

KECE471 Computer Vision

# Stereo

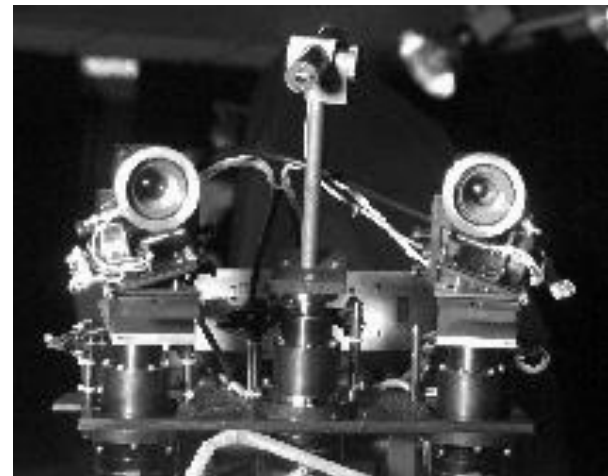
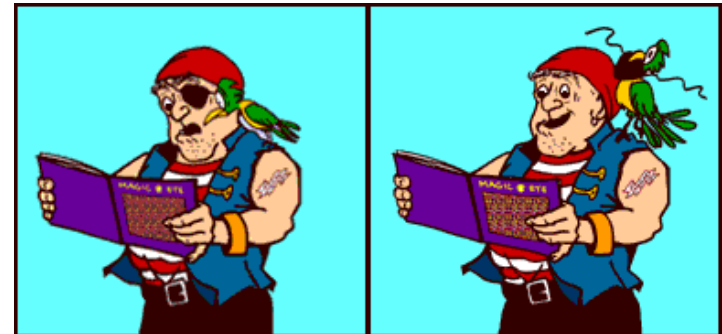
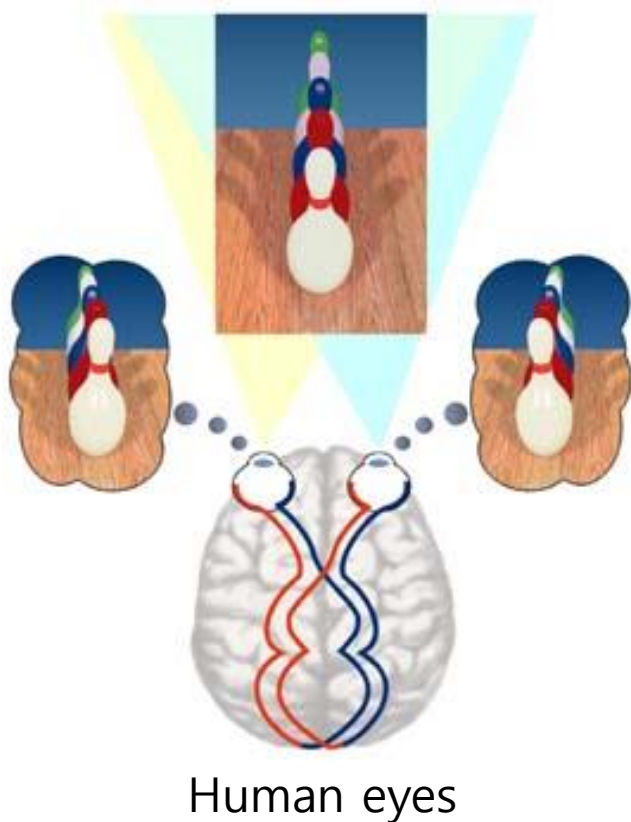
*Chang-Su Kim*

Chapter 11, Computer Vision by Forsyth and Ponce

Note: Most contents were copied from the lecture notes of Prof. Kyeong Mu Lee in SNU

# Stereo

- Inferring depth information using two cameras like a human
- Two eyes perceives three-dimension



Robot eyes

# Stereo



(a)



(b)



(c)



Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923





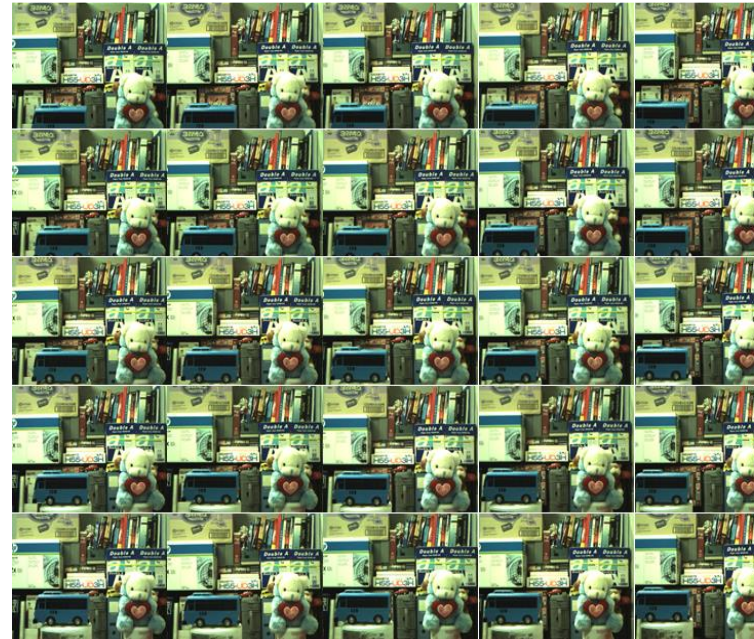
**Teesta suspension bridge-Darjeeling, India**

# Stereo

- Inferring depth information using two eyes or cameras
- Two eyes perceive 3<sup>rd</sup> dimension

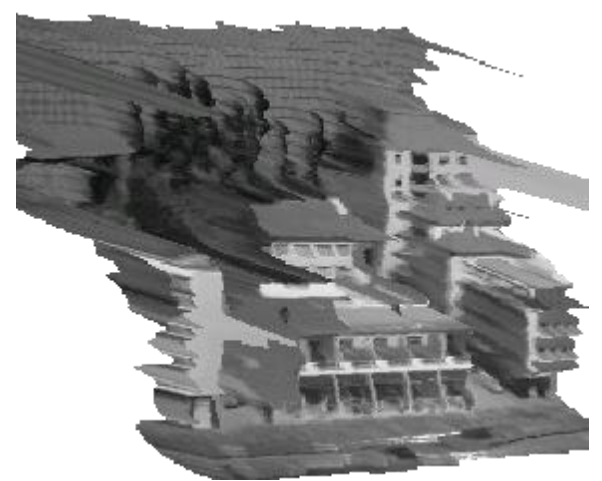


(a)



(b)

# Applications



[Matthies,Szeliski,Kanade'88]

# Applications



input image



317 images  
(hemisphere)



ground truth model

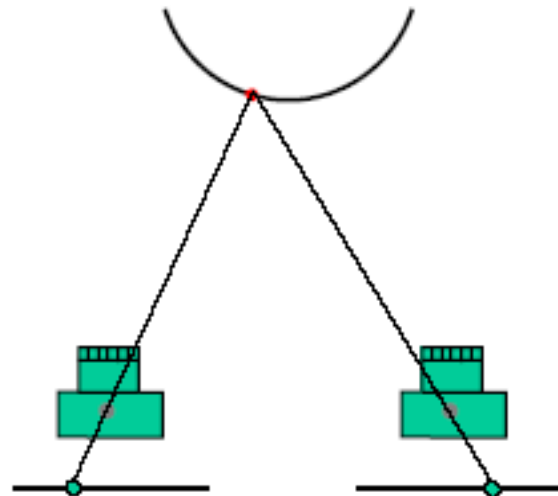
[Goesele, Curless, Seitz, 2006](#)



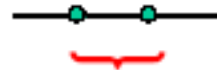
# Binocular Stereo

From known geometry of the cameras and estimated disparity, recover depth in the scene

Left

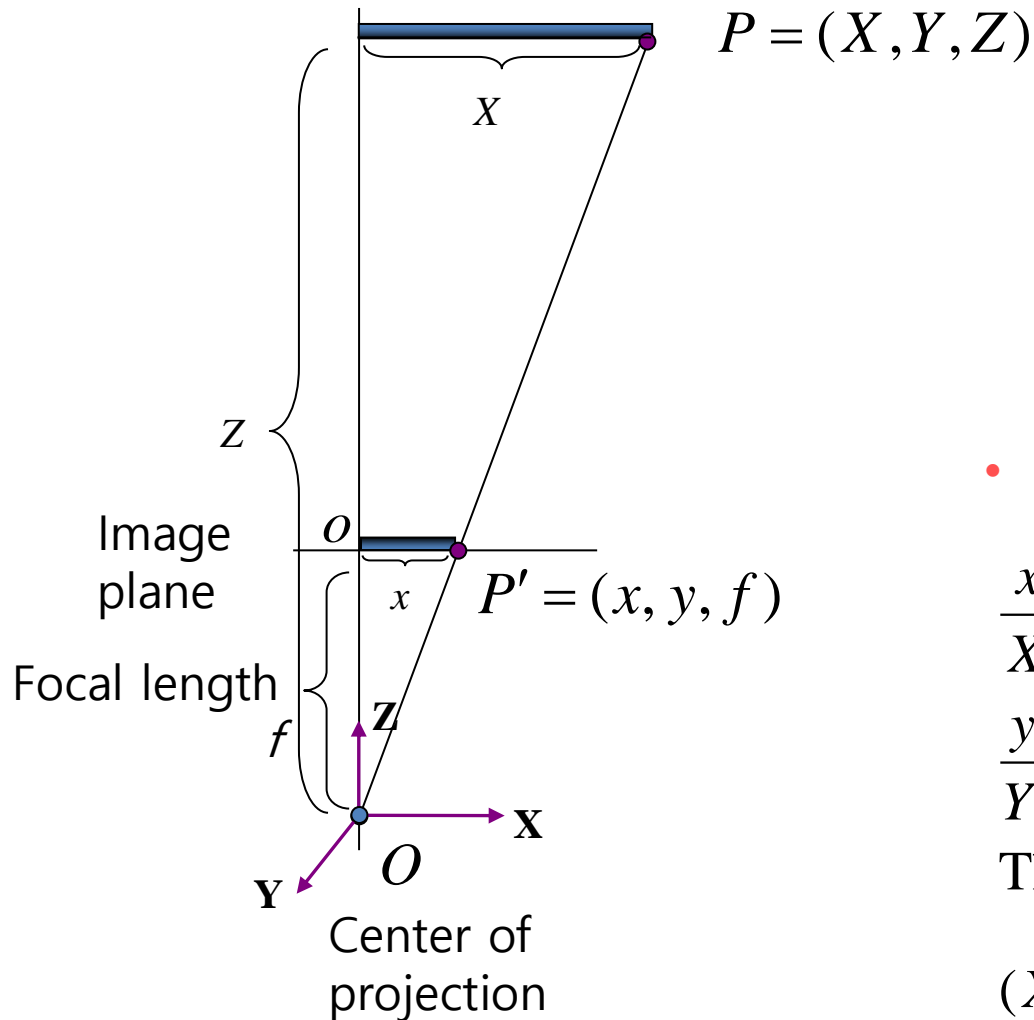


Right



binocular disparity

# Pinhole Camera Model



- 3D to 2D projection:

