Multiple Random Walkers and Their Application to Image Cosegmentation

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Contents

Multiple Random Walkers

- Single Random Walker

  $\mathbf{p}^{(t+1)} = \mathbf{A}\mathbf{p}^{(t)} \longrightarrow$ Stationary distribution $\boldsymbol{\pi}$

- Single Random Walker with Restart (RWR)

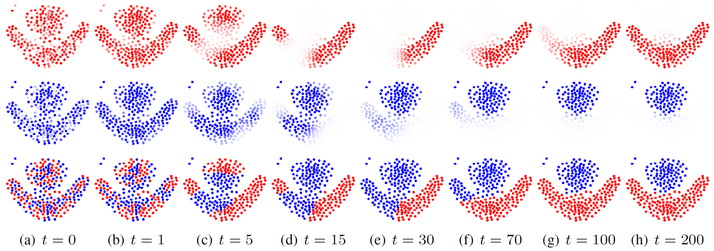
  $\mathbf{p}^{(t+1)} = (1-\epsilon)\mathbf{A}\mathbf{p}^{(t)} + \epsilon\mathbf{r} \longrightarrow \boldsymbol{\pi} = \epsilon\left(\mathbf{I} - (1-\epsilon)\mathbf{A}\right)^{-1}\mathbf{r} = \mathbf{S}\mathbf{r}$

- Multiple Random Walkers (MRW)

  $\mathbf{p}_k^{(t+1)} = (1-\epsilon)\mathbf{A}\mathbf{p}_k^{(t)} + \epsilon\mathbf{r}_k^{(t)} \longrightarrow \boldsymbol{\pi}_k$

  - Interactions by time-varying restart distribution $\mathbf{r}_k^{(t)} \simeq \phi\left(\{\mathbf{p}_k^{(t)}\}_{k=1}^{K}\right)$

MRW with Repulsive Restart Rule



(a) $t=0$ (b) $t=1$ (c) $t=5$ (d) $t=15$ (e) $t=30$ (f) $t=70$ (g) $t=100$ (h) $t=200$

- Designing restart rule $\phi(\cdot)$ for clustering
- Agent k should have large restart at the dominant nodes – posterior

  $\phi_{k,i} \propto p(\omega_k|\mathbf{x}_i)$

- Agent k should have smooth probability transition in the recursion – current probability

  $\phi_{k,i} \propto p(\mathbf{x}_i|\omega_k)$

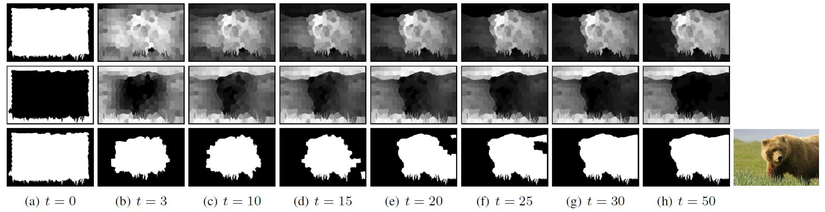
- Repulsive restart rule for MRW clustering

  $\phi_{k,i} = \alpha \cdot p(\omega_k|\mathbf{x}_i) \cdot p(\mathbf{x}_i|\omega_k) \qquad \boldsymbol{\phi}_k = \alpha\mathbf{Q}_k\mathbf{p}_k$

- Decision by the MAP rule

  $l_i = \arg\max_k p\left(\omega_k|\mathbf{x}_i\right)$

Image Examples



(a) $t=0$ (b) $t=3$ (c) $t=10$ (d) $t=15$ (e) $t=20$ (f) $t=25$ (g) $t=30$ (h) $t=50$

- Image to graph
  - Super-pixel nodes
  - Boundary connected and extended edges
- Feature distances for the edge weight
  - Average RGB and LAB
  - Boundary cue
  - Bag-of-visual-word: RGB and LAB
- Double random walkers

  $\mathbf{p}_{\mathrm{f}}^{(t+1)} = (1-\epsilon)\mathbf{A}\mathbf{p}_{\mathrm{f}}^{(t)} + \epsilon\mathbf{r}_{\mathrm{f}}^{(t)}$

  $\mathbf{p}_{\mathrm{b}}^{(t+1)} = (1-\epsilon)\mathbf{A}\mathbf{p}_{\mathrm{b}}^{(t)} + \epsilon\mathbf{r}_{\mathrm{b}}^{(t)}$ $\longrightarrow \boldsymbol{\pi}_{\mathrm{f}}, \boldsymbol{\pi}_{\mathrm{b}}$

(a) Input (b) Ncut (c) SKM (d) RWR (e) MRW

- Different characteristics

Ncut	SKM	RWR	MRW
No initial prior		Fixed	Interactive
Divide images along the strongest edges	Dependent on initials	Meaningful foregrounds	

Image Cosegmentation

- Inter-image transfer matrix $\mathbf{A}_{uv}$
  - SIFT, LAB, and texton
- Foreground estimation by RWR

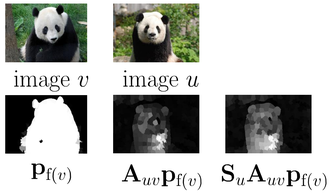
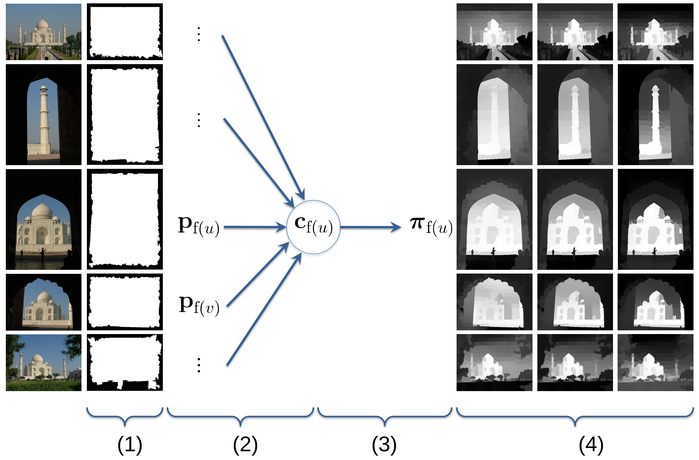


image v image u

$\mathbf{p}_{\mathrm{f}(v)}$ $\mathbf{A}_{uv}\mathbf{p}_{\mathrm{f}(v)}$ $\mathbf{S}_u\mathbf{A}_{uv}\mathbf{p}_{\mathrm{f}(v)}$

- Multi-Pass Refinement



1) Initials with the center priors
2) Inter-image concurrence computation
   - Similarity of each node in image u to foreground objects in the other images

   $\mathbf{c}_{\mathrm{f}(u)} = \frac{1}{Z}\mathbf{S}_u \sum_v \mathbf{A}_{uv}\mathbf{p}_{\mathrm{f}(v)}$
3) Intra-image MRW clustering

   $\boldsymbol{\phi}_{\mathrm{f}(u)}\left(\{\mathbf{p}_{\mathrm{f}(u)}, \mathbf{p}_{\mathrm{b}(u)}\}\right) = \gamma\,\alpha\,\mathbf{Q}_{\mathrm{f}(u)}\mathbf{p}_{\mathrm{f}(u)} + (1-\gamma)\mathbf{c}_{\mathrm{f}(u)}$

   Hybrid restart rule — Repulsive (Interactive) — Concurrence (Fixed)
4) Repeat until the overall foreground distance stops decreasing

   $\sum_{u,v} d_{\mathrm{f}}(C_u, C_v)$