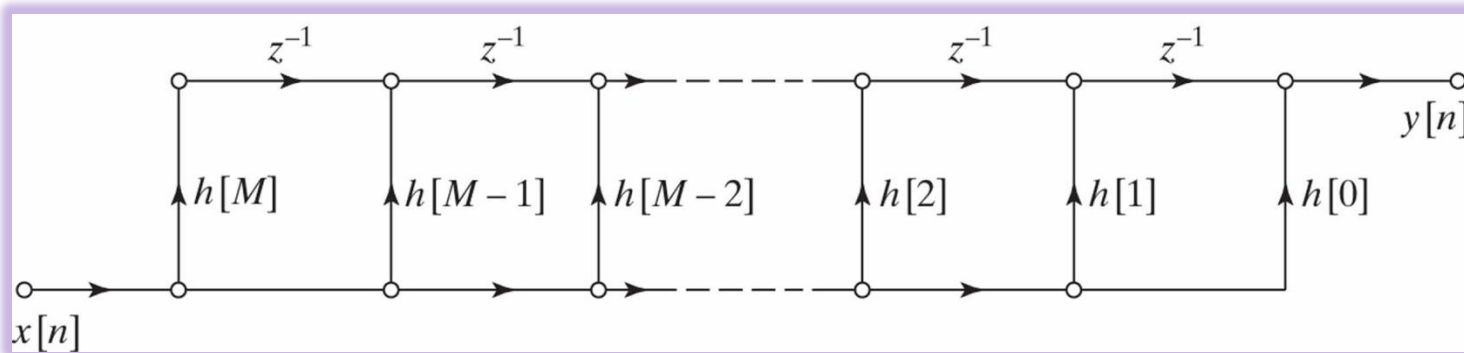
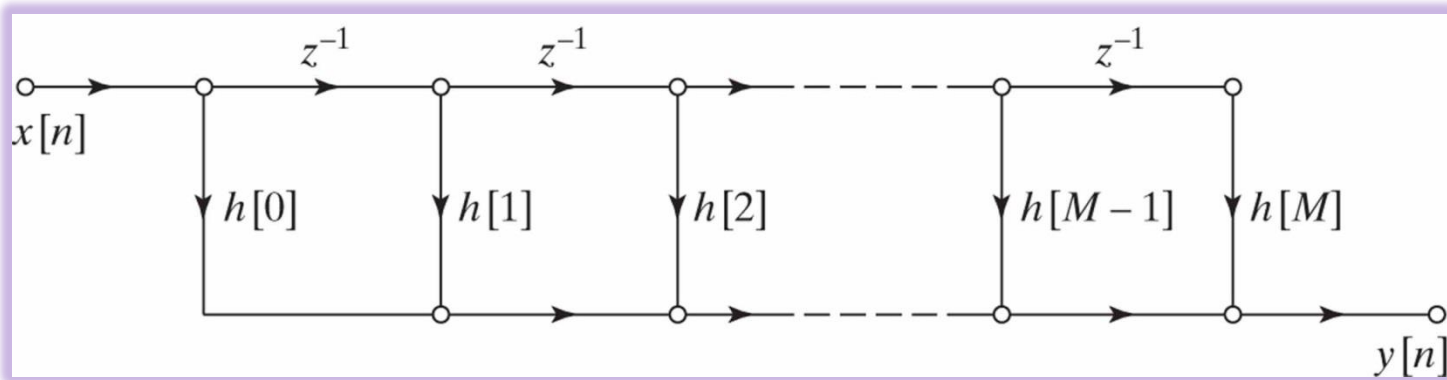


- Transposed direct form II



FIR Systems

$$y[n] = \sum_{k=1}^N \alpha_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

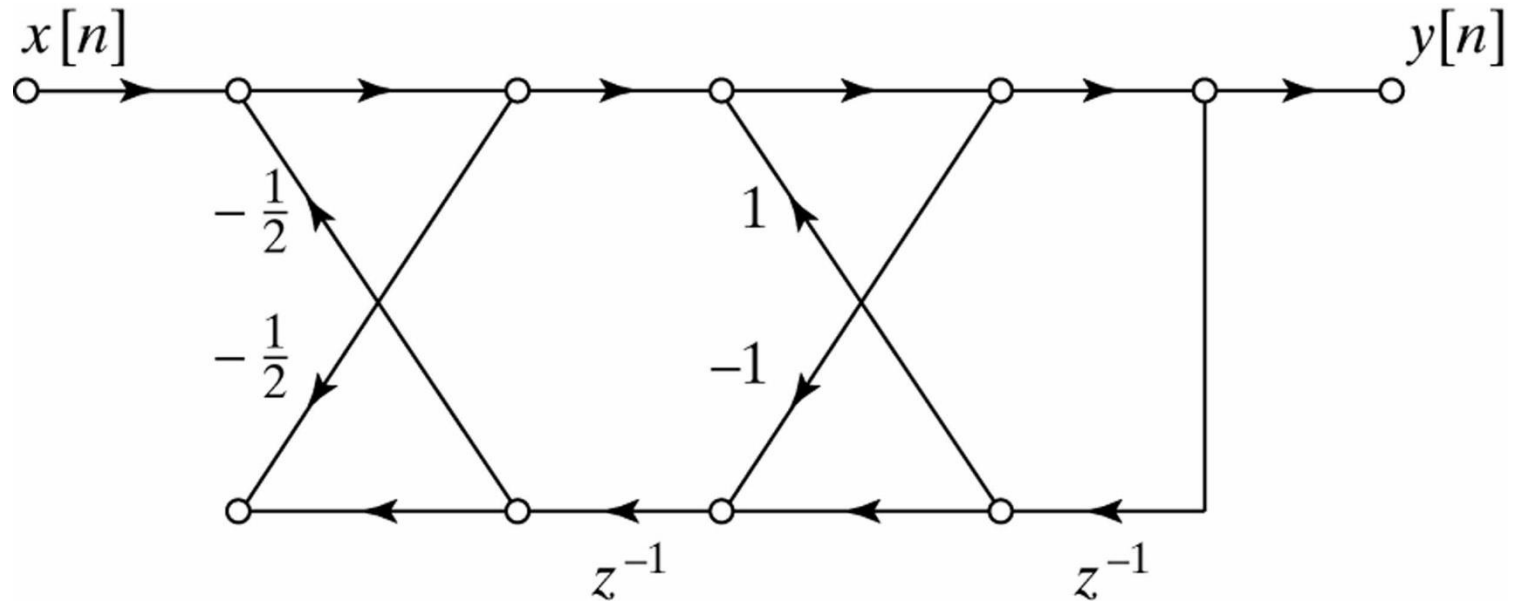


Linear-Phase FIR Systems

- Ex 1) $H(z) = 0.5 - 2z^{-1} + 3z^{-2} - 2z^{-3} + 0.5z^{-4}$

- Ex 2) $H(z) = 0.5 - 2z^{-1} + 2z^{-2} - 0.5z^{-3}$

Ex) IIR Lattice Filter



- Determine the transfer function.
- Determine $y[1]$ for input $x[n] = \delta[n]$.