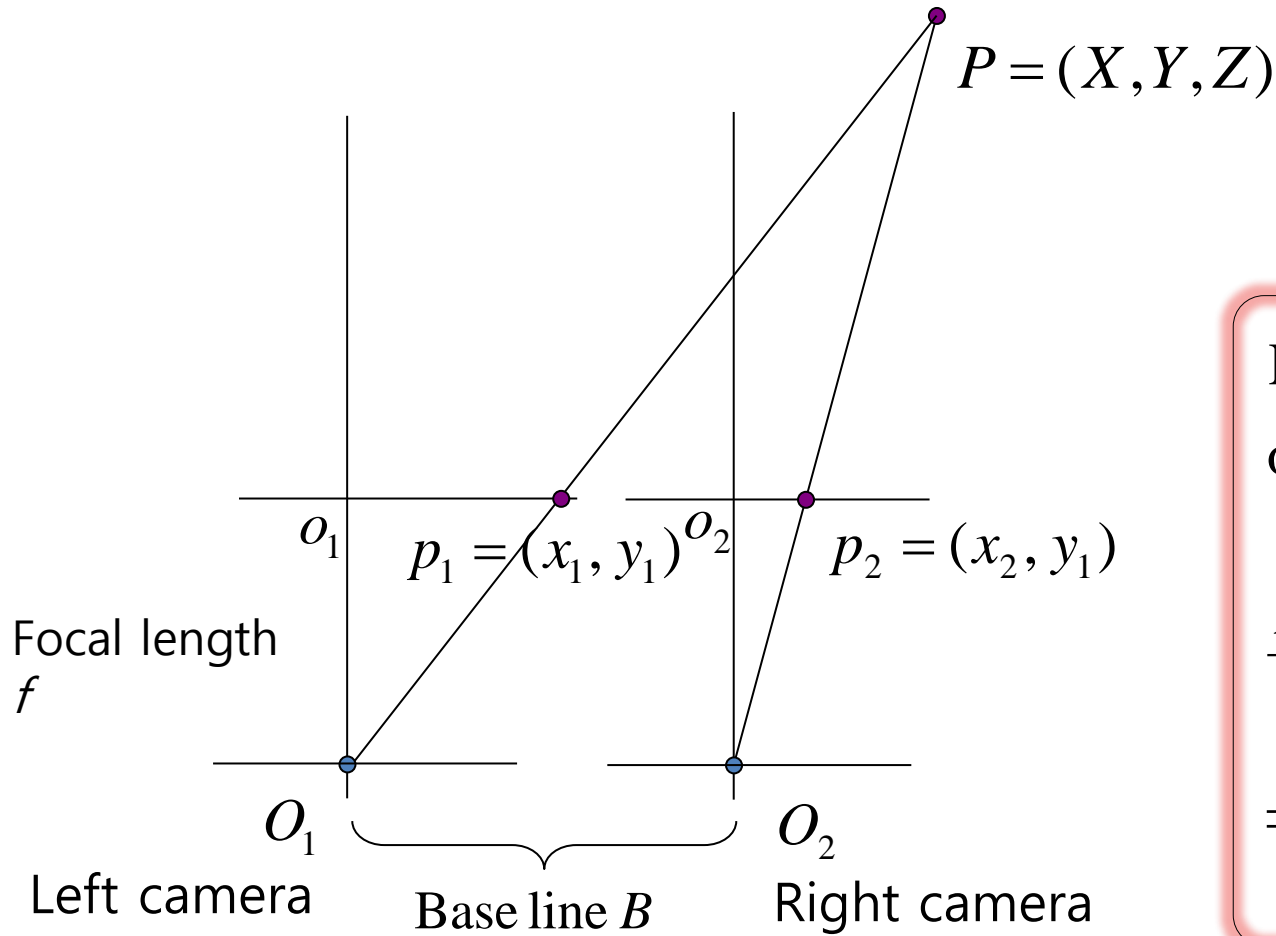
$$\frac{x}{X} = \frac{f}{Z} \Rightarrow x = f \frac{X}{Z}$$

$$\frac{y}{Y} = \frac{f}{Z} \Rightarrow y = f \frac{Y}{Z}$$

Thus

$$(X, Y, Z) \rightarrow (x, y) = \left( f \frac{X}{Z}, f \frac{Y}{Z} \right)$$

# Basic Stereo Model

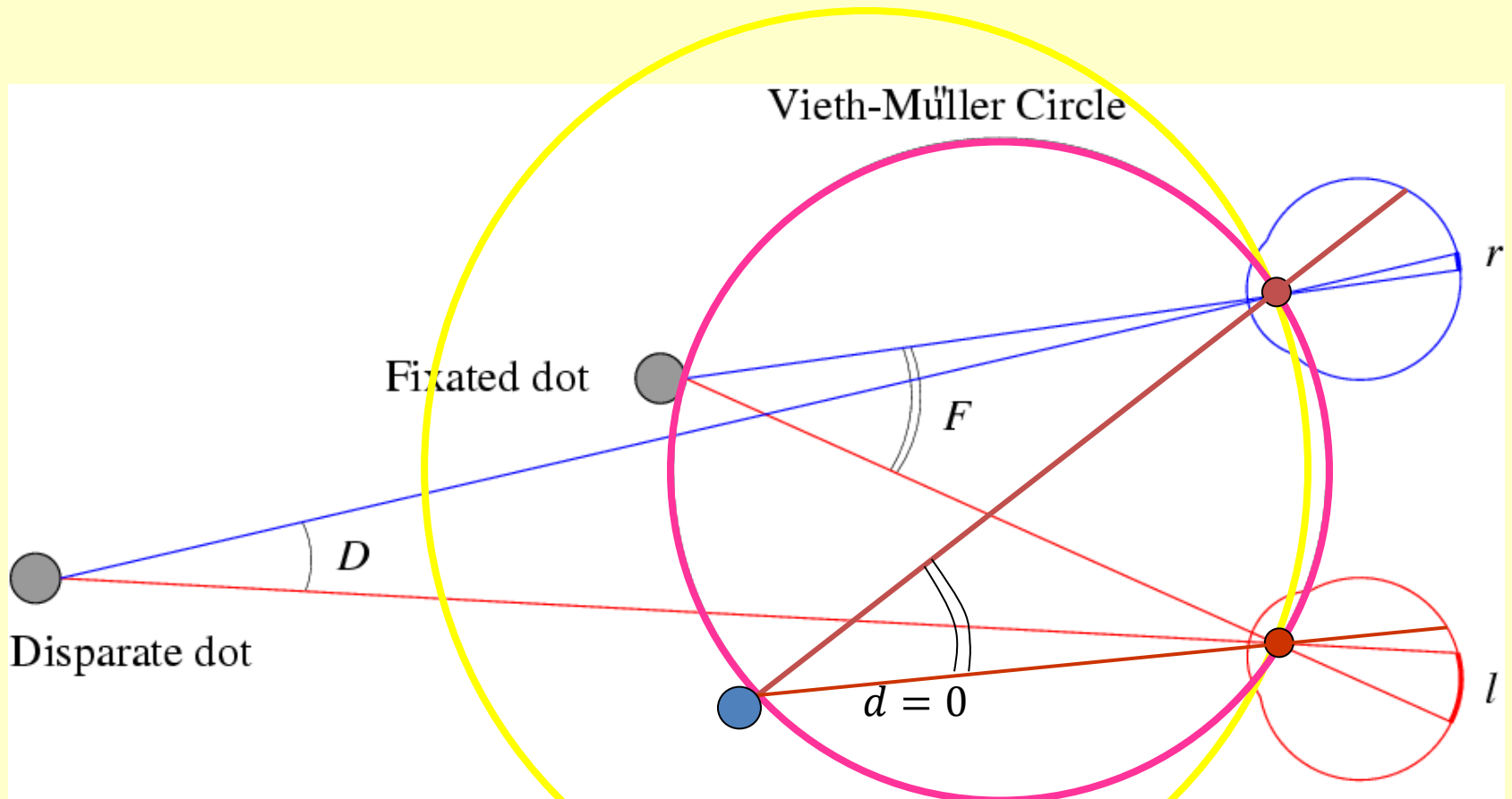


Express  $Z$  as a function  
of  $x_1, x_2, f, B$

$$\frac{X}{Z} = \frac{x_1}{f} \quad \text{and} \quad \frac{X - B}{Z} = \frac{x_2}{f}$$

$$\Rightarrow Z = \frac{fB}{x_1 - x_2} = \frac{fB}{d(p_1)}$$

# Human Stereopsis: Reconstruction



Disparity:  $d = r - l = D - F.$

$d < 0$

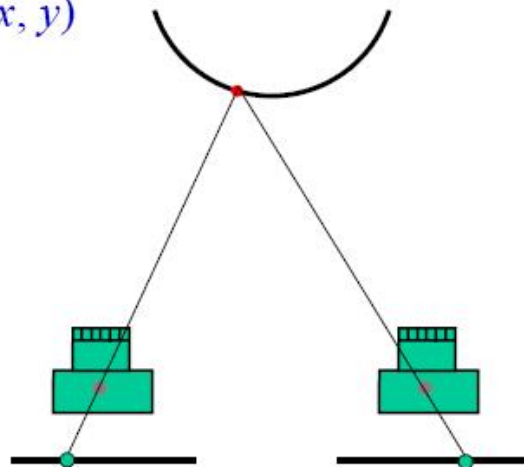
# Finding Correspondence

$Z(x, y)$  is depth at pixel  $(x, y)$   
 $d(x, y)$  is disparity

Estimate:

$$Z(x, y) = \frac{fB}{d(x, y)}$$

Left



Right



Search for best match  
along the same scan line

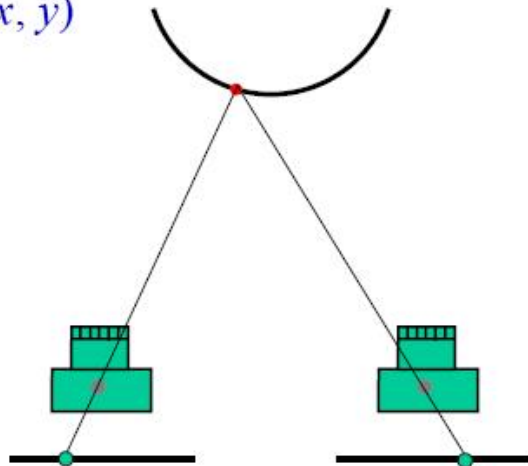
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Left



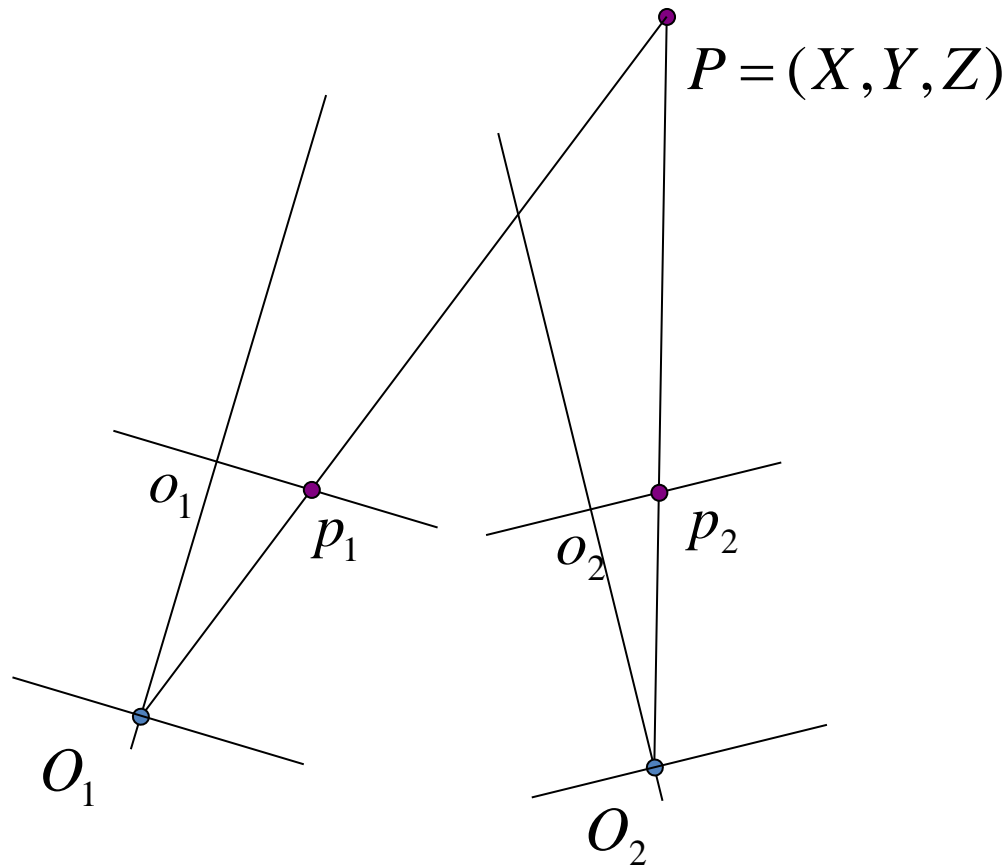
Right



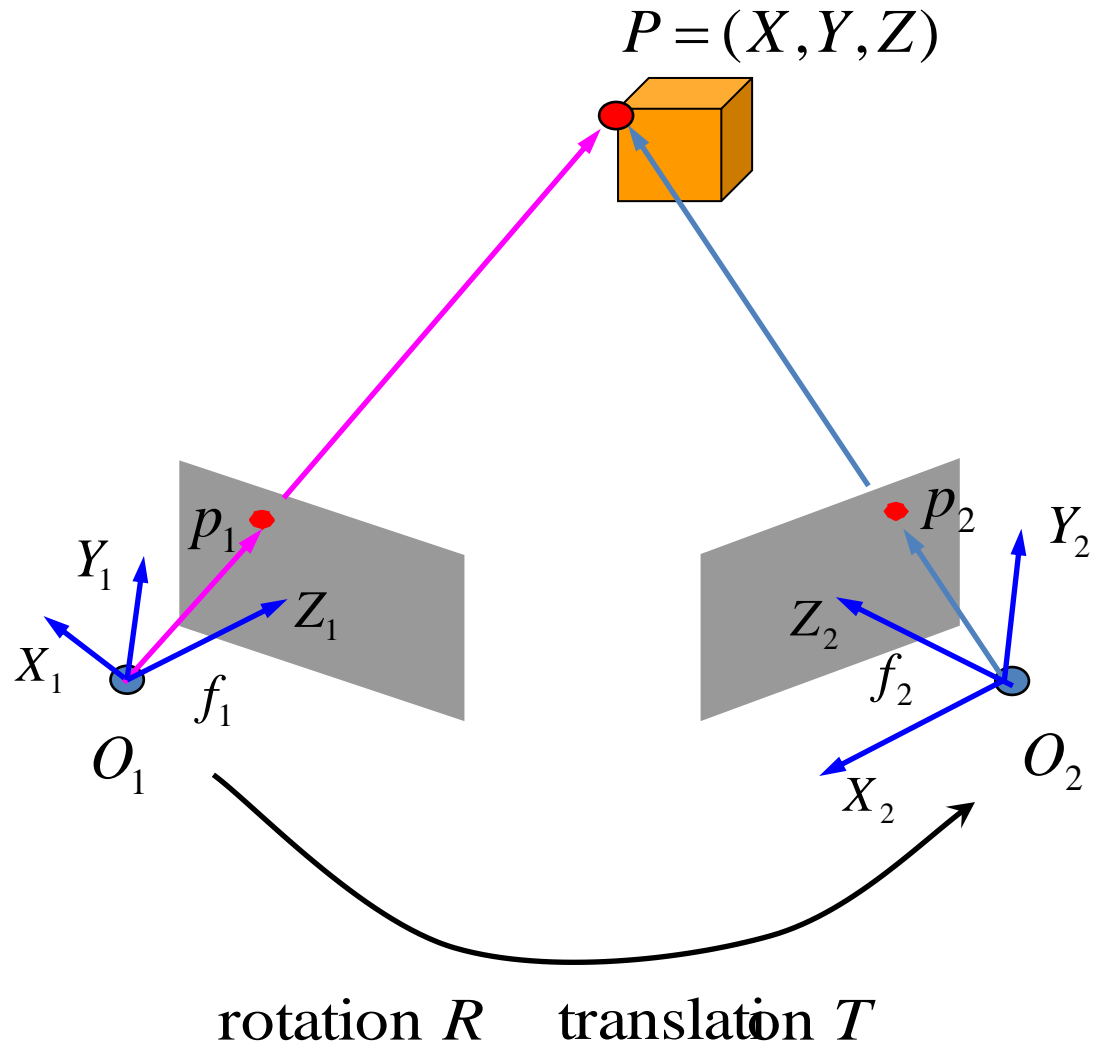
Do I need to consider  
this region?

# General stereo

- What if two cameras are not parallel?

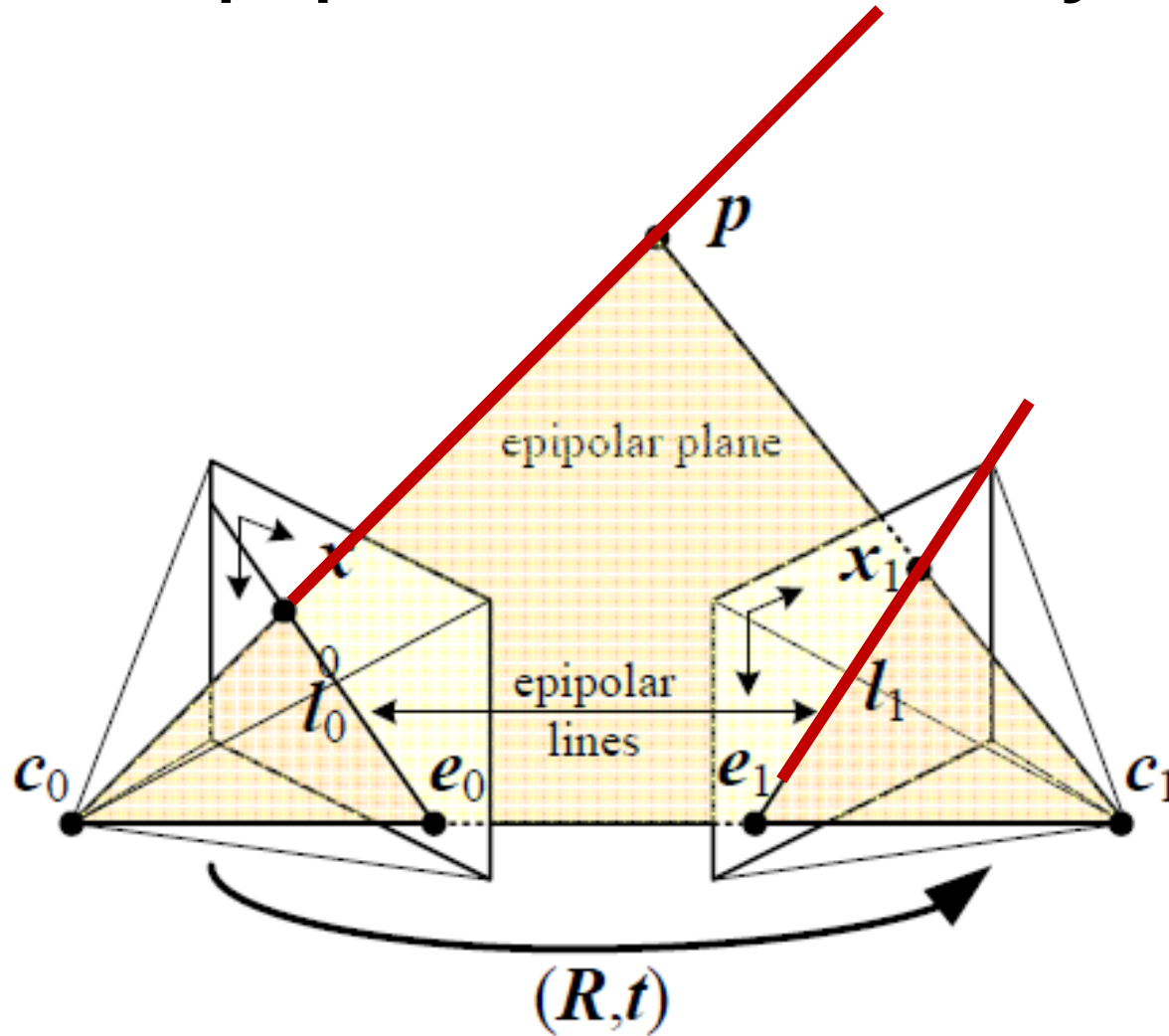


# Epipolar Geometry



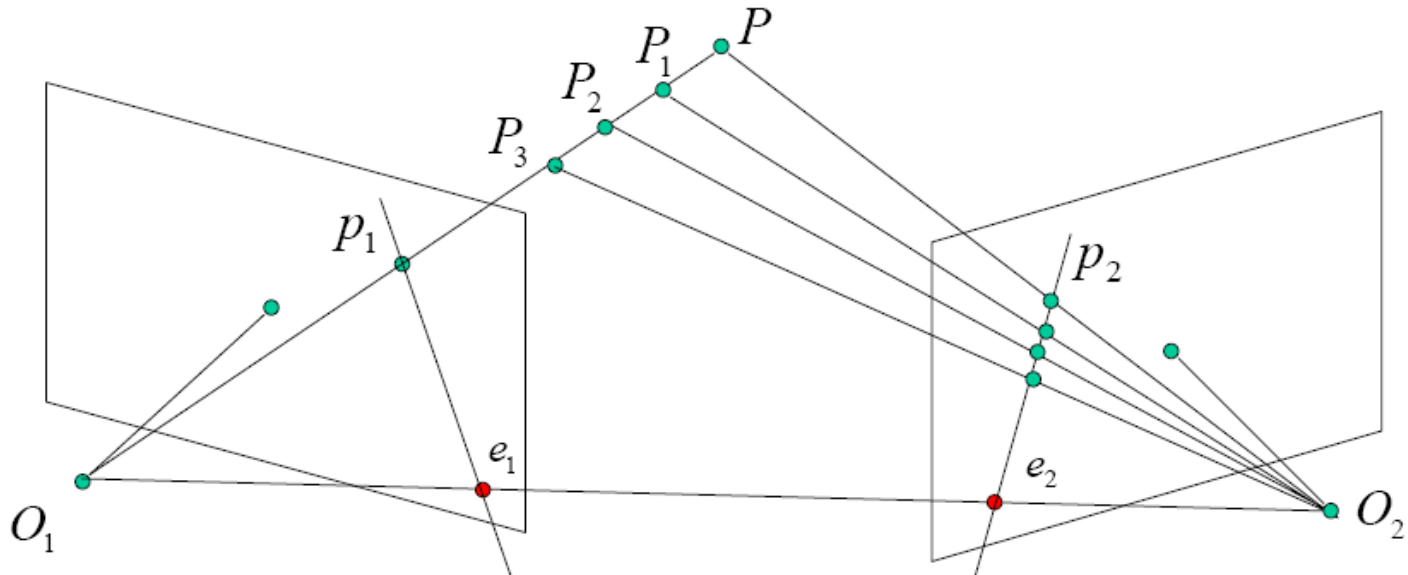


# Epipolar Geometry



# Epipolar Geometry

- Epipolar Constraint
  - A matching points lies on the associated epipolar line
  - It reduces the correspondence problem to 1D search along the epipolar line
  - It reduces the cost and ambiguity of matching

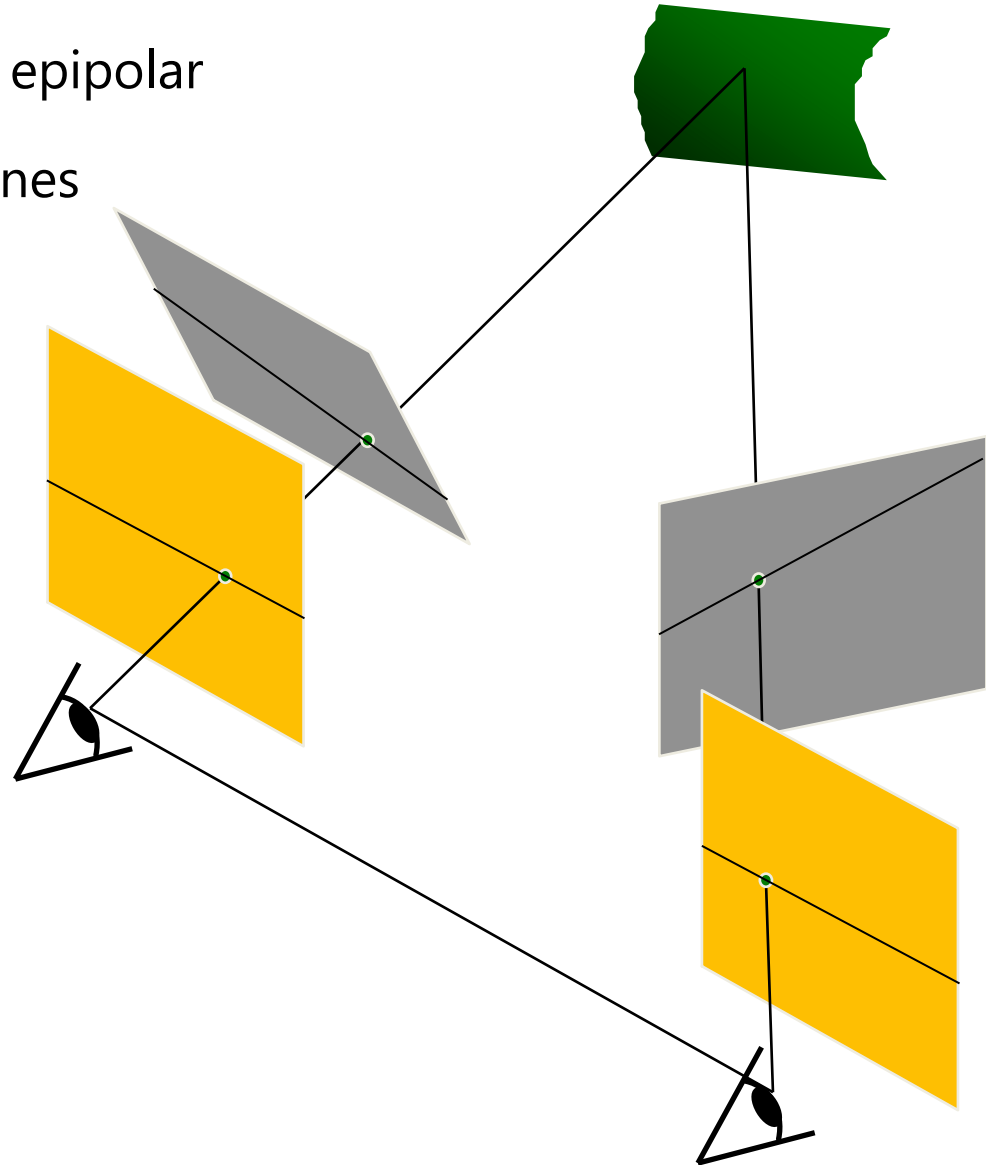


# Rectification

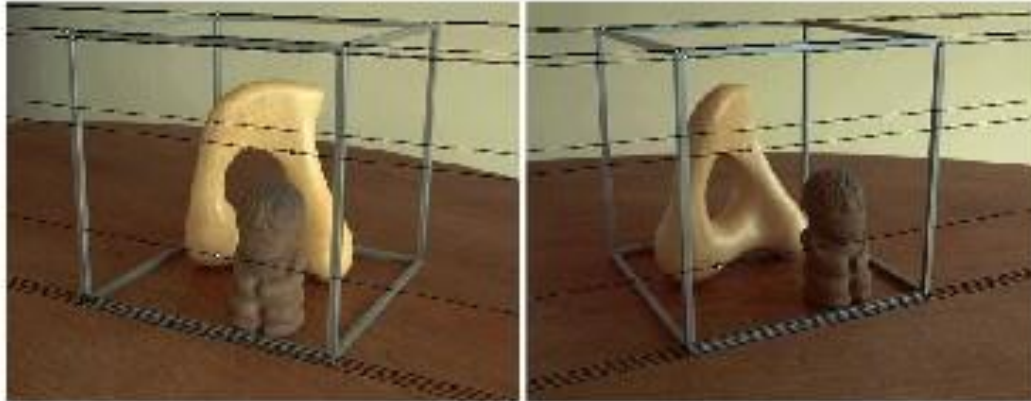
- Simple case
  - Cameras are parallel
  - Focal lengths are the same
  - Two image planes lie on the same plane
- Then, epipolar lines correspond to scan lines
- Rectification is a procedure to convert images so that the assumptions are satisfied
  - It simplifies algorithms
  - It improves efficiency

# Rectification

- Reproject (warp) images so that epipolar lines are aligned with the scan lines



# Rectification



(a) Original image pair overlaid with several epipolar lines.

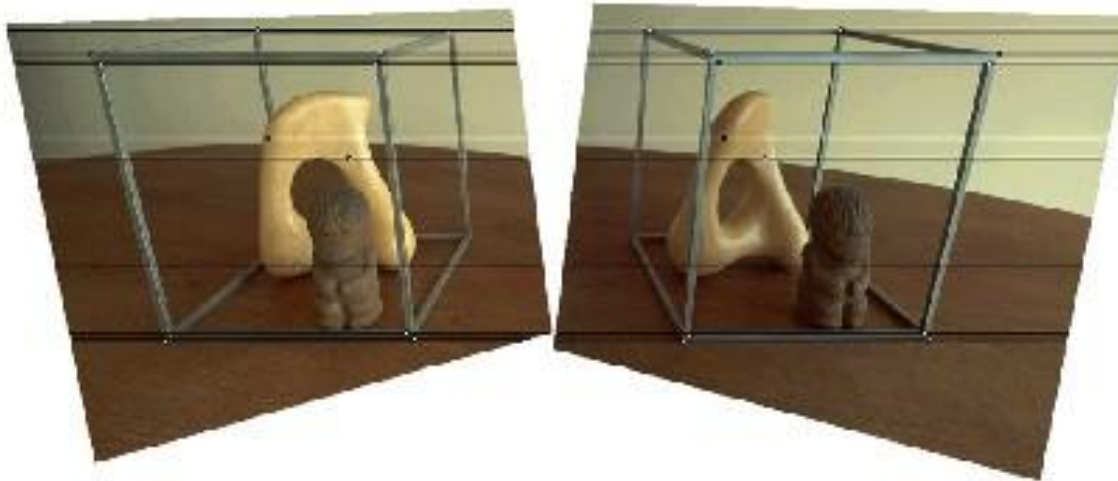


(b) Image pair transformed by the specialized projective mapping  $H_p$  and  $H'_p$ . Note that the epipolar lines are now parallel to each other in each image.

# Rectification



(c) Image pair transformed by the similarity  $H_r$  and  $H'_r$ . Note that the image pair is now rectified (the epipolar lines are horizontally aligned).



(d) Final image rectification after shearing transform  $H_s$  and  $H'_s$ . Note that the image pair remains rectified, but the horizontal distortion is reduced.

# Correspondence: What to Match?

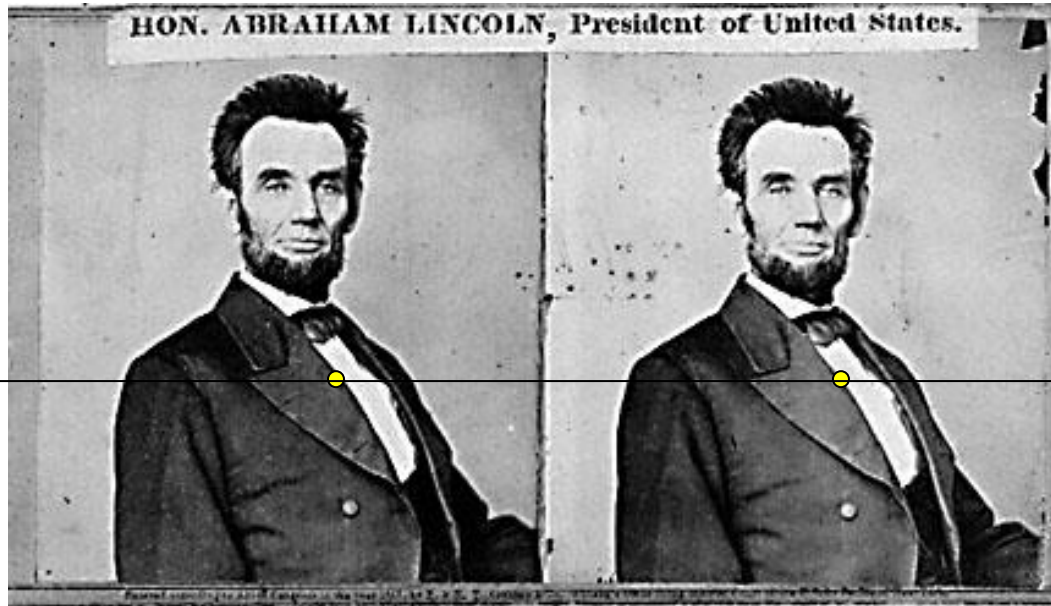
- Objects?
  - More identifiable, but difficult to compute
- Pixels?
  - Easier to handle, but maybe ambiguous
- Edges?
- Collections of pixels (regions)?

# Correspondence: Photometric Constraint

- Assume that the same world point has the same intensity in both images.
  - However, it is not true in general
    - Noise
    - Illumination
    - Camera calibration



# Pixel Matching



What if ?

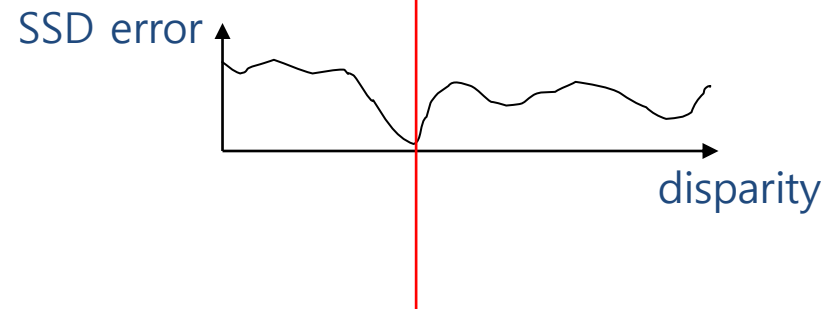
For each scanline , for each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost
- This will never work, so: **match windows**

# Correspondence Using Window Matching

Left

Right

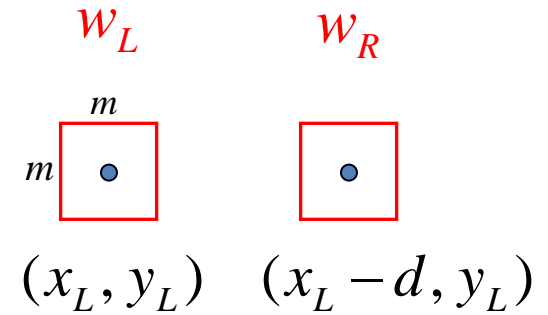
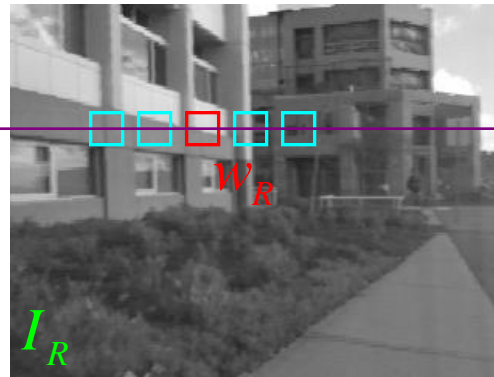


# SSD

Left



Right



- Two blocks  $\mathbf{w}_L$  and  $\mathbf{w}_R$
- $SSD = \|\mathbf{w}_L - \mathbf{w}_R\|^2$

# Normalization

- There can be differences in gain and sensitivity
- Normalize the pixels in each window

$$\tilde{\mathbf{w}} = \frac{\mathbf{w} - \mu \mathbf{1}}{\|\mathbf{w} - \mu \mathbf{1}\|}$$

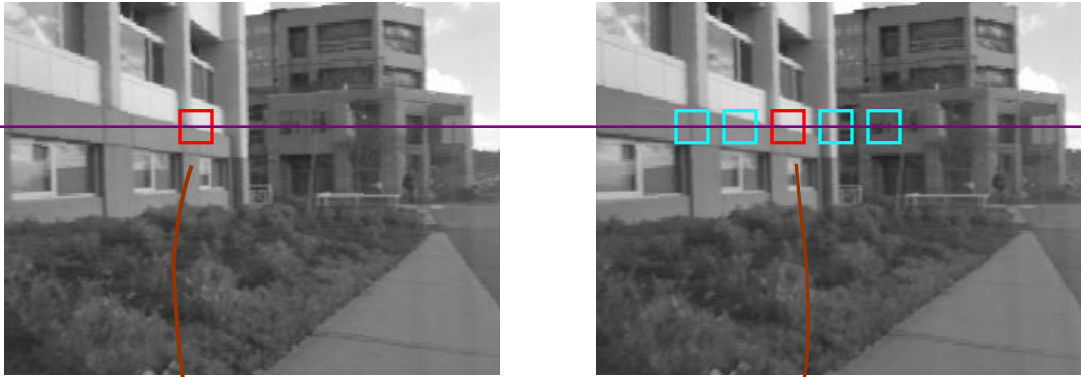
- Minimizing SSD becomes maximizing NCC (normalized cross correlation)

$$\|\tilde{\mathbf{w}}_L - \tilde{\mathbf{w}}_R\|^2 = 2 - 2\tilde{\mathbf{w}}_L \cdot \tilde{\mathbf{w}}_R$$

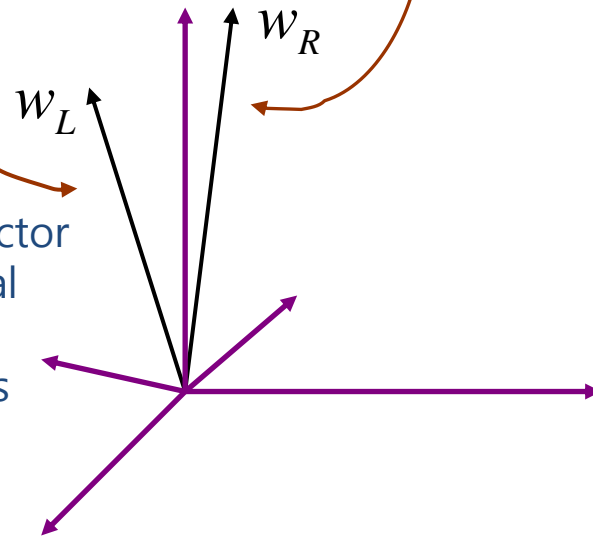
# Normalization

Left

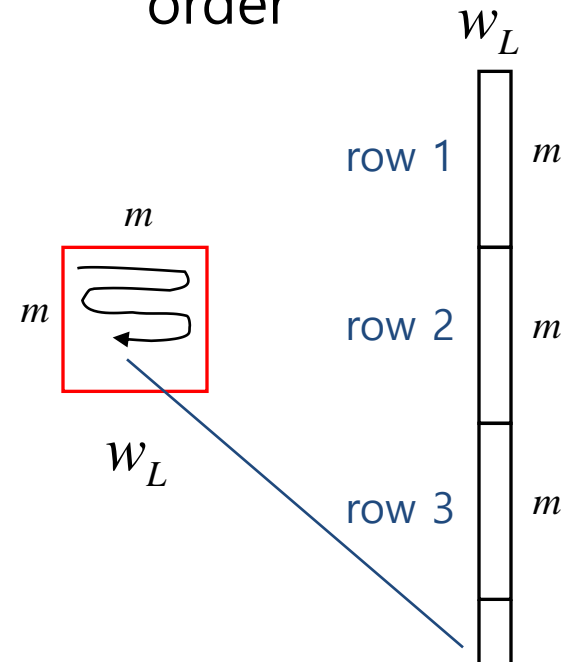
Right



“Unwrap”  
image to form  
vector, using  
raster scan  
order



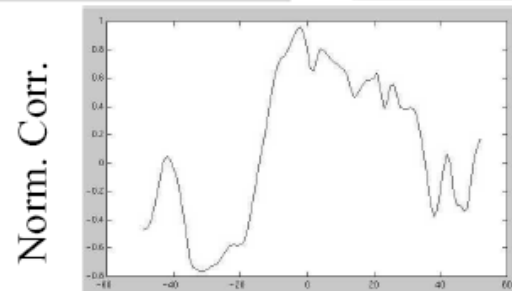
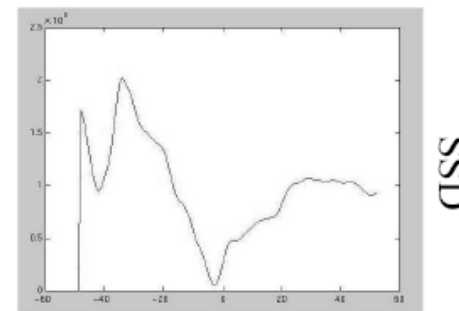
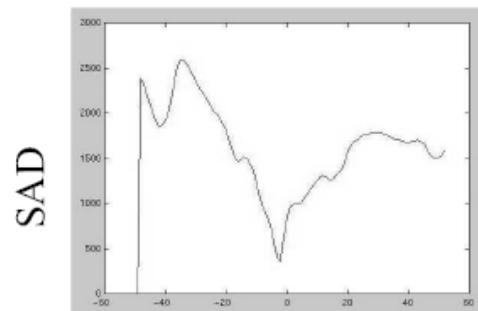
Each window is a vector  
in an  $m^2$  dimensional  
vector space.  
Normalization makes  
them unit length.



# Distance Metrics

Left

Right



# Stereo Results



Images courtesy of Point Grey Research



Disparity Map

# Problems with Window-Based Matching

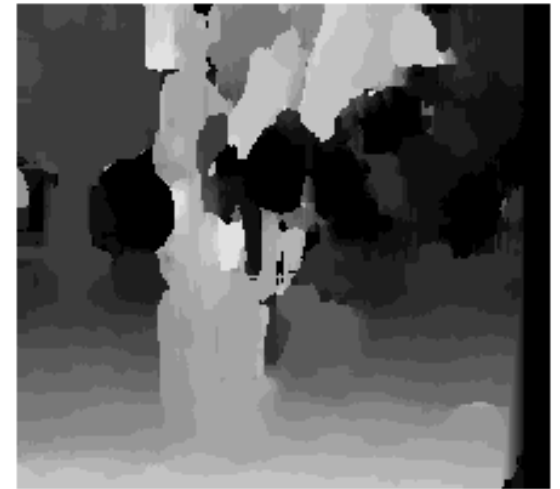
- Disparity within the window may not be constant
- Blur across depth discontinuities
- Poor performance in textureless regions
- Erroneous results in occluded regions



# Window Size



$W = 3$



$W = 20$

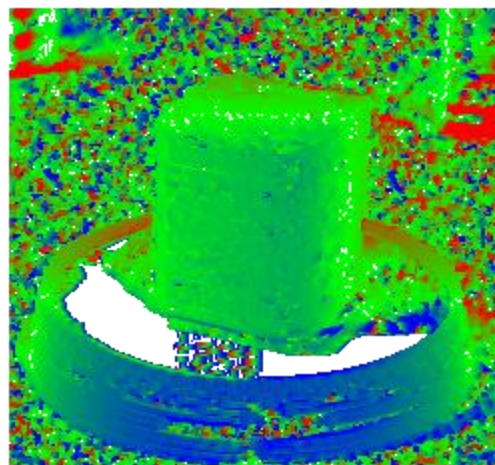
- The results depend on the window size
- Some approaches have been developed to use an adaptive window size (try multiple sizes and select best match)

# Certainty Modeling

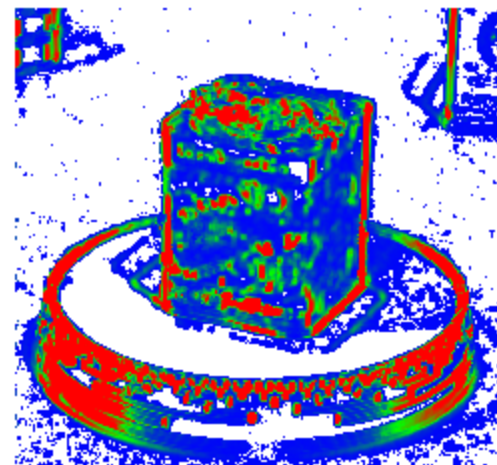
- Compute certainty map from correlations



input



depth map



certainty map

[Szeliski, 1991]

# Hierarchical Stereo Matching

Allows faster computation

Deals with large disparity ranges

Downsampling  
(Gaussian pyramid)



Disparity propagation

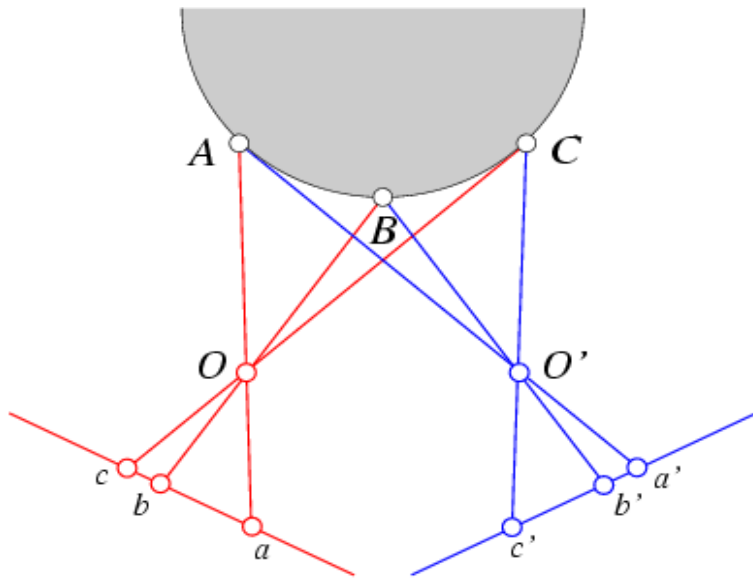


(Falkenhagen '97; Van Meerbergen, Vergauwen, Pollefeys, VanGool IJCV'02)

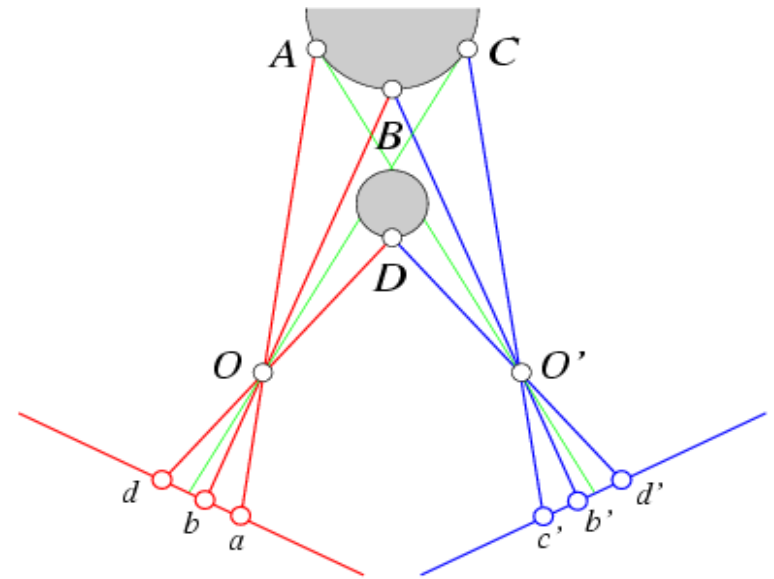
# Stereo Matching Using Dynamic Programming

# Ordering Constraint

- Points on the epipolar lines appear in the same order
- It may not be true in some cases, but can be assumed for most cases
- This is the basic assumption of the stereo matching using dynamic programming

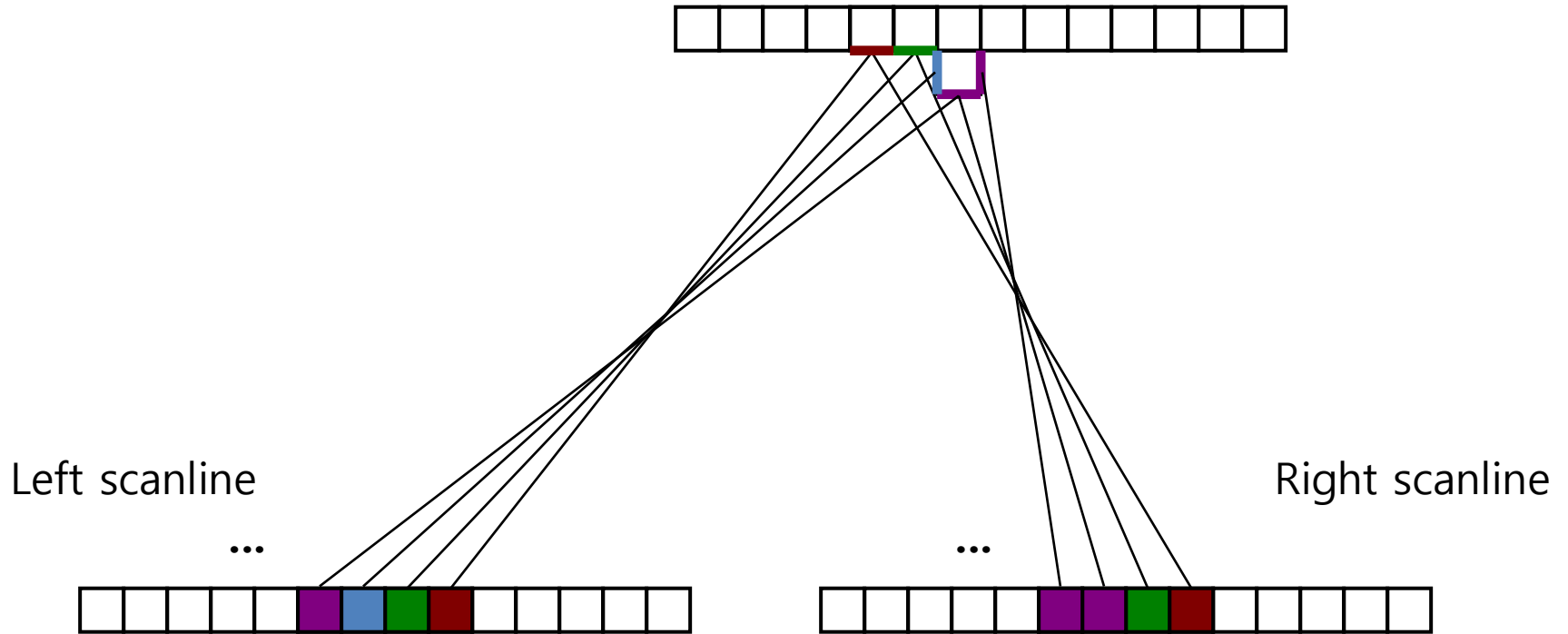


Ordering constraint...

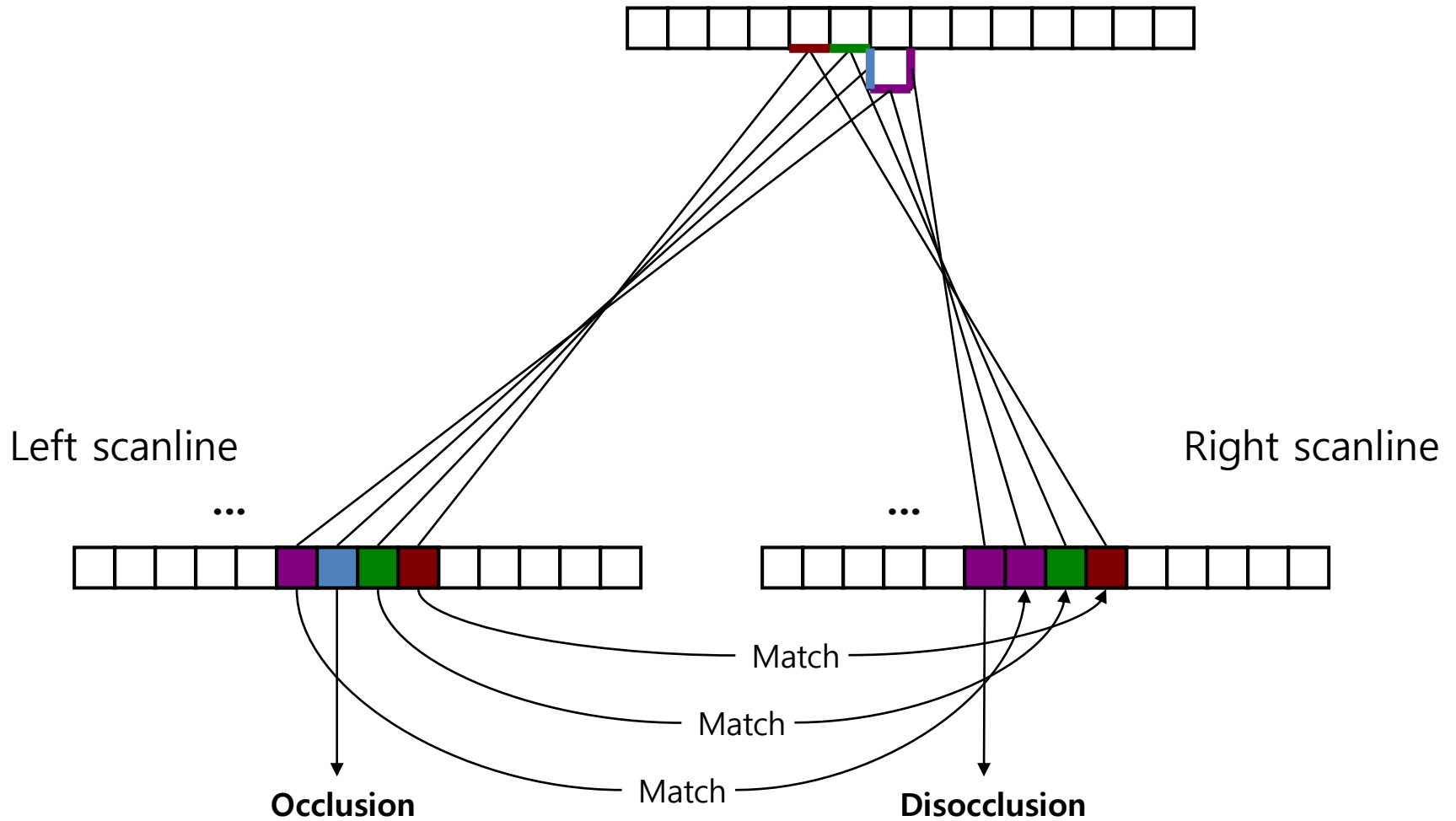


...and its failure

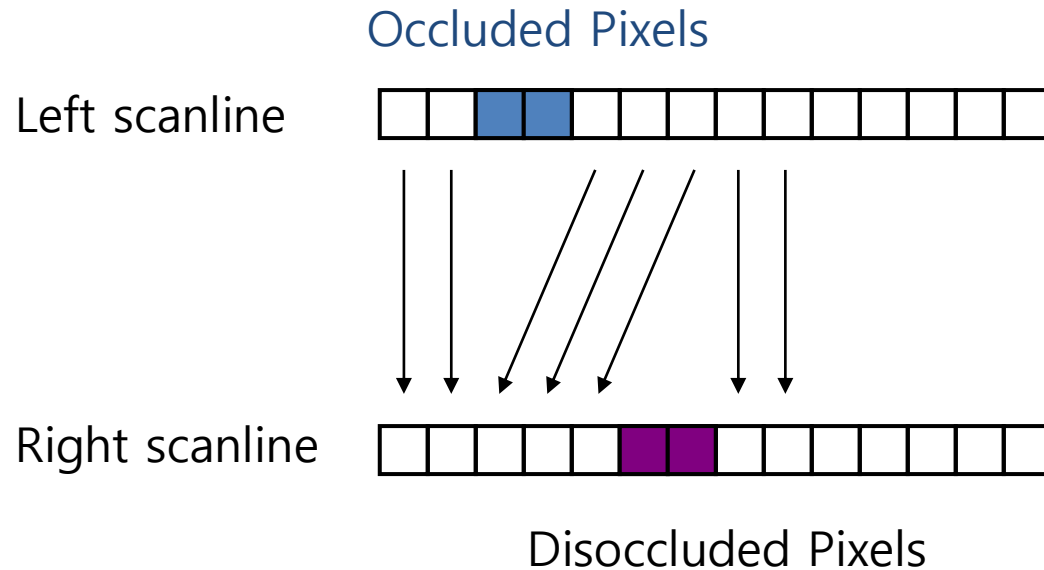
# Occlusion and Disocclusion



# Occlusion and Disocclusion



# Search over Correspondences

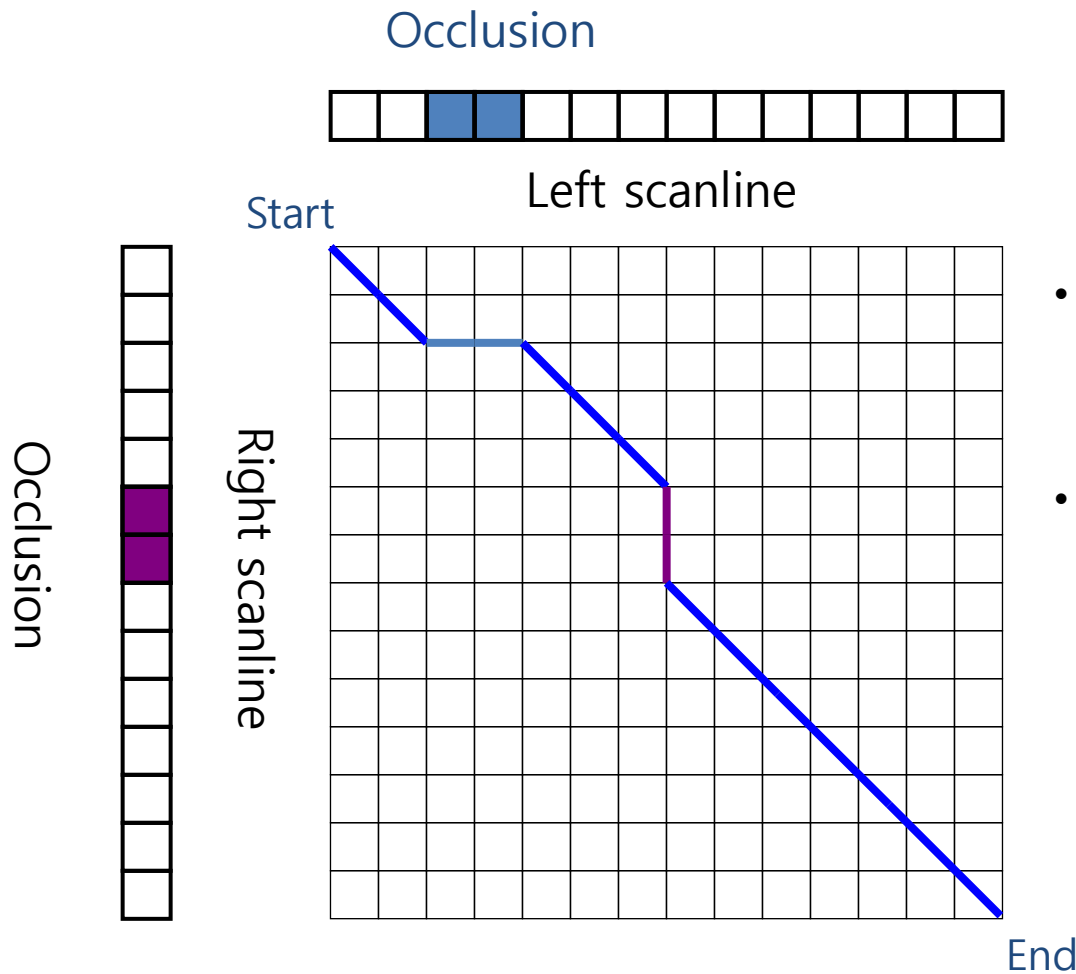


Three cases:

- Sequential – add cost of match (small if intensities agree)
- Occluded – add cost of no match (large cost)
- Disoccluded – add cost of no match (large cost)

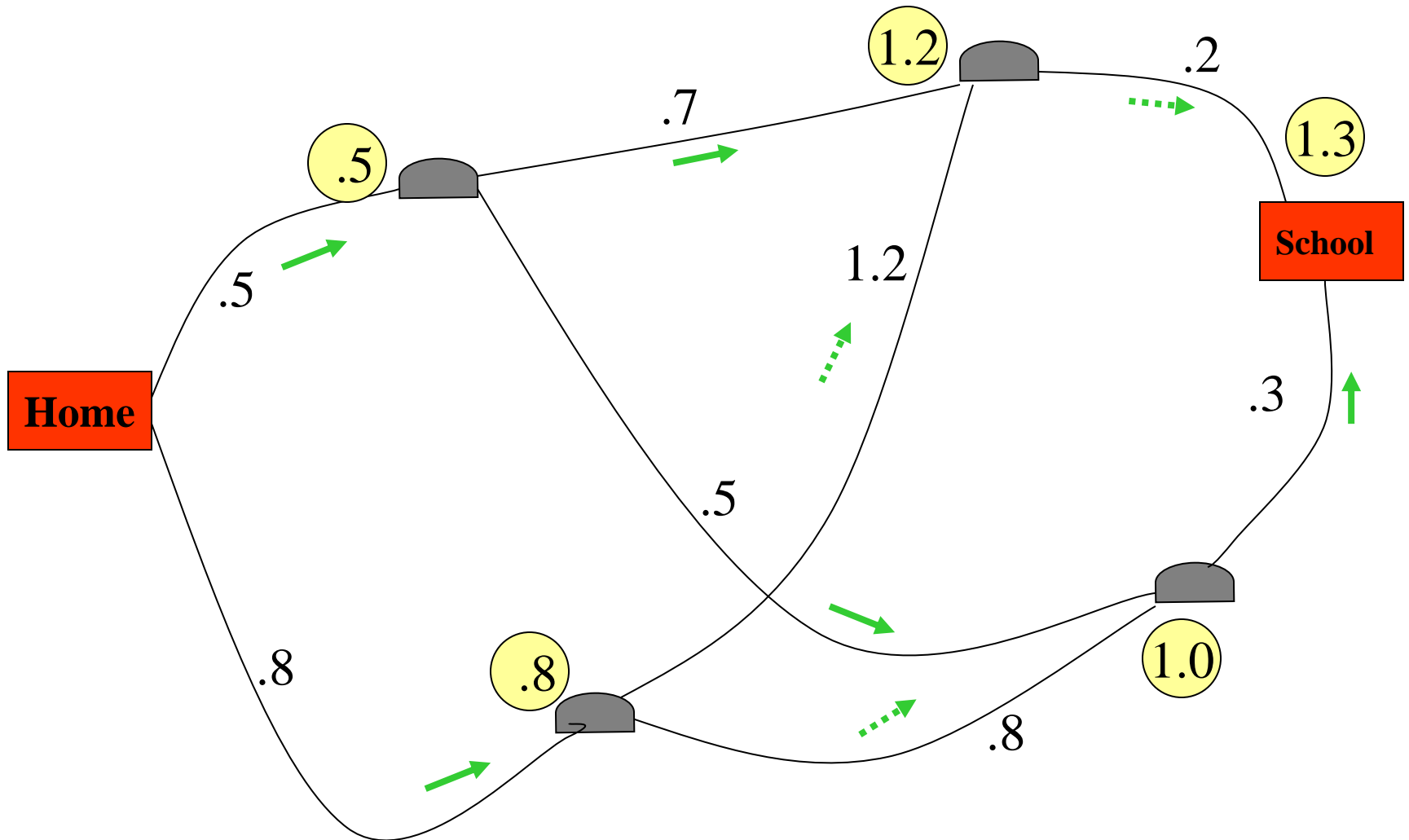


# Dynamic Programming Approach



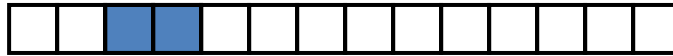
- Dynamic programming yields the optimal path, satisfying the ordering constraint
- Every segment on each scan line will be labeled as either matching or occlusion
  - Diagonal arc: matching
  - Horizontal arc: left occlusion
  - Vertical arc: right occlusion

# Bellman's Optimality Principle



# Dynamic Programming Approach

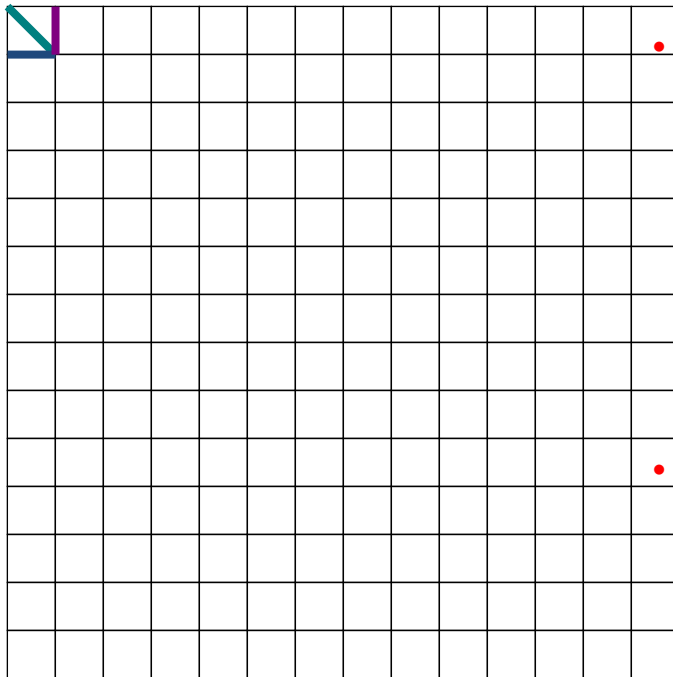
Occlusion



Left scanline



Right scanline



Cost function  $C(i, j)$ : the optimal cost up to node  $(i, j)$ .

$$C(i, j) = \min\{$$

$$C(i - 1, j - 1) + \text{matching cost},$$

$$C(i - 1, j) + \text{left occlusion penalty},$$

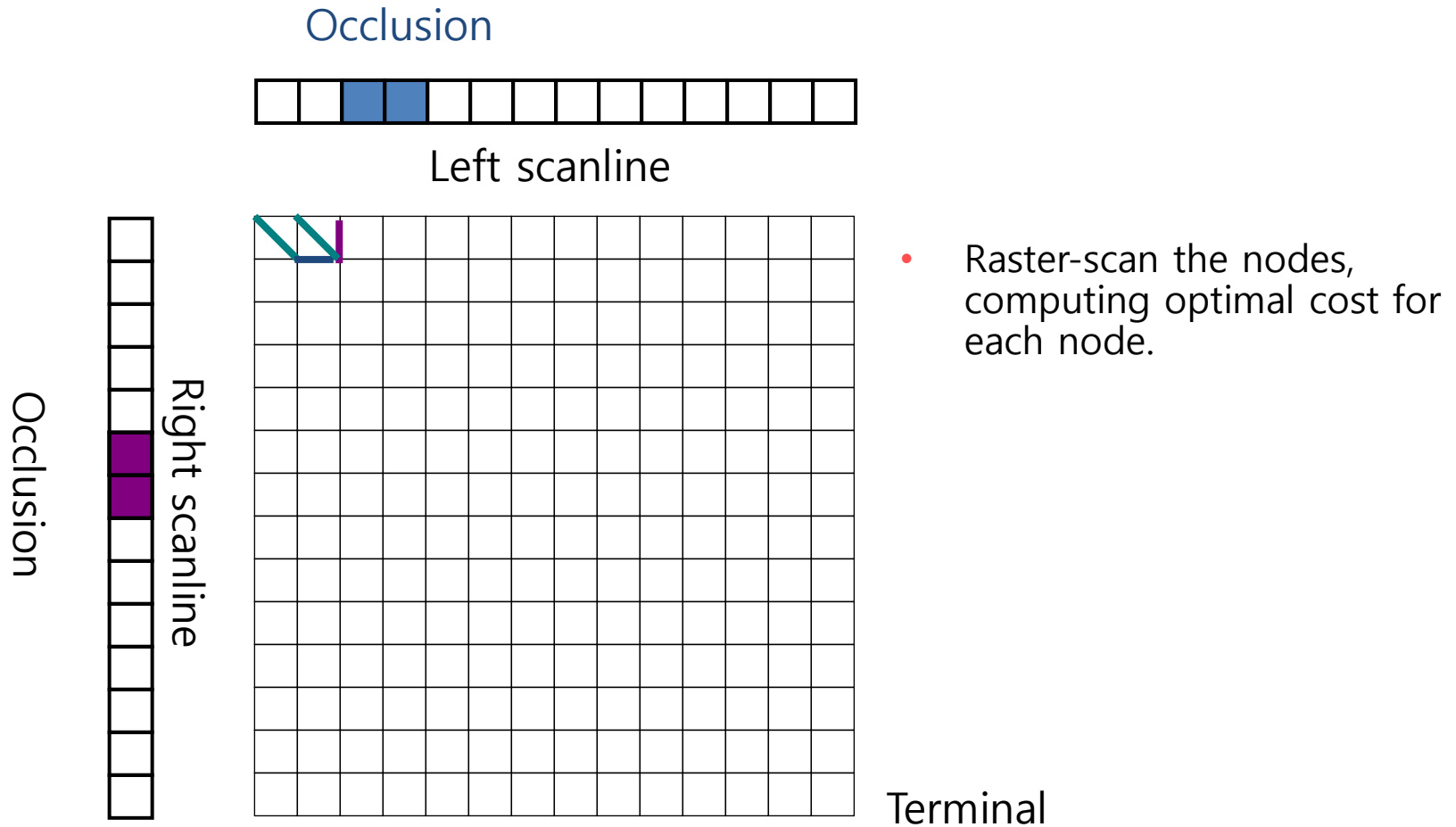
$$C(i, j - 1) + \text{right occlusion penalty}$$

$$\}$$

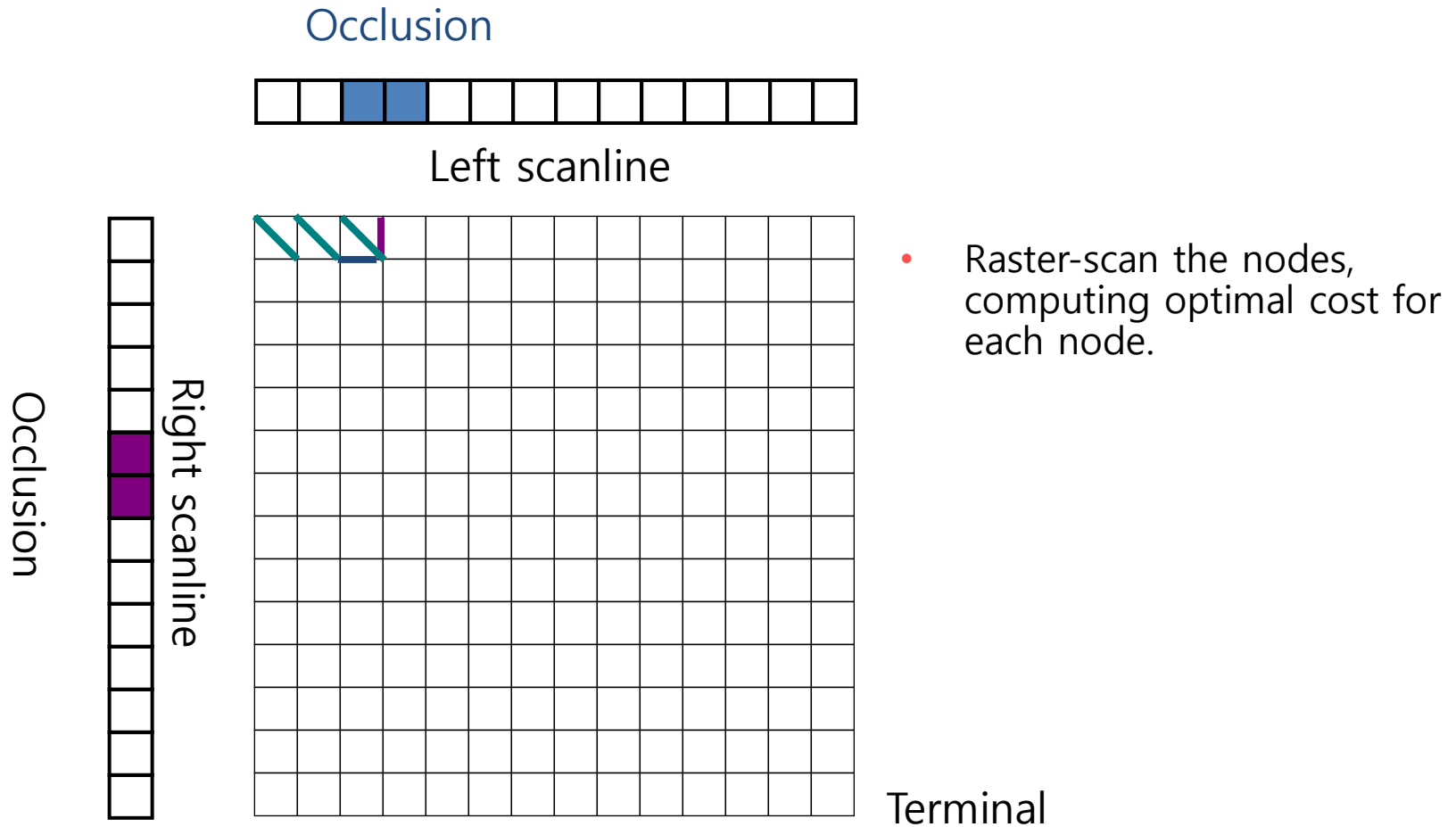
While computing the cost, we record how node  $(i, j)$  is connected to one of the three candidates

Terminal

# Dynamic Programming Approach

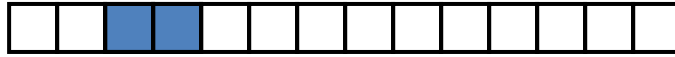


# Dynamic Programming Approach

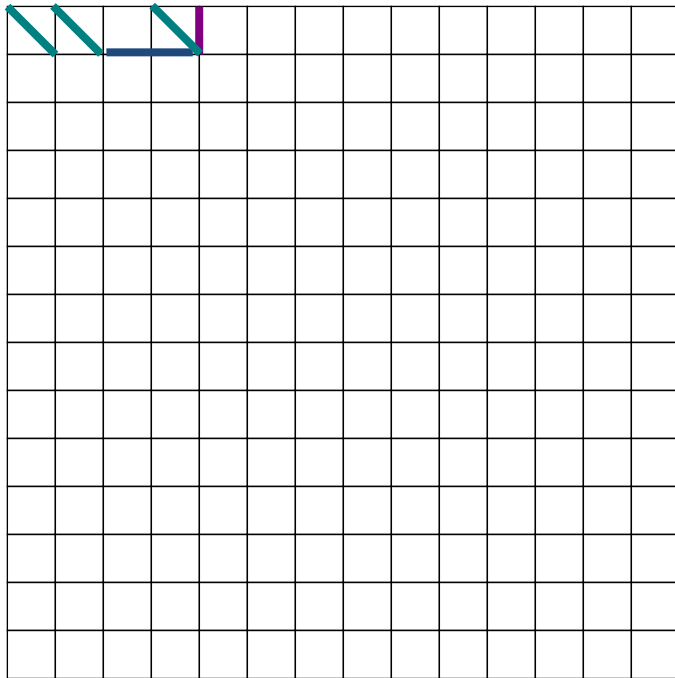


# Dynamic Programming Approach

Occlusion



Left scanline



Occlusion

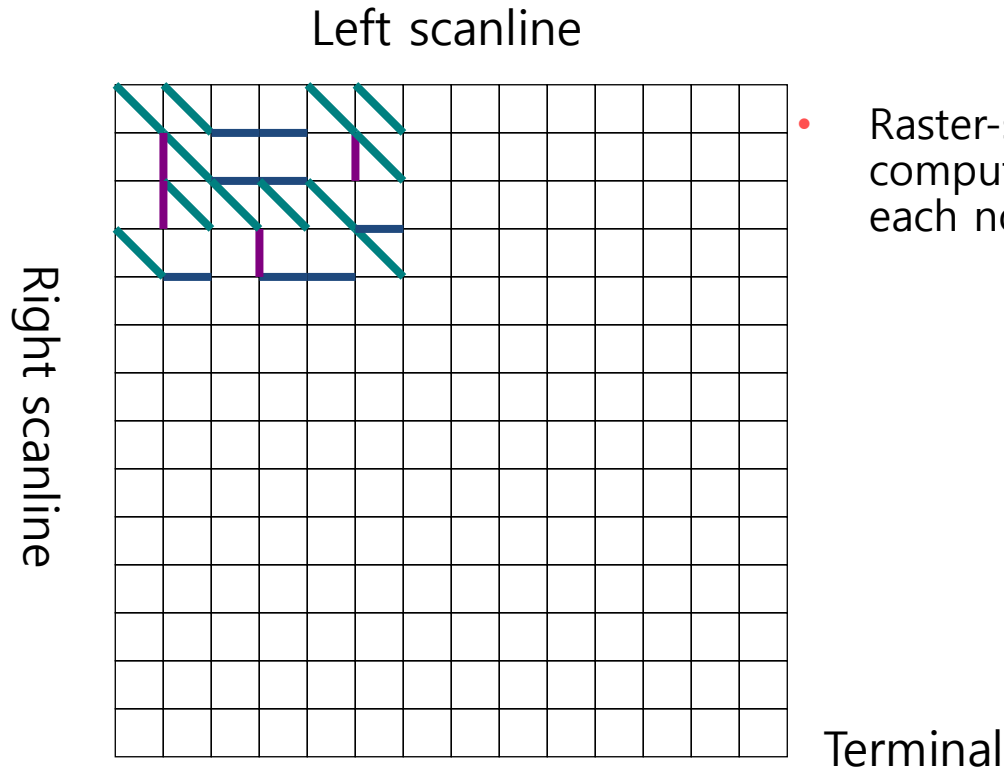


Right scanline

- Raster-scan the nodes, computing optimal cost for each node.

Terminal

# Dynamic Programming Approach



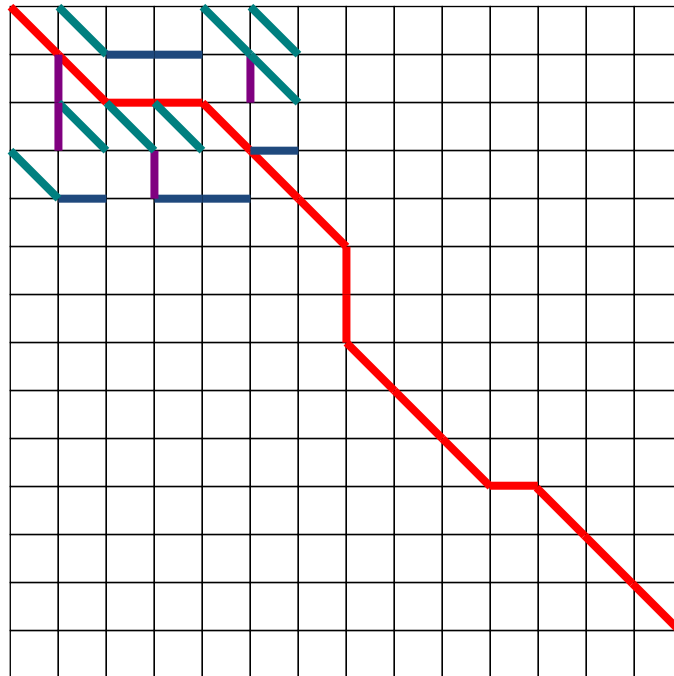
- Raster-scan the nodes, computing optimal cost for each node.

# Dynamic Programming Approach

Occlusion

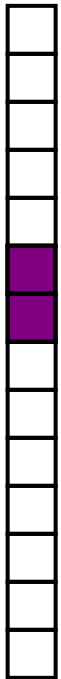


Left scanline



• It's done

Occlusion



Right scanline

Terminal



# Dynamic Programming Approach

- It treats each scan line independently and thus may generate streaking artifacts
- An error can propagate



Streaking artifacts